A Mathematical Approach for Three Stage Flow Shop Production Schedule with Jobs in a String of Disjoint Job Block

Deepak Gupta
Department of Mathematics
M. M. University. Mullana
Ambala, India

Sameer Sharma
Department of Mathematics
D. A. V. College, Jalandhar
Punjab, India

Shefali Aggarwal
Department of Mathematics
M. M. University. Mullana
Ambala, India

ABSTRACT
The aim of this paper is to introduce the concept of disjoint job blocks in n-jobs, three machines flow shop scheduling problem to minimize the total elapsed time and rental cost of the machines under a specified rental policy in which the processing time associated with probabilities including transportation time. A heuristic approach for flow shop with a computational algorithm to find optimal or near optimal solution is described. A computer program followed by a numerical illustration is given to justify the proposed algorithm.

Keywords
Flow Shop Scheduling; Disjoint job Block; Transportation Time; Processing TIme.

1. INTRODUCTION

In most real-world environments, scheduling is an ongoing reactive process where the presence of real-time information continually forces reconsideration and revision of pre-established schedules. Scheduling literature addressing three machine flow shop with multiple criteria mainly focused on two criteria, also known as bicriteria. The bicriteria scheduling problems are motivated by the fact that they are more meaningful from practical point of view. Most machine scheduling models assume that jobs are delivered instantaneously from one location to another without transportation time involved. In this paper we relax this assumption, since there are practical scheduling situations when certain times are required by jobs for their transplantation from one machine to another machine. One of the earliest results in flow shop scheduling theory is an algorithm given by Johnson [7] for scheduling jobs in a two machine flow shop to minimize the time at which all jobs are completed. Smith [12] considered minimization of mean flow time and maximum tardiness. Van Wassenhove and Gelders [16] studied minimization of maximum tardiness and mean flow time explicitly as objective. Some of the noteworthy heuristic approaches are due to Maggu & Das [9], Sen and Gupta [14], Singh[15], Dileepan [3], Narain [10], Heydari [1], Chandramouli [2], Khodadadi [8], Pandian & Rajendran [11] by considering various parameters. Heydari [1] dealt with a flow shop scheduling problem where the jobs are processed in two disjoint job blocks in a string consists of one block in which order of jobs is fixed & other block in which order of job is arbitrary.

Gupta & Sharma [5] studied bicriteria in n × 3 flow shop scheduling under specified rental policy, processing time associated with probabilities including transportation time and job block criteria. We have made an attempt to extend the work by introducing the jobs in a string of disjoint job blocks. The paper considers a more practical scheduling situation in which certain ordering of jobs is prescribed either by technological constraints or by externally imposed policy.

2. PRACTICAL SITUATION
Various practical situations occur in real life when the machines on which jobs are to be processed are planted at different places, the transportation time (which includes loading time, moving time and unloading time etc.) has a significant role in production concern. Further the priority of one job over other may be significant due to the relative importance of the jobs. It may be because of urgency or demand of its relative importance, the job block criteria becomes important. The concept of machines on rent is one of the latest. Renting enables saving working capital, gives option for having the equipment, and allows upgradation to new technology.

When one has got the assignments but does not have one's own machine or does not have enough money or does not want to take risk of investing huge amount of money to purchase machine. Under such circumstances, the machine has to be taken on rent in order to complete the assignments. In his starting career, we find a medical practitioner does not buy expensive machines say X-ray machine, Ultra Sound Machine, Rotating Triple Head Single Positron Emission Computed Tomography Scanner, Patient Monitoring Equipment, and Laboratory Equipment etc., but instead takes on rent. Rental of medical equipment is an affordable and quick solution for hospitals, nursing homes, physicians, which are presently constrained by the availability of limited funds due to the recent global economic recession.

3. NOTATIONS
S : Sequence of jobs 1,2,3,….,n
M_j : Machine j, j= 1,2,3
M : Minimum makespan
a_{ij} : Processing time of i^{th} job on machine M_j
p_{ij} : Probability associated to the processing time a_{ij}
A_{ij} : Expected processing time of i^{th} job on machine M_j
α : Equivalent job for job – block
C_i : Rental cost of i^{th} machine

$L_j(S)$: The latest time when machine M_j is taken on rent for sequence S

$t_{ij}(S)$: Completion time of i^{th} job of sequence S on machine M_j

$t'_{ij}(S)$: Completion time of i^{th} job of sequence S on machine M_j when machine M_j start processing jobs at time $L_j(S)$

$T_{i,j\to k}$: Transportation time of i^{th} job from j^{th} machine to k^{th} machine

$I_{ij}(S)$: Idle time of machine M_j for job i in the sequence S

$U_j(S)$:Utilization time for which machine M_j is required, when M_j starts processing jobs at time $L_j(S)$

$R(S)$: Total rental cost for the sequence S of all machine

3.1 Definition

Completion time of i^{th} job on machine M_j is denoted by t_{ij} and is defined as

$$t_{ij} = max\ (t_{i-1,j}\ ,\ t_{i,j-1})\ +T_{i,(j-1)\to j} + a_{ij} \times p_{ij} \quad \text{for} \quad j \geq 2.$$

$$= max\ (t_{i-1,j}\ ,\ t_{i,j-1})\ +T_{i,(j-1)\to j} + A_{i,j}$$

where $A_{i,j}$= expected processing time of i^{th} job on machine j.

3.2 Definition

Completion time of i^{th} job on machine M_j when M_j starts processing jobs at time L_j is denoted by $t'_{i,j}$ and is defined as

$$t'_{i,j} = L_j + \sum_{k=1}^{i} A_{k,j} = \sum_{k=1}^{i} I_{k,j} + \sum_{k=1}^{i} A_{k,j}$$

$$= \max(t_{i,j-1}, t'_{i-1,j}) + A_{i,j}$$

4. RENTAL POLICY

The machines will be taken on rent as and when they are required and are returned as and when they are no longer required i.e. the first machine will be taken on rent in the starting of the processing the jobs, 2nd machine will be taken on rent at time when 1st job is completed on 1st machine and transported to 2nd machine, 3rd machine will be taken on rent at time when 1st job is completed on the 2nd machine and transported.

5. THEOREM

The processing of jobs on M_3 at time $L_3 = \sum_{i=1}^{n} I_{i,3}$ keeps $t_{n,3}$ unaltered.

Proof: Let $t'_{n,3}$ be the competition time of i^{th} job on machine M_3 when M_3 starts processing of jobs at time L_3. We shall prove the theorem with the help of Mathematical Induction.

Let $P(n): t'_{n,3} = t_{n,3}$

Basic Step: For n = 1

$$t'_{1,3} = L_3 + A_{1,3} = I_{1,3} + A_{1,3}$$

$$= (A_{1,1}+(T_{1,1\to 2} + A_{1,2}) + T_{1,2\to 3}) + A_{1,3} = t_{1,3}.$$

Therefore P(1) is true.

Induction Step: Let P (k) be true.

i.e. $t'_{k,3} = t_{k,3}$.

Now, we shall show that P(k+1) is also true.

i.e. $t'_{k+1,3} = t_{k+1,3}$

But $t'_{k+1,3} = \max(t_{k+1,2}, t'_{k,3}) + T_{k+1,2\to 3} + A_{k+1,3}$ (As per Definition 2)

$$\therefore t'_{k+1,3} = \max(t_{k+1,2}, L_3 + \sum_{i=1}^{k} A_{i,3}) + T_{k+1,2\to 3} + A_{k+1,3}$$

$$= \max(t_{k+1,2}, \sum_{i=1}^{k+1} I_{1,3} + \sum_{i=1}^{k} A_{i,3}) + T_{k+1,2\to 3} + A_{k+1,3}$$

$$= \max(t_{k+1,2}, \sum_{i=1}^{k} I_{1,3} + \sum_{i=1}^{k} A_{i,3} + I_{k+1,3}) + T_{k+1,2\to 3} + A_{k+1,3}$$

$$= \max(t_{k+1,2}, t_{k,3} + I_{k+1,3}) + T_{k+1,2\to 3} + A_{k+1,3}$$

$$= \max(t_{k+1,2}, t'_{k,3} + I_{k+1,3}) + T_{k+1,2\to 3} + A_{k+1,3}$$

(by assumption)

$$= \max\left(t_{k+1,2}, t'_{k,3} + \max\left(\left(t_{k+1,2} - t_{k,3}\right), 0\right)\right) + T_{k+1,2\to 3} + A_{k+1,3}$$

$$= \max\left(t_{k+1,2}, t_{k,3}\right) + T_{k+1,2\to 3} + A_{k+1,3} = t_{k+1,3}$$

$\Rightarrow$ *P(k+1)* is true .

Hence by principle of mathematical induction P(n) is true for all n, .i.e. $t'_{n,3} = t_{n,3}$.

5.1 LEMMA

If M_3 starts processing jobs at $L_3 = \sum_{i=1}^{n} I_{i,3}$ then

(i). $L_3 > t_{1,2}$

(ii). $t'_{k+1,3} \geq t_{k,2}$, $k > 1$.

6. THEOREM

The processing of jobs on M_2 at time $L_2 = \min_{i\leq k\leq n} \{Y_k\}$ keeps total elapsed time unaltered where

$Y_1 = L_3 - A_{1,2} - T_{1,2\to 3}$ and

$$Y_k = t'_{k-1,3} - \sum_{i=1}^{k} A_{i,2} - \sum_{i=1}^{k} T_{i,2\to 3}; k > 1.$$

Proof: Since $L_2 = \min_{i\leq k\leq n} \{Y_k\} = Y_r$ (say)

In particular for k =1

$$Y_r \leq Y_1$$

$$\Rightarrow Y_r + A_{1,2} + T_{1,2\to 3} \leq Y_1 + A_{1,2} + T_{1,2\to 3}$$

$$\Rightarrow Y_r + A_{1,2} + T_{1,2\to 3} \leq L_3 \qquad \text{-----(1)}$$

$$\left(\because Y_1 = L_3 - A_{1,2} - T_{1,2\to 3}\right)$$

By Lemma 1; we have

$$t_{1,2} \leq L_3 \qquad \text{---- (2)}$$

Also, $t'_{1,2} = \max\left(Y_r + A_{1,2} + T_{1,2\to3}, t_{1,2}\right)$

On combining, we get $t'_{1,2} \le L_3$

For k >1, As $Y_r = \min_{i\le k\le n}\{Y_k\}$

$\Rightarrow Y_r \le Y_k$; $k = 2,3\ldots\ldots,n$

$$\Rightarrow Y_r + \sum_{i=1}^{k} A_{i,2} + \sum_{i=1}^{k} T_{i,2\to3} \le Y_k + \sum_{i=1}^{k} A_{i,2} + \sum_{i=1}^{k} T_{i,2\to3}$$

$$\Rightarrow Y_r + \sum_{i=1}^{k} A_{i,2} + \sum_{i=1}^{k} T_{i,2\to3} \le t'_{k-1,3} \quad \text{---- (3)}$$

By Lemma 1; we have

$$t_{k,2} \le t'_{k-1,3} \quad \text{---- (4)}$$

Also, $t'_{k,2} = \max\left(Y_r + \sum_{i=1}^{k} A_{i,2} + \sum_{i=1}^{k} T_{i,2\to3}, t_{k,2}\right)$

Using (3) and (4) , we get $t'_{k,2} \le t'_{k-1,3}$

Taking $k = n$, we have

$$t'_{n,2} \le t'_{n-1,3} \quad \text{---- (5)}$$

Total time elapsed = $t_{n,3}$

$= \max\left(t'_{n,2}, t'_{n-1,3}\right) + A_{n,3} + T_{n,2\to3}$

$= t'_{n-1,3} + A_{n,3} + T_{n,2\to3}$ (using 5)

$= t'_{n,3}$.

Hence, the total time elapsed remains unaltered if M_2 starts processing jobs at time

7. PROBLEM FORMULATION

Let some job i (i = 1,2,……..,n) is to be processed on three machines M_j (j = 1,2,3) under the specified rental policy P. Let a_{ij} be the processing time of i^{th} job on j^{th} machine with probabilities p_{ij} . . Let A_{ij} be the expected processing time of i^{th} job on j^{th} machine. Let $T_{i,j\to k}$ be the transportation time of i^{th} job from j^{th} machine to k^{th} machine. Let $\alpha = (i_k, i_m)$ be an equivalent job for job block in which job i_k is given priority over job i_m . Take two job blocks α and β such that block α consists of m jobs out of n jobs in which the order of jobs is fixed and β consists of r jobs out of n in which order of jobs is arbitrary such that $m + r = n$. Let $\alpha \cap \beta = \emptyset$ i.e. the two job blocks α & β form a disjoint set in the sense that the two blocks have no job in common. A string S of job blocks α and β is defined as $S = (\alpha, \beta)$. Our objective is to find an optimal sequence S of the jobs which minimize the rental cost of all the three machines while minimizing total elapsed time.

Mathematically, the problem is stated as:

Minimize $U_j(S)$ and

Minimize $R(S) = \sum_{i=1}^{n} A_{i1} \times C_1 + U_2(S) \times C_2 + \sum_{i=1}^{n} A_{i3} \times C_3$

Subject to constraint: Rental Policy (P)

Our objective is to minimize rental cost while minimizing total elapsed time.

Table 1 the mathematical model of the problem in matrix form

Jobs	Machine M_1				$T_{i,1\to2}$	Machine M_2				$T_{i,2\to3}$	Machine M_3			
i	a_{i1}	p_{i1}	s_{i1}	q_{i1}		a_{i2}	p_{i2}	s_{i2}	q_{i2}		a_{i3}	p_{i3}	s_{i3}	q_{i3}
1	a_{11}	p_{11}	s_{11}	q_{11}	$T_{1,1\to2}$	a_{12}	p_{12}	s_{12}	q_{12}	$T_{1,2\to3}$	a_{13}	p_{13}	s_{13}	q_{13}
2	a_{21}	p_{21}	s_{21}	q_{21}	$T_{2,1\to2}$	a_{22}	p_{22}	s_{22}	q_{22}	$T_{2,2\to3}$	a_{23}	p_{23}	s_{23}	q_{23}
3	a_{31}	p_{31}	s_{31}	q_{31}	$T_{3,1\to2}$	a_{32}	p_{32}	s_{32}	q_{32}	$T_{3,2\to3}$	a_{33}	p_{33}	s_{33}	q_{33}
4	a_{41}	p_{41}	s_{41}	q_{41}	$T_{4,1\to2}$	a_{42}	p_{42}	s_{42}	q_{42}	$T_{4,2\to3}$	a_{43}	p_{43}	s_{43}	q_{43}
-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
n	a_{n1}	p_{n1}	s_{n1}	q_{n1}	$T_{n,1\to2}$	a_{n2}	p_{n2}	s_{n2}	q_{n2}	$T_{n,2\to3}$	a_{n3}	p_{n3}	s_{n3}	q_{n3}

8. ALGORITHM

Step 1: Calculate the expected processing times $A_{ij} = a_{ij} \times p_{ij} \;\; \forall i, j$ =1, 2, 3

Step 2: Check the structural condition

Either $\text{Min}\{A_{i1} + T_{i,1\to2}\} \ge \text{Max}\{A_{i2} + T_{i,1\to2}\}$

or $\text{Min}\{A_{i3} + T_{i,2\to3}\} \ge \text{Max}\{A_{i2} + T_{i,2\to3}\}$ or both for all i

If the conditions are satisfied then go to step 3, else the data is not in the standard form.

Step 3: Introduce the two fictitious machines G and H with processing times G_i and H_i as

$G_i = A_{i1} + T_{i,1\to2} + A_{i2} + T_{i,2\to3}$ and

$H_i = A_{i2} + T_{i,1\to2} + A_{i3} + T_{i,2\to3}$ for all i.

Step 4: Take equivalent job $\alpha = (i_k , i_m)$ for the given job block (i_k , i_m) and define its processing time on the lines of Maggu & Das [9] defined as follows:

$G_\alpha = G_k + G_m - \min(G_m ,H_k)$ and $H_\alpha = H_k + H_m - \min(G_m ,H_k)$

Step 5: Obtain the order of jobs in the job block β in an optimal manner using Johnson's [7] technique by treating job block β as sub flow shop scheduling problem of the main problem. Let β' be the new job block. Define its processing time $G_{\beta'}$ & $H_{\beta'}$ on the lines of Maggu & Das [9] as defined in step 4.

Now, the given problem reduce into new problem replacing m jobs by job block α with processing times G_α & H_α on machine G & H respectively as defined in step 4 and r jobs of job block β by β' with processing times $G_{\beta'}$ & $H_{\beta'}$ on machine G & H respectively as defined in step 5.

Table 2: The new problem

Jobs *(i)*	Machine G *(G_i)*	Machine H *(H_i)*
α	G_α	H_α
β'	$G_{\beta'}$	$H_{\beta'}$

Step 6: Let S_1 denote set of all the processing time G_i when $G_i \le H_i$ and let S_2 denote the set of processing times which are not covered in set S_1.

Step 7: Let S'_1 denote a suboptimal sequence of jobs corresponding to non decreasing times in set S_1 & let S'_2 denote a suboptimal sequence of jobs co

Step 8: The augmented ordered sequence $S=(S'_1, S'_2)$ gives optimal sequence for processing the jobs for the original problem.

Step 9: Prepare In – Out tables for sequence S and compute total elapsed time $t_{n3}(S)$

Step 10: Compute latest time L_3 for machine M_3 for sequence S as

$$L_3(S) = t_{n3}(S) - \sum_{i=1}^{n} A_{i3}$$

Step 11: For the sequence S, compute

$t_{n2}(S)$

$Y_1(S) = L_3(S) - A_{1,2}(S) - T_{1,2\to 3}$

$Y_q(S) = L_3(S) - \sum_{i=1}^{q} A_{i2}(S) - \sum_{i=1}^{q} T_{i,2\to 3} + \sum_{i=1}^{q-1} A_{i,3} + \sum_{i=1}^{q-1} T_{i,1\to 2}; q = 2,3,......,n$

$L_2(S) = \min_{1\le q\le n} \{Y_q(S)\}$

$U_2(S) = t_{n2}(S) - L_2(S)$.

Step 12: Compute total rental cost of all the three machines for sequence S as:

$$R(S) = \sum_{i=1}^{n} A_{i1} \times C_1 + U_2(S) \times C_2 + \sum_{i=1}^{n} A_{i3} \times C_3$$

9. NUMERICAL ILLUSTRATION

Consider 5 jobs, 3 machine flow shop problem with processing time associated with their respective probabilities and transportation time as given in table and jobs 2 and 5 are processed as a group job (2, 5). The rental cost per unit time for machines M_1, M_2 and M_3 are 4 units, 6 units and 8 units respectively, under the specified rental policy P.

Table 3: Three machines with processing and transportation time

Jobs	Machine M_1		$T_{i,1\to 2}$	Machine M_2		$T_{i,2\to 3}$	Machine M_3	
i	a_{i1}	p_{i1}		a_{i2}	p_{i2}		a_{i3}	p_{i3}
1	27	0.2	2	7	0.3	2	17	0.2
2	30	0.2	2	20	0.2	1	18	0.3
3	42	0.2	2	22	0.2	2	12	0.2
4	24	0.2	3	24	0.1	3	23	0.1
5	22	0.2	4	9	0.2	4	25	0.2

Our objective is to obtain an optimal schedule for above said problem to minimize the total production time / total elapsed time subject to minimization of the total rental cost of the machines.

Solution: As per Step 1: The expected processing times for machines M_1, M_2 and M_3 are

Table 4: The expected processing times for machines

Jobs	A_{i1}	$T_{i,1\to 2}$	A_{i2}	$T_{i,2\to 3}$	A_{i3}
1	5.4	2	2.1	2	3.4
2	6.0	2	4.0	1	5.4
3	8.4	2	4.4	2	2.4
4	4.8	3	2.4	3	2.3
5	4.4	4	1.8	4	5.0

As per step 2 & 3: The expected processing times G_i and H_i for two fictitious machines

Table 5: Expected processing times for two fictitious machines G & H

Jobs	G_i	H_i
1	11.5	9.5
2	13.0	12.4
3	16.8	10.8
4	13.2	10.7
5	14.2	14.8

As per step 4:Here $\alpha = (2, 5)$

Therefore, $G_\alpha = 13.0 + 14.2 - 12.4 = 14.8$ and $H_\alpha = 12.4 + 14.8 - 12.4 = 14.8$

As per step 5: Here $\beta = (1, 3, 4)$

Now, using Johnson [1] technique by treating job block β as sub flow shop scheduling problem of the main problem. Let β' be the new job block. Here we get $\beta' = (3, 4, 1)$

Also $\beta' = (3, 4, 1) = ((3, 4), 1) = (\alpha', 1)$, where $\alpha' = (3, 4)$

$G_{\beta'} = 19.2 + 11.5 - 10.7 = 20$ and $H_{\beta'} = 10.7 + 9.5 - 10.7 = 9.5$

As per step 6: $S_1 = 14.8$, $S_2 = 20$

As per step 7: $S'_1 = \alpha$, $S'_2 = \beta'$

As per step 8: Optimal sequence is S = 2 – 5 – 3 – 4 – 1

As per step 9: The In – Out flow table for the optimal sequence S is

Table 6: The In-Out table

J	M_1	$T_{i,1\to 2}$	M_2	$T_{i,2\to 3}$	M_3
i	In – Out		In – Out		In - Out
2	0 – 6.0	2	8.0 – 12.0	1	13.0 – 18.4

5	6.0 – 10.4	4	14.4 – 16.2	4	20.2 – 25.2
3	10.4 – 18.8	2	20.8 – 25.2	2	27.2 – 29.6
4	18.8 – 23.6	3	26.6 – 29.0	3	32.0 – 34.3
1	23.6 – 29	2	31.0 – 33.1	2	35.1 – 38.5

Total elapsed time $t_{n3}(S)$ = 38.5 units

As per Step 10: $L_3\left(S\right)=t_{n3}\left(S\right)-\sum_{i=1}^{n}A_{i,3}\left(S\right)$ = 38.5 – 18.5

= 20 units

As per Step 11: For sequence S, we have

$$Y_1=20-4.0-1=15$$
$$Y_2=20-10.8+7.4=16.6$$
$$Y_3=20-17.2+16.4=19.2$$
$$Y_4=20-22.6+20.8=18.2$$
$$Y_5=20-26.7+26.1=19.4$$
$$L_2\left(S\right)=Min\left\{Y_k\right\}=15$$
$$U_2\left(S\right)=t_{n2}\left(S\right)-L_2\left(S\right)=33.1-15=18.1$$

The new reduced Bi-objective In – Out table is

J	M_1	$T_{i,1\to2}$	M_2	$T_{i,2\to3}$	M_3
i	In – Out		In – Out		In - Out
2	0 – 6.0	2	15.0 – 19.0	1	20.0 – 25.4
5	6.0 – 10.4	4	19.0 – 20.8	4	25.4 – 30.4
3	10.4 – 18.8	2	20.8 – 25.2	2	30.4 – 32.8
4	18.8 – 23.6	3	26.6 – 29.0	3	32.8 – 35.1
1	23.6 – 29.0	2	31.0 – 33.1	2	35.1 – 38.5

The latest possible time at which machine M_2 should be taken on rent = $L_2(S)$ = 15 units.

Also, utilization time of machine M_2 = $U_2(S)$ = 18.1 units.

Total Minimum rental cost =

$$R(S)=\sum_{i=1}^{n}A_{i1}\times C_1+U_2(S)\times C_2+\sum_{i=1}^{n}A_{i3}\times C_3$$

= 372.6 Units.

10. APPENDIX

Programme

```
#include<iostream.h>
#include<stdio.h>
#include<conio.h>
#include<process.h>

int n,e;
float a1[16],b1[16],c1[16],g[16],h[16],T12[16],T23[16];
float macha[16],machb[16],machc[16];
float cost_a,cost_b,cost_c,cost;
int f=1;
float minval,minv,maxv1[16],maxv2[16];
int group[16];//variables to store two job blocks
int gg=0;

float gcal,hcal;
float gbeta=0.0,hbeta=0.0;float galfa=0.0,halfa=0.0;
char s1[5];char s2[5];

void ghcal(int k,int m)
{
float minv;
if(g[m]>h[k])
minv=h[k];
else
minv=g[m];gcal=g[k]+g[m]-minv;hcal=h[k]+h[m]-minv;
//return(c);
}

void main()
{
clrscr();
int a[16],b[16],c[16],j[16];float p[16],q[16],r[16];
cout<<"How many Jobs (<=15) : ";cin>>n;
if(n<1 || n>15)
{
cout<<endl<<"Wrong input, No. of jobs should be less than
15..\n Exitting";
getch();exit(0);}
for(int i=1;i<=n;i++)
{
j[i]=i;
        cout<<"\nEnter the processing time and its
probability of "<<i<<" job for machine A and
Transportation time from Machine A to B :
";cin>>a[i]>>p[i]>>T12[i];
        cout<<"\nEnter the processing time and its
probability of "<<i<<" job for machine B and
Transportation time from Machine B to C :
";cin>>b[i]>>q[i]>>T23[i];
cout<<"\nEnter the processing time and its probability of
"<<i<<"job for machine C: ";
cin>>c[i]>>r[i];
a1[i] = a[i]*p[i];b1[i] = b[i]*q[i];c1[i] = c[i]*r[i];
}
cout<<"\nEnter the rental cost of Machine M1:";cin>>cost_a;
cout<<"\nEnter the rental cost of Machine M2:";cin>>cost_b;
cout<<"\nEnter the rental cost of Machine M3:";cin>>cost_c;
cout<<endl<<"Expected processing time of machine A, B and
C: \n";
for(i=1;i<=n;i++)
{
cout<<j[i]<<"\t"<<a1[i]<<"\t"<<b1[i]<<"\t"<<c1[i]<<"\t";cou
t<<endl;
}
//Function for two ficticious machine G and H
//Finding smallest in a1
float mina1;
mina1=a1[1]+T12[1];
for(i=2;i<n;i++)
{
        if(a1[i]+T12[i]<mina1)
        mina1=a1[i]+T12[i];
}
//For finding largest in b1
float maxb1;maxb1=b1[1]+T23[1];
for(i=2;i<n;i++)
{
        if(b1[i]+T23[i]>maxb1)
        maxb1=b1[i]+T23[i];
```

```
}
//Finding smallest in c1
float minc1;
minc1=c1[1]+T23[1];
for(i=2;i<n;i++)
{
if(c1[i]+T23[i]<minc1)
minc1=c1[i]+T23[i];
}

if(mina1>=maxb1||minc1>=maxb1)
{
for(i=1;i<=n;i++)
{
g[i]=a1[i]+T12[i]+b1[i]+T23[i];
h[i]=T12[i]+b1[i]+T23[i]+c1[i];
}
}
else
{
cout<<"\n data is not in Standard
Form...\nExitting";getch();exit(0);
}
cout<<endl<<"Expected processing time for two fictious
machines G and H: \n";
 for(i=1;i<=n;i++)
 {
 cout<<endl;cout<<j[i]<<"\t"<<g[i]<<"\t"<<h[i];cout<<endl;
 }
 cout<<"\nEnter the number of fixed jobs in job block alpha
<="<<n<<":";cin>>e;
cout<<"\nEnter the fixed job blocks ("<<e<<" numbers from
1 to "<<n<<") alpha : ";
for(int y=1;y<=e;y++)
{
cin>>group[y];
}
cout<<"\nEnter the jobs having disjoint job block ( numbers
from 1 to "<<n<<" other
 than the fixed job block) beta:";
for(int u= e+1;u<=n;u++)
{
cin>>group[u];
}
float btj[16],btg[16],bth[16];
cout<<"Expected processing time for two fictious machines G
and H for Beta: \n";
for(i=1,u=e+1;u<=n;i++,u++)
{
btj[i]=group[u];btg[i]=g[group[u]];bth[i]=h[group[u]];
cout<<endl<<btj[i]<<"\t"<<btg[i]<<"\t"<<bth[i];
}
float mingh[16];
char ch[16];
for(i=1;i<=n-e;i++)
{
if(btg[i]<bth[i])
{
mingh[i]=btg[i];
ch[i]='g';
 }
else
{
mingh[i]=bth[i];
ch[i]='h';
 }
}
for(i=1;i<=n-e;i++)
{
for(int j=1;j<=n-e;j++)

if(mingh[i]<mingh[j])
{
float temp=mingh[i]; int temp1=btj[i]; char d=ch[i];
mingh[i]=mingh[j]; btj[i]=btj[j]; ch[i]=ch[j];
mingh[j]=temp; btj[j]=temp1; ch[j]=d;
                              }
}
// calculate beta scheduling
int sbeta[16];int t=1,s=0;
for(i=1;i<=n-e;i++)
{
if(ch[i]=='h')
{
 sbeta[(n-s-e)]=btj[i]; s++;
}
else if(ch[i]=='g')
{
sbeta[t]=btj[i];t++;
}
}
cout<<endl<<endl<<"Beta Scheduling:"<<"\t";
for(i=1;i<=n-e;i++)
{
cout<<sbeta[i]<<"  ";
}
//calculate G_Alfa and H_Alfa
ghcal(group[1],group[2]);
galfa=gcal;halfa=hcal;
i=3;
while(i<=e)
{
if(i>e)
break;
else
{
if(g[group[i]]<halfa)
minval=g[group[i]];
else
minval=halfa;galfa=galfa+g[group[i]]-
minval;halfa=halfa+h[group[i]]-minval;
}
i++;
}
cout<<endl<<endl<<"G_Alfa="<<galfa;
cout<<endl<<"H_Alfa="<<halfa;
//calculate G_Beta and H_Beta
ghcal(sbeta[1],sbeta[2]);
gbeta=gcal;hbeta=hcal;
i=3;
while(i<=(n-e))
{
if(i>(n-e))
break;
else
{
if(g[sbeta[i]]<hbeta)
minval=g[sbeta[i]];
else
minval=hbeta;
gbeta=gbeta+g[sbeta[i]]-minval;hbeta=hbeta+h[sbeta[i]]-
minval;
}
```

```
i++;
}
cout<<endl<<endl<<"G_Beta="<<gbeta;cout<<endl<<"H_Be
ta="<<hbeta;
//calculate optimal sequence
if(galfa<=halfa)
{
s1[1]='a';s2[1]='\0';
}
else
{
s2[1]='a';s1[1]='\0';
}
if(gbeta<=hbeta)
{
s1[2]='b';s2[2]='\0';
}
else
{
s2[2]='b';s1[2]='\0';
}
//cout<<endl<<endl<<"Optimal Sequence:"<<"\t";
int arr[16];
if(s1[1]=='a')
{
//cout<<"\n a";
for(i=1;i<=e;i++)
{
//cout<<group[i]<<"\t";
arr[i]=group[i];
}
gg=gg+e;
}
if(s1[2]=='b')
{
//cout<<"\n b";
for(i=1;i<=n-e;i++)
{
//cout<<endl<<sbeta[i]<<"\t";
arr[i+gg]=sbeta[i];
}
gg=gg+(n-e)+1;
}
if(s2[1]=='a')
{
//cout<<"\n a";
for(i=1;i<=e;i++)
{
//cout<<endl<<group[i]<<"\t";
arr[i+gg]=group[i];
}
gg=gg+e;
}
if(s2[2]=='b')
{
//cout<<"\n b";
for(i=1;i<=(n-e);i++)
{
//cout<<sbeta[i]<<"\t";
arr[i+gg]=sbeta[i];
}
}
//calculating total computation sequence
float time=0.0;macha[1]=time+a1[arr[1]];
for(i=2;i<=n;i++)
{
macha[i]=macha[i-1]+a1[arr[i]];
}
machb[1]=macha[1]+b1[arr[1]]+T12[arr[1]];
for(i=2;i<=n;i++)
{
if((machb[i-1])>(macha[i]+T12[arr[i]]))
maxv1[i]=machb[i-1];
else
maxv1[i]=macha[i]+T12[arr[i]];machb[i]=maxv1[i]+b1[arr[i]
];
}
machc[1]=machb[1]+c1[arr[1]]+T23[arr[1]];
for(i=2;i<=n;i++)
{
if((machc[i-1])>(machb[i]+T23[arr[i]]))
maxv2[i]=machc[i-1];
else
maxv2[i]=machb[i]+T23[arr[i]];machc[i]=maxv2[i]+c1[arr[i]]
;
}
//displaying solution
cout<<"\n\n\n\n\n\t\t\t    #####THE SOLUTION##### ";
cout<<"\n\n\t***************************************
***********************";
cout<<"\n\n\n\t    Optimal Sequence is : ";
for(i=1;i<=n;i++)
{
cout<<" "<<arr[i];
}
cout<<endl<<endl<<"In-Out Table is:"<<endl<<endl;
cout<<"Jobs"<<"\t"<<"MachineM1"<<"\t"<<"\t"<<"Machine
M2" <<"\t"<<"\t"<<"Machine M3"<<endl;
cout<<arr[1]<<"\t"<<time<<"--"<<macha[1]<<"
\t"<<"\t"<<(macha[1]+T12[arr[1]])<<"--"<<machb[1]<<"
\t"<<"\t"<<(machb[1]+T23[arr[1]])<<"--"<<machc[1]<<endl;
for(i=2;i<=n;i++)
{
cout<<arr[i]<<"\t"<<macha[i-1]<<"--"<<macha[i]<<"
"<<"\t"<<maxv1[i]<<"--"<<machb[i]<<"
"<<"\t"<<maxv2[i]<<"--"<<machc[i]<<endl;
}
cout<<"\n\n\nTotal Elapsed Time (T) = "<<machc[n];
float L3,Y[16],min,u2;
float sum1=0.0,sum2=0.0,sum3=0.0;
for(i=1;i<=n;i++)
{
sum1=sum1+a1[i];sum2=sum2+b1[i];sum3=sum3+c1[i];
}
L3=machc[n]-sum3;
cout<<"\n\nLatest Time When Machine M3 is Taken on
Rent:"<<L3;
cout<<"\n\nTotal Completion Time of Jobs on Machine
M2:"<<machb[n];
Y[1]=L3-b1[arr[1]]-T23[arr[1]];
cout<<"\n\n\tY[1]\t="<<Y[1];
float sum_2,sum_3;
for(i=2;i<=n;i++)
{
sum_2=0.0,sum_3=0.0;
for(int j=1;j<=i-1;j++)
{
sum_3=sum_3+c1[arr[j]]+T12[arr[j]];
}
for(int k=1;k<=i;k++)
{
sum_2=sum_2+b1[arr[k]]+T23[arr[k]];
}
Y[i]=L3+sum_3-sum_2;
```

```
cout<<"\n\n\tY["<<i<<"]\t="<<Y[i];
}
min=Y[1];
for(i=2;i<n;i++)
{
if(Y[i]<min)
min=Y[i];
}
cout<<"\n\nMinimum of Y[i]="<<min;
u2=machb[n]-min;
cout<<"\n\nUtilization Time of Machine M2="<<u2;
cost=(sum1*cost_a)+(u2*cost_b)+(sum3*cost_c);
cout<<"\n\nThe Minimum Possible Rental Cost is="<<cost;
cout<<"\n\n\t*************************************
***********************";
getch();
}
```

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