Uncertain 2D Continuous Systems with State Delay: Filter Design using an H_∞ Polynomial Approach

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ABSTRACT
This paper proposes a methodology to design filters that extract information from noisy signals. From a mathematical point of view, a method is used based on homogeneous polynomially parameter-dependent (HPPD) matrices of arbitrary degree. The optimal H_∞ filter is then obtained by solving a convex optimization problem using off-the-self software. To show the effectiveness of the proposed filter design methodology some examples are solved, and the solution is illustrated using computer simulations.

Keywords
Systems theory, uncertainty, delays, filtering, linear matrix inequalities (LMI).

1. INTRODUCTION
Designing filters and observers is a well-studied problem in one-dimensional systems (see, for example, [1], [2], and references therein), and some two-dimensional systems in image processing applications (see [3] and references therein. More precisely, a solution to the H_∞ filtering problem is given in this paper for the class of two dimensional (2-D) continuous systems that are described by a Roesser state space model with both state delays and parameter uncertainties. Delays are considered L_2 as they appear frequently in practical problems (see [5] and references therein). Similarly, uncertainties are inherent to any practical implementation (see [6] and references therein).

The H_∞ estimation problem has attracted much interest in the past decades within the systems theory community [24], [38]. One of the reasons is the fact that it does not require a precise knowledge of the statistics of the noisy signals, as required by alternatives approaches. This estimation procedure just ensures that the L_2-induced gain from the noise to the estimation error is smaller than a prescribed level, with the noise signals described as energy-bounded signals. Many results on the H_∞ filtering problem have been proposed in the literature, in both the deterministic and stochastic contexts: see, e.g., [4], [11], [15], [24], [27], [32], [37], [38] and references therein. In practice, system parameters are never perfectly known. When parameter uncertainties affect a system, the corresponding robust H_∞ filtering has also been investigated: see, e.g., [9], [21], [36]; in the particular case of for state-delayed systems, we can cite [13], [14], [22] and [26]. Note that all these mentioned H_∞ filtering results are obtained in the context of one-dimensional (1-D) system. The study of two-dimensional (2-D) filters has received much attention in past decades: [7], [10], [12], [16], [17], [19], [23], [30], [33], [34], [35]. For example, the 2-D H_∞ filtering problem for Roesser models was solved in [10], although in the absence of uncertainties and delays, with the parallel results for the 2D Fornasini-Marchesini second model reported in [33] and [34]. We point out that these H_∞ filtering results were obtained for 2D discrete systems. However, as it is well known, partial differential equations actually correspond to 2-D or n-D continuous systems [23]. Therefore, the study of 2-D continuous systems is of practical and theoretical importance.

It is worth noting that most of the results regarding this topic only deal with 2-D systems without delays. However, delays are frequent in systems described by partial differential equations, for example in signal transmissions and biological systems. Examples of 2-D systems with time delays include the material rolling process [31] and systems described by delayed lattice differential equations [20] and partial difference equations [39], [40]. In addition, certain 2-D systems containing digital processors that need finite numerical computation time [8], [28] display also the delay phenomenon. The stability and control problems of uncertain 2-D discrete state-delayed systems have been studies in [28], [29], whereas the H_∞ filtering problem for 2-D continuous state-delayed systems (albeit with norm bounded uncertainties) was considered in [18].In this paper, motivated by the underlying idea in [25], we present a new approach, the structured polynomially parameter-dependent method, for designing the robust H_∞ filters for uncertain 2D state-delayed systems described by the Roesser state-space model. Assuming parameter uncertainties in a polytope, the focus is on designing a filter such that the filtering error system is robustly asymptotically stable and the H_∞ norm of the filtering error system for the entire uncertainty domain minimized. This new polynomially parameter-dependent idea is based on using homogeneous polynomially parameter-dependent matrices: by increasing its degree, less conservative filters are obtained. Moreover, the obtained conditions are expressed in terms of linear matrix inequalities which can be easily solved using computers and off-the-self software. This methodology includes as a particular case the quadratic framework, and the linearly parameter-dependent framework, special cases for zeroth degree and first degree, respectively.

Notation: Throughout this paper, for real symmetric matrices *X* and *Y*, the notation $X \geq Y$ (respectively, $X > Y$) means that the matrix *X-Y* is positive semi-definite (respectively, positive definite). I is the identity matrix with appropriate dimension (All matrices, if not explicitly stated, are assumed to have compatible dimensions). The superscript *T* represents the transpose of a matrix, with $her(S) = S + S^T$. The symbol $\sigma_{\max}(.)$ denotes the spectral norm of a matrix.

The symmetric term in a symmetric matrix is denoted by *, e.g., $\begin{bmatrix} X & Y \\ * & Z \end{bmatrix} = \begin{bmatrix} X & Y \\ Y^T & Z \end{bmatrix}$.

2. PROBLEM FORMULATION

Consider a 2-D continuous system described by the following Roesser's state-space model with delays in the states:

$$(\Sigma):\begin{cases}\dot{x}(t_1,t_2)=A(\alpha)x(t_1,t_2)+A_d(\alpha)x(t_1-\tau_1,t_2-\tau_2)\\ \qquad +B(\alpha)w(t_1,t_2)\\ y(t_1,t_2)=C_1(\alpha)x(t_1,t_2)+C_{1d}(\alpha)x(t_1-\tau_1,t_2-\tau_2)\\ \qquad +D_1(\alpha)w(t_1,t_2)\\ z(t_1,t_2)=C(\alpha)x(t_1,t_2)+D(\alpha)w(t_1,t_2)\end{cases}$$

with

$x(0,t_2)=f(t_2)$ for $t_2\in[-\tau_2,0]$, $x(t_1,0)=g(t_1)$ for $t_1\in[-\tau_1,0]$, $x(t_1,t_2)=\begin{bmatrix}x^h(t_1,t_2)\\ v^v(t_1,t_2)\end{bmatrix}$, $\dot{x}(t_1,t_2)=\begin{bmatrix}\dfrac{\partial x^h(t_1,t_2)}{\partial t_1}\\ \dfrac{\partial x^v(t_1,t_2)}{\partial t_2}\end{bmatrix}$,

$x(t_1-\tau_1,t_2-\tau_2)=\begin{bmatrix}x^h(t_1-\tau_1,t_2)\\ v^v(t_1,t_2-\tau_2)\end{bmatrix}$, where $x^h(t_1,t_2)\in\mathbb{R}^{n_h}$ and $x^v(t_1,t_2)\in\mathbb{R}^{n_v}$ are the horizontal and vertical states, respectively, $y(t_1,t_2)\in\mathbb{R}^p$ is the measured output, $z(t_1,t_2)\in\mathbb{R}^r$ is the signal to be estimated, $w(t_1,t_2)\in\mathbb{R}^m$ is the exogenous input, and $\tau_1,\tau_2>0$ are constant time delays.

All matrices are assumed to be real, belonging to the polytope

$$\mathcal{P}\triangleq\left\{\begin{bmatrix}A(\alpha) & A_d(\alpha)\\ B(\alpha) & C_1(\alpha)\\ C_{1d}(\alpha) & D_1(\alpha)\\ C(\alpha) & D(\alpha)\end{bmatrix}=\sum_{i=1}^N\alpha_i\begin{bmatrix}A_i & A_{di}\\ B_i & C_{1i}\\ C_{1di} & D_{1i}\\ C_i & D_i\end{bmatrix},\sum_{i=1}^N\alpha_i=1,\ \alpha_i\ge 0\right\} \tag{1}$$

Here, we are interested in estimating the signal $z(t_1,t_2)$ by a robust HPPD filter of the form

$$(\Sigma_f):\begin{cases}\dot{\hat{x}}(t_1,t_2)=A_f(\alpha)\hat{x}(t_1,t_2)+B_f(\alpha)y(t_1,t_2)\\ \hat{z}(t_1,t_2)=C_f(\alpha)\hat{x}(t_1,t_2),\end{cases}$$

where

$\hat{x}(t_1,t_2)=\begin{bmatrix}\hat{x}^h(t_1,t_2)\\ \hat{v}^v(t_1,t_2)\end{bmatrix}$, $\hat{x}^h(t_1,t_2)\in\mathbb{R}^{n_h}$ and $\hat{x}^v(t_1,t_2)\in\mathbb{R}^{n_v}$ are the horizontal and vertical states of the filter, respectively, $\hat{z}(t_1,t_2)\in\mathbb{R}^r$ is the estimate of $z(t_1,t_2)$. $A_f(\alpha)$, $B_f(\alpha)$ and $C_f(\alpha)$ are filter parameter-dependent matrices to be determined.

By defining an augmented state vector and the filtering error output signal:

$$\tilde{x}^h(t_1,t_2)=\begin{bmatrix}x^h(t_1,t_2)^T & \hat{x}^h(t_1,t_2)^T\end{bmatrix}^T,$$

$$\tilde{x}^v(t_1,t_2)=\begin{bmatrix}x^v(t_1,t_2)^T & \hat{x}^v(t_1,t_2)^T\end{bmatrix}^T,$$

$$\tilde{x}^h(t_1-\tau_1,t_2)=\begin{bmatrix}x^h(t_1-\tau_1,t_2)\\ \hat{x}^h(t_1-\tau_1,t_2)\end{bmatrix},$$

$$\tilde{x}^v(t_1,t_2-\tau_2)=\begin{bmatrix}x^v(t_1,t_2-\tau_2)\\ \hat{x}^v(t_1,t_2-\tau_2)\end{bmatrix},$$

$$\tilde{x}(t_1,t_2)=\begin{bmatrix}\tilde{x}^h(t_1,t_2)^T & \tilde{x}^v(t_1,t_2)^T\end{bmatrix}^T,$$

$$\tilde{x}(t_1-\tau_1,t_2-\tau_2)=\begin{bmatrix}\tilde{x}^h(t_1-\tau_1,t_2)\\ \tilde{x}^v(t_1,t_2-\tau_2)\end{bmatrix},$$

$$\tilde{z}(t_1,t_2)=z(t_1,t_2)-\hat{z}(t_1,t_2),$$

the following augmented system can be obtained:

$$(\Sigma_e):\begin{cases}\dot{\tilde{x}}(t_1,t_2)=\tilde{A}(\alpha)\tilde{x}(t_1,t_2)+\tilde{A}_d(\alpha)\tilde{x}(t_1-\tau_1,t_2-\tau_2)\\ \qquad +\tilde{B}(\alpha)w(t_1,t_2)\\ \tilde{z}(t_1,t_2)=\tilde{C}(\alpha)\tilde{x}(t_1,t_2)+\tilde{D}(\alpha)w(t_1,t_2).\end{cases}$$

where

$$\tilde{A}(\alpha)=\Phi\tilde{A}_f(\alpha)\Phi^T,\ \tilde{A}_d(\alpha)=\Phi\tilde{A}_{df}(\alpha)\Phi^T,$$
$$\tilde{B}(\alpha)=\Phi\tilde{B}_f(\alpha),\ \tilde{C}(\alpha)=\tilde{C}_f(\alpha)\Phi^T,\ \tilde{D}(\alpha)=D(\alpha) \tag{2}$$

and the augmented matrices are given by

$$\tilde{A}_f(\alpha)=\begin{bmatrix}A(\alpha) & 0\\ B_f(\alpha)C_1(\alpha) & A_f(\alpha)\end{bmatrix},$$
$$\tilde{A}_{df}(\alpha)=\begin{bmatrix}A_d(\alpha) & 0\\ B_f(\alpha)C_{1d}(\alpha) & 0\end{bmatrix},\qquad \tilde{B}_f(\alpha)=\begin{bmatrix}B(\alpha)\\ B_f(\alpha)D_1(\alpha)\end{bmatrix},$$
$$\tilde{C}_f(\alpha)=\begin{bmatrix}C(\alpha) & -C_f(\alpha)\end{bmatrix}, \tag{3}$$

$$\Phi=\begin{bmatrix}I_{n_h} & 0 & 0 & 0\\ 0 & 0 & I_{n_h} & 0\\ 0 & I_{n_v} & 0 & 0\\ 0 & 0 & 0 & I_{n_v}\end{bmatrix}, \tag{4}$$

The robust H_∞ filtering problem to be addressed in this paper can be formulated as follows : Given a scalar $\gamma>0$ and the 2D continuous system with delays (Σ), find matrices $A_f(\alpha)\in\mathbb{R}^{n\times n}$, $B_f(\alpha)\in\mathbb{R}^{n\times p}$ and $C_f(\alpha)\in\mathbb{R}^{r\times n}$ of the filter realization (Σ_f) such that the filtering error system (Σ_e) is asymptotically stable and the transfer function of the error system given as

$$T_{\tilde{z}w}(s_1,s_2)=\tilde{C}(\alpha)\left[I(s_1,s_2)-\tilde{A}(\alpha)-\tilde{A}_d(\alpha)I(e^{-s_1\tau_1},e^{-s_2\tau_2})\right]^{-1}$$
$$\times\tilde{B}(\alpha)+\tilde{D}(\alpha) \tag{5}$$

satisfies

$$\|T_{\tilde{z}w}\|_\infty < \gamma \qquad (6)$$

for all admissible uncertainties and with null initial conditions where

$$I(\sigma_1,\sigma_2) = diag(\sigma_1 I_{n_h}, \sigma_2 I_{n_v}), \qquad (7)$$

and

$$\|T_{\tilde{z}w}(s_1,s_2)\|_\infty = \sup_{\theta_1,\theta_2\in\mathbb{R}} \sigma_{\max}\left[T_{\tilde{z}w}(j\theta_1, j\theta_2)\right], \quad (8)$$

In order to solve the filtering problem, we first introduce the following Theorem which considers a parameter independent structure for $P(\alpha)$, i.e., $P(\alpha)=P=P^T$.

Theorem 1: Given a scalar $\gamma > 0$, the continuous system with delays (Σ_0) is asymptotically stable and satisfies the H_∞ performance $\|T_{zw}\|_\infty < \gamma$ if there exist matrices $P = diag(P_h, P_v) > 0$ and $Q = diag(Q_h, Q_v) > 0$ such that the following LMI holds:

$$\begin{bmatrix} A(\alpha)^T P + PA(\alpha) & PA_d(\alpha) & PB(\alpha) & C(\alpha)^T \\ * & -Q & 0 & 0 \\ * & * & -\gamma I & D(\alpha)^T \\ * & * & * & -\gamma I \end{bmatrix} < 0 \qquad (9)$$

Proof: First, from (9), it is easy the see that

$$\begin{bmatrix} A(\alpha)^T P + PA(\alpha) + Q & PA_d(\alpha) \\ A_d(\alpha)^T P & -Q \end{bmatrix} < 0$$

which by Theorem 2, gives that system (Σ) is asymptotically stable. Next, we show the H_∞ performance, by applying the Schur complement formula to (9), we obtain $V := \gamma^2 I - D(\alpha)^T D(\alpha) > 0$ and

$$her(A^T P) + Q + \gamma^{-1} C^T C + PA_d Q^{-1} A_d^T P + \left[PB + \gamma^{-1} C^T D\right] V^{-1} \left[B^T P + \gamma^{-1} D^T C\right] < 0$$

Multiplying this inequality by γI yields

$$her(A^T(\gamma P)) + (\gamma Q) + C^T C + (\gamma P) A_d (\gamma Q)^{-1} A_d^T (\gamma P) + \left[(\gamma P)B + C^T D\right] V^{-1} \left[B^T(\gamma P) + D^T C\right] < 0 \qquad (10)$$

Let $\tilde{P} = \gamma P > 0$ and $\tilde{Q} = \gamma Q > 0$; then, (10) can be rewritten as

$$A^T \tilde{P} + \tilde{P} A + \tilde{Q} + C^T C + \tilde{P} A_d \tilde{Q}^{-1} A_d^T \tilde{P} + \left[\tilde{P} B + C^T D\right] V^{-1} \left[B^T \tilde{P} + D^T C\right] < 0$$

Therefore, there exists a matrix $U > 0$ such that

$$-her(A^T \tilde{P}) - \tilde{Q} - C^T C - \tilde{P} A_d \tilde{Q}^{-1} A_d^T \tilde{P} > \left[\tilde{P} B + C^T D\right] V^{-1} \left[B^T \tilde{P} + D^T C\right] + U \qquad (11)$$

Set

$$\Omega(j\theta_1, j\theta_2) = I(j\theta_1, j\theta_2) - A - A_d I(e^{-j\theta_1}, e^{-j\theta_2})$$

and $z(j\theta_1, j\theta_2) = \tilde{P} A_d I(e^{-j\theta_1}, e^{-j\theta_2})$ recalling that for any matrices K_1, K_2 and K_3 of appropriate dimension with $K_2 > 0$

$$K_1^* K_3 + K_3^* K_1 \le K_1^* K_2 K_1 + K_3^* K_2^{-1} K_3 \qquad (12)$$

Therefore,

$$z(j\theta_1, j\theta_2) + z(j\theta_1, j\theta_2)^* \le \tilde{P} A_d \tilde{Q}^{-1} A_d^T \tilde{P} + \tilde{Q} \qquad (13)$$

Then, it can be verified that

$$\tilde{P} I(j\theta_1, j\theta_2) + I(-j\theta_1, -j\theta_2)^T \tilde{P} = 0 \qquad (14)$$

By (12), (13) and (14), we have

$$\begin{aligned} &\Omega(-j\theta_1, -j\theta_2)^T \tilde{P} + \tilde{P}\Omega(j\theta_1, j\theta_2) - C^T C = \\ &-her(A^T \tilde{P}) - z(j\theta_1, j\theta_2) - z^*(j\theta_1, j\theta_2) - C^T C \\ &> (\tilde{P} B + C^T D) V^{-1} (B^T \tilde{P} + D^T C) + U \end{aligned} \qquad (15)$$

Since system (Σ) is asymptotically stable, we have $\det\left[I(j\theta_1, j\theta_2) - A - A_d I(e^{-j\theta_1}, e^{-j\theta_2})\right] \ne 0,$ for all $\theta_1, \theta_2 \in \mathbf{R}$. Therefore, $\Omega(j\theta_1, j\theta_2)^{-1}$ is well defined for all $\theta_1, \theta_2 \in \mathbf{R}$. Now, pre-and post multiplying (15) by $B^T \Omega(j\theta_1, j\theta_2)^{-T}$ and $\Omega(j\theta_1, j\theta_2)^{-1} B$ respectively, we have that for all $\theta_1, \theta_2 \in \mathbf{R}$

$$\begin{aligned} &B^T \Omega(j\theta_1, j\theta_2)^{-T} \\ &\times\left[\Omega(-j\theta_1, -j\theta_2)^T \tilde{P} + \tilde{P}\Omega(j\theta_1, j\theta_2) - C^T C\right] \\ &\times\Omega(j\theta_1, j\theta_2)^{-1} B \\ &\ge B^T \Omega(j\theta_1, j\theta_2)^{-T} \Lambda \Omega(j\theta_1, j\theta_2)^{-1} B, \end{aligned} \qquad (16)$$

with

$$\Lambda = (\tilde{P} B + C^T D) V^{-1} (B^T \tilde{P} + D^T C) + U.$$

Then, by noting (5), we have

$$\gamma^2 I - T_{zw}(-j\theta_1,-j\theta_2)^T T_{zw}(j\theta_1, j\theta_2) = \gamma^2 I$$
$$-\left[B^T\Omega(-j\theta_1,-j\theta_2)^{-T}C^T + D^T\right]$$
$$\times\left[C\Omega(j\theta_1, j\theta_2)^{-1}B + D^T\right]$$
$$= \gamma^2 I - D^T D + B^T\Omega(-j\theta_1,-j\theta_2)^{-T}$$
$$\times\left[\tilde{P}\Omega(j\theta_1, j\theta_2) + \Omega(-j\theta_1,-j\theta_2)^{-T}\tilde{P} - C^T C\right]$$
$$\times\Omega(j\theta_1, j\theta_2)^{-1}B$$
$$-B^T\Omega(-j\theta_1,-j\theta_2)^{-T}(\tilde{P}B + C^T D)$$
$$-(B^T\tilde{P} + D^T C)\Omega(j\theta_1, j\theta_2)^{-1}B$$
$$\geq V + B^T\Omega(-j\theta_1,-j\theta_2)^{-T}\Lambda\Omega(j\theta_1, j\theta_2)^{-1}B$$
$$-B^T\Omega(-j\theta_1,-j\theta_2)^{-T}(\tilde{P}B + C^T D)$$
$$-(B^T\tilde{P} + D^T C)\Omega(j\theta_1, j\theta_2)^{-1}B \tag{17}$$

By using the relation (16), we obtain

$$\begin{aligned}&\gamma^2 I - T_{zw}(-j\theta_1,-j\theta_2)^T T_{zw}(j\theta_1, j\theta_2)\\ &\quad\geq V - (B^T\tilde{P} + D^T C)\Lambda^{-1}(\tilde{P}B + C^T D)\end{aligned} \tag{18}$$

Now, observe that

$$\Lambda - (\tilde{P}B + C^T D)V^{-1}(B^T\tilde{P} + D^T C) = U > 0$$

Then, by the Schur complement formula, we have

$$\begin{bmatrix} V & B^T\tilde{P} + D^T C \\ \tilde{P}B + C^T D & \Lambda \end{bmatrix} > 0$$

which, by the Schur complement formula again, gives

$$V - (B^T\tilde{P} + D^T C)\Lambda^{-1}(\tilde{P}B + C^T D) > 0. \tag{19}$$

Then, it follows from (18) and (19) that for all $\theta_1, \theta_2 \in \mathbf{R}$

$$\gamma^2 I - T_{zw}(-j\theta_1,-j\theta_2)^T T_{zw}(j\theta_1, j\theta_2) > 0. \tag{20}$$

Hence, by (20), we have. This completes the proof. □

3. MAIN RESULTS

In this section, an LMI approach will be developed to solve the Robust H_∞ filtering problem formulated in the previous section.

3.1 Parameter-dependent LMIs

In this section, we develop the parameter-dependent LMIs conditions stated in Theorem 1 in terms of generic parameter-dependent matrix solutions.

Theorem 2: Given a scalar $\gamma > 0$, the 2-D robust H_∞ filtering problem is solvable if the 2-D system (Σ) is asymptotically stable with γ performance, that is, if there exist matrices $Z(\alpha)$, $\Theta(\alpha)$, $\Psi(\alpha)$, $X = diag(X_h, X_v) > 0$, $Y = diag(Y_h, Y_v) > 0$, and $S = diag(S_h, S_h) > 0$ with X_h, Y_h, $S_h \in \mathbf{R}^{n_h\times n_h}$ and X_v, Y_v, $S_v \in \mathbf{R}^{n_v\times n_v}$ such that the following LMIs hold

$$\begin{bmatrix} YA(\alpha) + A(\alpha)^T Y + Y & J_{12} & YA_d(\alpha) & YA_d(\alpha) & YB(\alpha) & C(\alpha)^T - \Theta(\alpha)^T \\ * & J_{22} & XA_d(\alpha) + \Psi(\alpha)C_{1d}(\alpha) & XA_d(\alpha) + \Psi(\alpha)C_{1d}(\alpha) & XB(\alpha) + \Psi(\alpha)D_1(\alpha) & C(\alpha)^T \\ * & * & -Y & -Y & 0 & 0 \\ * & * & * & -S & 0 & 0 \\ * & * & * & * & -\gamma I & D(\alpha)^T \\ * & * & * & * & * & -\gamma I \end{bmatrix} < 0 \tag{21}$$

$$X - Y > 0 \tag{22}$$

$$S - Y > 0 \tag{23}$$

where

$$J_{12} = YA(\alpha) + A(\alpha)^T X + C_1(\alpha)^T\Psi(\alpha)^T + Z(\alpha)^T + Y,$$
$$J_{22} = XA(\alpha) + A(\alpha)^T X + \Psi(\alpha)C_1(\alpha) + C_1(\alpha)^T\Psi(\alpha)^T + S,$$
$$J_{23} = XA_d(\alpha) + X_{12}B_f(\alpha)C_{1d}(\alpha)$$

Then, a desired 2-D continuous filter in the form of (Σ_f) can be chosen with the following matrices:

$$A_f(\alpha) = X_{12}^{-1}Z(\alpha)Y^{-1}Y_{12}^{-T} \tag{24}$$

$$B_f(\alpha) = X_{12}^{-1}\Psi(\alpha) \tag{25}$$

$$C_f(\alpha) = \Theta(\alpha)Y^{-1}Y_{12}^{-T} \tag{26}$$

where

$$X_{12}=\begin{bmatrix} X_{h_{12}} & 0 \\ 0 & X_{v_{12}} \end{bmatrix},\ Y_{12}=\begin{bmatrix} Y_{h_{12}} & 0 \\ 0 & Y_{v_{12}} \end{bmatrix},$$
$$S_{12}=\begin{bmatrix} S_{h_{12}} & 0 \\ 0 & S_{v_{12}} \end{bmatrix}, \tag{27}$$

in which $X_{h_{12}}$, $X_{v_{12}}$, $Y_{h_{12}}$, $Y_{v_{12}}$, $S_{h_{12}}$ and $S_{v_{12}}$ are nonsingular matrices satisfying

$$X_{12}Y_{12}^T = I - XY^{-1} \tag{28}$$
$$S_{12}Y_{12}^T = I - SY^{-1} \tag{29}$$

Proof: Let $\bar{Y}_h = Y_h^{-1}$, $\bar{Y}_v = Y_v^{-1}$, $\bar{Y} = Y^{-1}$ then the relations (22)-(23), can be written as

$$\begin{bmatrix} X & I \\ I & \bar{Y} \end{bmatrix} > 0,\ \begin{bmatrix} X & I \\ I & \bar{Y} \end{bmatrix} > 0. \tag{30}$$

By the Schur complement formula, it follows from (30) that

$\bar{Y} - X^{-1} > 0,\quad \bar{Y} - S^{-1} > 0,$ which implies that $I - X\bar{Y}$ and $I - S\bar{Y}$ are nonsingular. Therefore, by noting the structure of X and Y, we have that there always exist nonsingular matrices $X_{h_{12}}$, $X_{v_{12}}$, $Y_{h_{12}}$, $Y_{v_{12}}$, $S_{h_{12}}$ and $S_{v_{12}}$ such that (28) and (29) is satisfied, that is

$$X_{h_{12}}Y_{h_{12}}^T = I - X_h\bar{Y}_h,\quad X_{v_{12}}Y_{v_{12}}^T = I - X_v\bar{Y}_v \tag{31}$$
$$S_{h_{12}}Y_{h_{12}}^T = I - S_h\bar{Y}_h,\quad S_{v_{12}}Y_{v_{12}}^T = I - S_v\bar{Y}_v. \tag{32}$$

Set

$$\Pi_{h_1}=\begin{bmatrix} \bar{Y}_h & I \\ Y_{h_{12}}^T & 0 \end{bmatrix},\quad \Pi_{v_1}=\begin{bmatrix} \bar{Y}_v & I \\ Y_{v_{12}}^T & 0 \end{bmatrix},\quad \Pi_{h_2}=\begin{bmatrix} I & X_h \\ 0 & X_{h_{12}}^T \end{bmatrix},$$
$$\Pi_{v_2}=\begin{bmatrix} I & X_v \\ 0 & X_{v_{12}}^T \end{bmatrix},\quad \Pi_{h_3}=\begin{bmatrix} I & S_h \\ 0 & S_{h_{12}}^T \end{bmatrix},\quad \Pi_{v_3}=\begin{bmatrix} I & S_v \\ 0 & S_{v_{12}}^T \end{bmatrix},$$
$$\Pi_1=\begin{bmatrix} \Pi_{h_1} & 0 \\ 0 & \Pi_{v_1} \end{bmatrix},\ \Pi_2=\begin{bmatrix} \Pi_{h_2} & 0 \\ 0 & \Pi_{v_2} \end{bmatrix},\ \Pi_3=\begin{bmatrix} \Pi_{h_3} & 0 \\ 0 & \Pi_{v_3} \end{bmatrix}.$$

Then, by some calculation, it can be verified that

$$P := \Pi_2\Pi_1^{-1} = \begin{bmatrix} P_h & 0 \\ 0 & P_v \end{bmatrix},\ Q := \Pi_3\Pi_1^{-1} = \begin{bmatrix} Q_h & 0 \\ 0 & Q_v \end{bmatrix} \tag{33}$$

where

$$P_h=\begin{bmatrix} X_h & X_{h_{12}} \\ X_{h_{12}}^T & X_{h_{12}}^T(X_h - Y_h)^{-1}X_{h_{12}} \end{bmatrix},$$
$$P_v=\begin{bmatrix} X_v & X_{v_{12}} \\ X_{v_{12}}^T & X_{v_{12}}^T(X_v - Y_v)^{-1}X_{v_{12}} \end{bmatrix},$$
$$Q_h=\begin{bmatrix} S_h & S_{h_{12}} \\ S_{h_{12}}^T & S_{h_{12}}^T(S_h - Y_h)^{-1}S_{h_{12}} \end{bmatrix},$$
$$Q_v=\begin{bmatrix} S_v & S_{v_{12}} \\ S_{v_{12}}^T & S_{v_{12}}^T(S_v - Y_v)^{-1}S_{v_{12}} \end{bmatrix}.$$

Observe that

$$X_h - X_{12}\left[X_{h_{12}}^T(X_h - Y_h)^{-1}X_{h_{12}}\right]^{-1}X_{h_{12}}^T = Y_h > 0,$$
$$X_v - X_{12}\left[X_{v_{12}}^T(X_v - Y_v)^{-1}X_{v_{12}}\right]^{-1}X_{v_{12}}^T = Y_v > 0,$$
$$S_h - S_{12}\left[S_{h_{12}}^T(S_h - Y_h)^{-1}S_{h_{12}}\right]^{-1}S_{h_{12}}^T = Y_h > 0,$$
$$S_v - S_{12}\left[S_{v_{12}}^T(S_v - Y_v)^{-1}S_{v_{12}}\right]^{-1}S_{v_{12}}^T = Y_v > 0.$$

Therefore, it is easy to see that $P_h > 0$, $P_v > 0$, $Q_h > 0$ and $Q_v > 0$. Now, pre- and post-multiplying (21) by $diag\{\bar{Y}, I, \bar{Y}, I, I, I\}$, we obtain

$$\begin{bmatrix} \bar{Y}(YA(\alpha)+A(\alpha)^TY+Y)\bar{Y} & \bar{Y}J_{12} & \bar{Y}YA_d(\alpha)\bar{Y} & \bar{Y}YA_d(\alpha) & \bar{Y}YB(\alpha) & \bar{Y}C(\alpha)^T - \bar{Y}YY_{12}C_f(\alpha)^T \\ * & J_{22} & J_{23}\bar{Y} & XA_d(\alpha)+X_{12}B_f(\alpha)C_{1d}(\alpha) & XB(\alpha)+\Psi(\alpha)D_1(\alpha) & C(\alpha)^T \\ * & * & -\bar{Y}Y\bar{Y} & -\bar{Y}Y & 0 & 0 \\ * & * & * & -S & 0 & 0 \\ * & * & * & * & -\gamma I & D(\alpha)^T \\ * & * & * & * & * & -\gamma I \end{bmatrix} < 0 \tag{34}$$

$$\begin{bmatrix} her(\Phi^T\Pi_1^T P\Phi\tilde{A}_f\Phi^T\Pi_1\Phi)+\Phi^T\Pi_1^T Q\Pi_1\Phi & \Phi^T\Pi_1^T P\Phi\tilde{A}_{df}\Phi^T\Pi_1\Phi & \Phi^T\Pi_1^T P\Phi\tilde{B}_f & \Phi^T\Pi_1^T\Phi C_f \\ * & -\Phi^T\Pi_1^T Q\Pi_1\Phi & 0 & 0 \\ * & * & -\gamma I & D^T \\ * & * & * & -\gamma I \end{bmatrix} < 0 \quad (35)$$

A_f, B_f and C_f are given in (24)–(26), Φ is given in (4). By (33), the inequality (34) can be rewritten as (35), Pre- and post-multiplying (35) by $diag(\Pi_1^{-T}\Phi^{-T},\Pi_1^{-T}\Phi^{-T},I,I)$ and $diag(\Phi^{-1}\Pi_1^{-1},\Phi^{-1}\Pi_1^{-1},I,I)$ we have

$$\begin{bmatrix} P\tilde{A}(\alpha)+\tilde{A}(\alpha)^T P+Q & * & * & * \\ \tilde{A}_d(\alpha)^T P & -Q & * & * \\ \tilde{B}(\alpha)^T P & 0 & -\gamma I & * \\ \tilde{C}(\alpha) & 0 & \tilde{D}(\alpha) & -\gamma I \end{bmatrix} < 0 \quad (36)$$

Finally, by Theorem 2, it follows that the error system (Σ_e) is asymptotically stable, and the transfer function of the error system satisfies (6). This completes the proof. □

Remark 1: From Theorem 2, it is easy to see that the minimal value of the H_∞ norm $\gamma > 0$, which, satisfies the LMIs in (21)-(23), can be determined by solving the following optimization problem :

$$\min_{S,X,Y,Z(\alpha),\Theta(\alpha),\Psi(\alpha)} \gamma$$

subject to

$S > 0, X > 0, Y > 0$ and LMIs in (21)-(23).

In the case when there is no parameter uncertainty and no delay in system (Σ), Theorem 2 reduces to Corollary 1 in [35].

3.2 HPPD filtering

In what follows, based on Theorem 2, we propose a new method for designing robust H_∞ filters via a structured polynomially parameter-dependent approach. Now before presenting the Theorem 2 in HPPD, some definitions and preliminaries are needed to represent and to handle products and sums of homogeneous polynomials. First, define the HPPD matrices of arbitrary degree *g* by

$$\Psi_g(\alpha) = \sum_{j=1}^{J(g)} \alpha_1^{k_1}\alpha_2^{k_2}...\alpha_N^{k_N}\Psi_{\Re_j}(\alpha) \quad (37)$$

$$\Theta_g(\alpha) = \sum_{j=1}^{J(g)} \alpha_1^{k_1}\alpha_2^{k_2}...\alpha_N^{k_N}\Theta_{\Re_j}(\alpha) \quad (38)$$

$$Z_g(\alpha) = \sum_{j=1}^{J(g)} \alpha_1^{k_1}\alpha_2^{k_2}...\alpha_N^{k_N}Z_{\Re_j}(\alpha) \quad (39)$$

with $k_1k_2...k_N = \Re_j(g)$

The notations in the above are explained as follows. $\alpha_1^{k_1}\alpha_2^{k_2}...\alpha_N^{k_N}$, $\alpha\in\Omega$, $k_i\in$ □, $i=1,...,N$ are the monomials, $\Psi_{\Re_j(g)}$, $\Theta_{\Re_j(g)}$, and $Z_{\Re_j(g)}$, are matrices valued coefficients. Here, by definition, $\Re_j(g)$ is the jth N-tuples of $\Re(g)$ which is lexically ordered, $j=1,...,\Im(g)$ and $\Re(g)$ is the set of N-tuples obtained as all possible combinations of $k_1k_2...k_N$, $k_i\in$ □, $i=1,...,N$ such that $k_1+k_2+...+k_N=g$. Since the number of vertices in the polytope $\mathcal{P}$ is equal to *N*, the number of elements in $\Re(g)$ is given by $\Im(g)=(N+g-1)!/(g!(N-1)!)$.

For each $i=1,...,N$ define the *N*-tuples $\Re_j^i(g)$, that are equal to $\Re_j(g)$, but with $k_i>0$ replaced by k_i-1. Note that the N-tuples $\Re_j^i(g)$ are defined only in the cases where the corresponding k_i is positive. Note also that, when applied to the elements of $\Re(g+1)$, the *N*-tuples $\Re_j^i(g+1)$ define subscripts $k_1k_2...k_N$ of matrices $\Psi_{k_1k_2...k_N}$, $\Theta_{k_1k_2...k_N}$ and $Z_{k_1k_2...k_N}$ associated to homogeneous polynomial parameter-dependent matrices of degree *g*. Finally, define the scalar constant coefficients $\beta_j^i(g+1)=g!/(k_1!k_2!...k_N!)$, with $[k_1,k_2,...,k_N]\in\Re_j^i(g+1)$.

To clarify this notation, consider as an example a polytope with N = 3 vertices and g = 2. Then, $J(2)=6$, $\Re(2)=\{002,011,020,101,110,200\}$ and

$$\Psi_2(\alpha)=\alpha_3^2\Psi_{002}+\alpha_2\alpha_3\Psi_{011}+\alpha_2^2\Psi_{020}+\alpha_1\alpha_3\Psi_{101} +\alpha_1\alpha_2\Psi_{110}+\alpha_1^2\Psi_{200}$$

$$\Theta_2(\alpha)=\alpha_3^2\Theta_{002}+\alpha_2\alpha_3\Theta_{011}+\alpha_2^2\Theta_{020}+\alpha_1\alpha_3\Theta_{101} +\alpha_1\alpha_2\Theta_{110}+\alpha_1^2\Theta_{200}$$

$$Z_2(\alpha)=\alpha_3^2Z_{002}+\alpha_2\alpha_3Z_{011}+\alpha_2^2Z_{020}+\alpha_1\alpha_3Z_{101} +\alpha_1\alpha_2Z_{110}+\alpha_1^2Z_{200}.$$

Moreover, $\mathrm{N}(2)=\{\{3\},\{2,3\},\{2\},\{1,3\},\{1,2\},\{1\}\}$, $\Re_1^3(2)=001$, $\Re_2^2(2)=001$, $\Re_2^3(2)=010$, $\Re_3^2(2)=010$, $\Re_4^1(2)=001$, $\Re_4^3(2)=100$, $\Re_5^1(2)=010$, $\Re_5^2(2)=100$ and $\Re_6^1(2)=100$ are the only possible triples $\Re_j^i(2)$, $j=1,...,\Im(2)$ associated to $\Re(2)$.

To facilitate the presentation of our main results, denote $\beta_j^i(g+1)$ by F. Using this notation we now present the following result.

Theorem 3: Given a scalar $\gamma > 0$ and the uncertain 2-D continuous system (Σ), then, the robust H_∞ filtering problem is solvable if there exist matrices $\Psi_{\Re_j(g)}$, $\Theta_{\Re_j(g)}$, $Z_{\Re_j(g)}$, $\Re_j(g) \in \Re(g)$, $j = 1,...,\Im(g)$, $X = diag(X_h, X_v) > 0$ and $Y = diag(Y_h, Y_v) > 0$ with $X_h, Y_h \in \mathbb{R}^{n_h}$ and $X_v, Y_v \in \mathbb{R}^{n_v}$, such that $\forall \Re_l(g+l) \in \Re(g+l)$, $l = 1,...,\Im(g+1)$ such that the following LMI holds :

$$\begin{bmatrix} \digamma YA_i + \digamma A_i^T Y + \digamma Y & J_{12} & \digamma YA_{di} & \digamma YA_{di} & \digamma YB_i & \digamma C_i^T - \Theta^T_{\Re^i_l(g+1)} \\ * & J_{22} & \digamma XA_{di} + \Psi_{\Re^i_{l(g+1)}} C_{1di} & \digamma XA_{di} + \Psi_{\Re^i_{l(g+1)}} C_{1di} & \digamma XB_i + \Psi_{\Re^i_l(g+1)} D_{1i} & \digamma C_i^T \\ * & * & -\digamma Y & -\digamma Y & 0 & 0 \\ * & * & * & -\digamma S & 0 & 0 \\ * & * & * & * & -\digamma \gamma I & \digamma D_i^T \\ * & * & * & * & * & -\digamma \gamma I \end{bmatrix} < 0 \qquad (40)$$

$$X - Y > 0 \qquad (41)$$

$$S - Y > 0 \qquad (42)$$

where

$$J_{12} = \digamma YA_i + \digamma A_i^T X + C_{1i}^T \Psi^T_{\Re^i_{l(g+1)}} + Z^T_{\Re^i_{l(g+1)}} + \digamma Y$$

$$J_{22} = \digamma XA_i + \digamma A_i^T X + C_{1i} \Psi_{\Re^i_{l(g+1)}} + \Psi^T_{\Re^i_{l(g+1)}} C_{1i}^T + \digamma S$$

then, the homogeneous polynomially parameter-dependent matrices given by (37)-(39) ensure (21)-(23) for all $\alpha \in \Omega$. Moreover, if the LMIs of (40)-(42) are fulfilled for a given degree g, then the LMIs corresponding to any degree $g > \hat{g}$ are also satisfied.

In this case, the matrices of the 2D continuous-time HPPD filter are given by

$$A_{fg}(\alpha) = \sum_{j=1}^{\Im(g)} \alpha^k A_{f\Re_j(g)} \qquad (43)$$

$$B_{fg}(\alpha) = \sum_{j=1}^{\Im(g)} \alpha^k B_{f\Re_j(g)} \qquad (44)$$

$$C_{fg}(\alpha) = \sum_{j=1}^{\Im(g)} \alpha^k C_{f\Re_j(g)} \qquad (45)$$

$$k_1 k_2 ... k_N = \Re_j(g), \quad \alpha^k = \alpha_1^{k_1} \alpha_2^{k_2} ... \alpha_N^{k_N} \qquad (46)$$

$$A_{f\Re_j(g)} = X_{12}^{-1} Z_{\Re_j(g)} Y^{-1} Y_{12}^{-T} \qquad (47)$$

$$B_{f\Re_j(g)} = X_{12}^{-1} \Psi_{\Re_j(g)} \qquad (48)$$

$$C_{f\Re_j(g)} = \Theta_{\Re_j(g)} Y^{-1} Y_{12}^{-T} \qquad (49)$$

Proof: Note that (21) for ($A(\alpha)$, $B(\alpha)$, $C(\alpha)$, $D(\alpha)$, $C_1(\alpha)$, $D_1(\alpha)$) $\in \mathcal{P}$ and $\Psi(\alpha)$, $\Theta(\alpha)$, $Z(\alpha)$ given by (40)-(42) are homogeneous polynomial matrices equations of degree g + 1 that can be written as

$$\sum_{l=1}^{J(g+1)} \alpha^k \left\{ \sum_{i \in N(g+1)} \begin{bmatrix} \digamma her(YA_i) + \digamma Y & J_{12} & \digamma YA_{di} & \digamma YA_{di} & \digamma YB_i & \digamma C_i^T - \Theta^T_{\Re^i_l(g+1)} \\ * & J_{22} & \digamma XA_{di} + \Psi_{\Re^i_{l(g+1)}} C_{1di} & \digamma XA_{di} + \Psi_{\Re^i_{l(g+1)}} C_{1di} & \digamma XB_i + \Psi_{\Re^i_l(g+1)} D_{1i} & \digamma C_i^T \\ * & * & -\digamma Y & -\digamma Y & 0 & 0 \\ * & * & * & -\digamma S & 0 & 0 \\ * & * & * & * & -\digamma \gamma I & \digamma D_i^T \\ * & * & * & * & * & -\digamma \gamma I \end{bmatrix} \right\} < 0$$

$$k_1 k_2, ..., k_N = \Re_l(g+1). \qquad (50)$$

Condition (40)-(42) imposed for all $l = 1,...,\Im(g+1)$ assure condition in (21) for all $\alpha \in \Omega$, and thus the first part is proved.

Suppose that the LMIs of (40)-(42) are fulfilled for a certain degree $\hat{g}$, that is, there exist $\Im(\hat{g})$ matrices $\Psi_{\Re_j(\hat{g})}$, $\Theta_{\Re_j(\hat{g})}$ and $\Psi_{\Re_j(\hat{g})}$, $j = 1,...,\Im(\hat{g})$ such that $\Psi_{\hat{g}}(\alpha)$, $\Theta_{\hat{g}}(\alpha)$ and $Z_{\hat{g}}(\alpha)$ are homogeneous polynomially parameter-dependent matrices assuring condition in (21)-(23). Then, the terms of the polynomial matrices $\Psi_{\hat{g}+1}(\alpha) = (\alpha_1 + ... + \alpha_N)\Psi_{\hat{g}}(\alpha)$, $\Theta_{\hat{g}+1}(\alpha) = (\alpha_1 + ... + \alpha_N)\Theta_{\hat{g}}(\alpha)$ and $Z_{\hat{g}+1}(\alpha) = (\alpha_1 + ... + \alpha_N) Z_{\hat{g}}(\alpha)$ satisfy the LMIs of Theorem 3 corresponding to the degree $\hat{g}+1$ which can be obtained in this case by linear combination of the LMIs of Theorem 3 for $\hat{g}$. □

4. ILLUSTRATIVE EXAMPLES

In this section, we provide some numerical examples to illustrate that the proposed approach ensures a smaller H_∞ performance when increasing the degree.

Example 1: First, consider an uncertain 2-D continuous system (Σ) with the following parameters:

$$A=\begin{bmatrix}-0.6 & 2\pm\alpha\\ -4 & -0.6\end{bmatrix},\quad A_d=\begin{bmatrix}0 & 0\\ 1.9\pm\alpha & 1.2\end{bmatrix},\quad B=\begin{bmatrix}0 & 0\\ 1.5 & 0\end{bmatrix},$$
$$C_1=\begin{bmatrix}0 & -1.2\\ 0.9 & 2\pm\beta\end{bmatrix},\ D_1=\begin{bmatrix}0.1 & 1.1\pm\alpha\\ 0.1 & 1.1\end{bmatrix},\quad C_{1d}=\begin{bmatrix}1.6 & 0.2\\ 0 & 0\end{bmatrix},$$
$$C=\begin{bmatrix}0 & 3\end{bmatrix},\ D=\begin{bmatrix}0 & 0\end{bmatrix}.$$

with α and β uncertain parameters, bounded as follows : $-1.2\le\alpha\le1.2$ and $-1.8\le\beta\le1.8$, which gives a four-vertices polytopic system.

To design a 2-D filter for this system, we apply Theorem 2, first with g = 0, (quadratic filtering), the LMIs are infeasible. Then, for g = 1 (linearly parameter-dependent approach), we get γ =27.0765, whereas for g = 2, we obtain a better noise attenuation level: γ =20.0570. The number of LMIs and the number of scalar variables are compared in Table 1.

Table 1

g	γ	K	L	Time
0	Infeasible	17	32	1.154
1	27.0765	47	74	1.575
2	20.0570	107	144	2.293
3	20.0570	207	249	3.760

K is the number of scalar variables, L is the number of LMI rows involved in the optimization problem, and the computational times is given in seconds.

Example 2: Taking the same parameters in example 1 except replacing A by

$$A=\begin{bmatrix}-0.6 & 4\pm\alpha\\ -4 & -0.6\end{bmatrix}\ \text{and}\ \begin{cases}-2.6\le\alpha\le2.6\\ -1.8\le\beta\le1.8\end{cases}$$

first for g = 0, (quadratic filtering), the LMIs are infeasible. Then, for g = 1 (linearly parameter-dependent approach), we get γ = 43.0885, whereas for g = 2, we obtain a better noise attenuation level: γ = 21.3200.

The number of LMIs and the number of scalar variables are compared in the following Table 2.

Table 2

g	γ	K	L	Time
0	Infeasible	17	32	1.154
1	43.0885	47	74	1.575
2	21.3200	107	144	2.293
3	21.3200	207	249	3.760

K is the number of scalar variables, L is the number of LMI rows involved in the optimization problem, and the computational times is given in seconds.

5. CONCLUSIONS

This paper has studied the robust H_∞ filtering problem for 2-D continuous systems described by Roesser state-space models with state delays and uncertainty of polytopic type. A design methodology has been proposed based on using homogeneous polynomially parameter-dependent matrices of arbitrary degree: with the increasing degree, the obtained H_∞ filter design is less conservative. Numerical examples illustrate the proposed methodology, showing that it is efficient for the design of parameter-dependent filters for this class of systems.

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