Markov Process for Service Facility systems with perishable inventory and analysis of a single server queue with reneging – Stochastic Model

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ABSTRACT
In this paper, we develop a supply network model for a service facility system with perishable inventory (on hand) by considering a two dimensional stochastic process of the form (L, X) = $\{(L(t), X(t)); t \geq 0\}$, where L (t) is the level of the on hand inventory and X (t) is the number of customers at time t. The inter-arrival time to the service station is assumed to be exponentially distributed with mean 1/λ. The service time for each customer is exponentially distributed with mean 1/ μ. The maximum inventory level is S and the maximum capacity of the waiting space is N. The replenishment process is assumed to be (S-1, S) with a replenishment of only one unit at any level of the inventory. Lead time is exponentially distributed with parameter β. The items are replenished at a rate of β whose mean replenishment time is 1/β. Item in inventory is perishable when it's utility drops to zero or the inventory item become worthless while in storage. Perishable of any item occurs at a rate of γ. Once entered a queue, the customer may choose to leave the queue at a rate of α if they have not been served after a certain time (reneging). The steady state probability distributions for the system states are obtained. A numerical example is provided to illustrate the method described in the model.

Keywords
Markov Process, Service facility system, stochastic model, Inventory control, Queue-inventory model, equilibrium distribution.

1. INTRODUCTION
The production-inventory systems or service-inventory systems can be described by queuing systems with inventory by considering the behavior of the customer in the queue and the perish nature of the product kept in stock. In a production/service-inventory model a queue may be formed at the service/production centre, because for such model each customer who succeeds to enter the system has a demand and satisfying each demand needs an on hand inventory and a process, where some amount of time is required for the process.
Even in a pure inventory system where there is no production, items in inventory require some time for retrieval, preparation, packing, and loading. Hence, it is reasonable to assume that every inventory system consists of a pure inventory section and a service/production facility section that can be properly modeled by a queuing system. A production/service-inventory system is in connection with the integrated supply chain management. Using queuing system with inventory every segment in a supply chain can be described. Recently many researchers have been done on production-inventory systems. As an early contribution to this field, Sigman and Simchi-Levi applied approximation procedures to find performance descriptions for an M/G/1 queue with limited inventory.
Berman, Kaplan and Shimshak (1993) [1] considered an inventory management system at a service facility which uses one item of inventory for each service provided by assuming that both demand and service rates are deterministic and constant and such queues are formed only during stock outs. They determined the optimal order quantity Q that minimizes the total cost rate. Berman and Kim (1999) [2] analyzed the situation in a stochastic environment where customers arrive at service facilities in a Poisson process and service times are exponentially distributed with mean inter-arrival times greater than the mean service time and each service require one item from inventory. A logically related model was studied by, He, Q-M, E.M. Jewes and Buzzacott, who analyzed a complete Markov production – inventory system, where demands arrive at a workshop and are processed one by one in order. Berman and Sapna (2000) [3] studied extensively an inventory control problem at a service facility system which uses one item of inventory for each service provided.

A continuous review perishable inventory system at service facilities was studied by Elango, (2002) [9]. The importance of inventory management for the quality of service of today's service system is generally accepted and optimization of system parameters such as inventory level and service rate in order to maximize quality of service is therefore a vital factor.

A few analytical models in this field have been developed up to now. He et al [12] derived optimal inventory policy for a make-to-order inventory-production system with Poisson arrival process of demands, exponentially distributed processing times and zero replenishment lead times of raw material. They used a Markov decision process approach to determine when and how much raw material should be ordered. Berman and Kim [4] considered a service system with an attached inventory, with Poisson customer arrival process, exponential service times and Erlang distribution of replenishment lead times. They formulated model as a Markov decision problem to characterize an optimal inventory policy as a monotonic threshold structure which minimizes system costs. Berman and Sapna [5] considered a similar model, with perishable products where replenishment is instantaneous and service times have general distribution. They derived optimal control of service rates which minimizes long-run average costs. As an extension of Berman and Kim [4], Berman and Kim [6] presented a similar model where revenue is generated upon the service. They found an optimal policy which maximizes the profit.

For the last two decades, research on service facility systems with inventory has found much attention. Mathematical methods associated with service facility system along with inventory are usually aggregation – disaggregation techniques or simulation or hybrid techniques. Analytical models are very rare. Schwarz and Daduna (2006) [20] have had a continuous review of M/M/1 queue with inventory along with back ordering in which the time between two successive orders follow an exponential distribution. Also the stationary distribution of the model has been processed in an explicit product form. Also Saffari & Haji (2009) [18] studied and applied the same procedure to a two-echelon supply chain. Later Saffari et al extended the above concept to several suppliers with lead time following a mixed exponential distribution.

In this article we consider a service facility system with perishable inventory for service completion in a queue having reneging behavior. The steady state probability distribution of the inventory level and queue size is obtained by using the Markov Decision process and the block transition matrix techniques. The content of the paper is presented as follows. Section 2 deals with the proposed model for service facility system and the notations used in the paper. The subsection of Section 2 deals with the analysis of the system and its solution procedure. In section 3 the equilibrium distribution of the system under consideration is discussed in detail. In section 4 a numerical example is provided to have a clear concept of the method analyzed. Section 5 provides a brief conclusion about the topic.

2. MODEL DESCRIPTION

Consider a service facility system in which inventory is maintained to perform service. The items are delivered to the demanding customers. The demand is for single item per customer. The maximum capacity of the inventory is S and the maximum number of customers allowed in the waiting space is N, i.e., an arriving customer who sees N customers already waiting in the system does not enter the system. The following assumptions are made:

- The time arrival of customer to the service station is a Poisson process with parameter λ (>0).
- The service time for each customer follows negative exponential distribution with parameter μ (>0).
- The replenishment process is (S-1, S) policy with exponential lead time having parameter β (>0).
- The items in inventory are perishable at a rate of γ (>0) when its utility drops to zero or the inventory items will become worthless while in storage.
- Once entered a queue, the customer may choose to leave the queue at a rate of α (>0) if their wait time is intolerable.
- Once entered a queue, the customer may choose to leave the queue at a rate of α (>0) if their wait time become intolerable.

2.1 Analysis

2.1.1 Notations

$E_1 = \{0, 1, 2, 3, \ldots, S\}$

$E_2 = \{0, 1, 2, \ldots, N\}$

$E = E_1 \times E_2$

λ – *arrival rate*

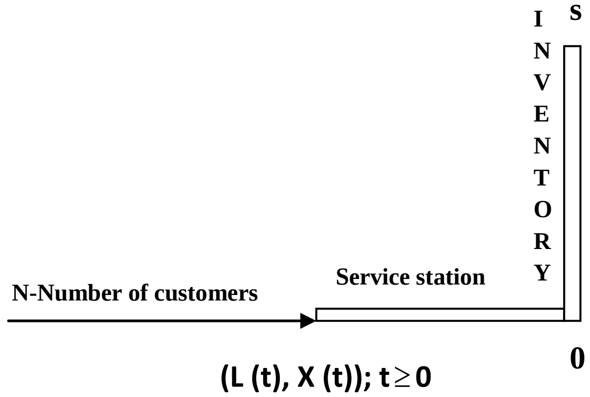
μ – *departure rate*

β – *replenishment rate*

γ – *perishable rate*

α – *reneging rate*

2.1.2 Steady state analysis



(L (t), X (t)); t ≥ 0

Let L (t) denote the inventory level and X (t), the number of customers (waiting and being served) in the system, at time t+. From the assumptions made on the input and output processes, it can be verified that (L, X) = $\{(L(t), X(t)); t \geq 0\}$ is a Markov process on the state space E. The infinitesimal generator of this process $A = ((a(i,q,j,r))),\ (i,q),\ (j,r) \in E$ can be obtained using the following arguments:

- The arrival of a customer makes a transition from (i, q) to (i, q + 1) with intensity of transition λ.
- Completion of service makes one customer leave the system and decrease the inventory level by 1. Thus a transition takes place from (i, q) to (i-1, q-1) with intensity of transition μ.
- Whenever an item in stock perishes, the transition takes place from (i, q) to (i-1, q) with intensity of transition γ.
- Whenever a customer reneges from the system, the transition takes place from (i, q) to (i, q-1) with intensity of transition α.
- If the number of customer is zero or one there is no reneging in the system.
- Whenever the vendor orders for the system, the transition takes place from (i, q) to (i + 1, q) with intensity of transition β.

$$Define\ \ A_{ij} = ((a(i, q, j, r)))\ ;\ q, r \in E_2\ ;\ i, j \in E_1 \quad where$$

$$a(i,q,j,r)=\begin{cases}\lambda,\ i=S,S-1,\ldots,1,0;\ q=0,1,\ldots,N-1;\ j=i;\ r=q+1;\\ \mu,\ i=S,S-1,\ldots,2,1;\ q=1,2,\ldots,N;\ j=i-1;\ r=q-1;\\ \beta,\ i=S-1,\ldots,1,0;\ q=0,1,\ldots,N;\ j=i+1;\ r=q;\\ \alpha,\ i=S,S-1,\ldots,1,0; q=2,\ldots,N; j=i; r=q-1;\\ \gamma,\ i=S,S-1,\ldots,1;\ q=0,1,2,\ldots,N;\ j=i-1;\ r=q;\\ -(\lambda+\beta),\ i=0;\ q=0,1;\ j=i;\ r=q;\\ -(\lambda+\gamma),\ i=S;\ q=0;\ j=i;\ r=q;\\ -(\beta+\alpha),\ i=0;\ q=N;\ j=i;\ r=q;\\ -(\gamma+\alpha+\mu),\ i=S;\ q=N;\ j=i;\ r=q;\\ -(\lambda+\gamma+\mu),\ \ i=S,\ \ q=1;\ \ j=i,\ \ r=q;\\ -(\lambda+\beta+\alpha);\ i=0,\ q=2,\ldots,N-1;\ j=i;\ r=q;\\ -(\lambda+\beta+\gamma),\ i=S-1,\ldots,2,1;\ q=0;\ j=i;\ r=q;\\ -(\lambda+\beta+\gamma+\mu),\ i=S-1,\ldots,1;\ q=1;\ j=i,\ \ r=q,\\ -(\beta+\gamma+\alpha+\mu),\ i=S-1,\ldots,2,1;\ q=N;\ j=i;\ r=q;\\ -(\lambda+\alpha+\mu+\gamma);\ i=S;\ q=2,\ldots,N-1;\ j=i;\ r=q;\\ -(\alpha+\beta+\gamma+\lambda+\mu);\ i=S-1,\ \ldots,2,1;\ q=2,\ldots,N-1;\\ \qquad\qquad j=i;\ r=q\,.\end{cases}$$

The infinitesimal generator A can be written in terms of sub matrices A $_{ij}$, namely A= $\left(\left(A_{ij}\right)\right)$ and is given by,

$$A_{ij}=\begin{cases}A_{S-i+1},\ j=i; i=0,1,2,\ldots,S-1,S;\\ B_{S-i+1}, i=0,1,\ 2,\ldots,\ S-1;\ j=i+1;\\ C_{S-j-1}, i=j+1;\ j=0,1,2,\ldots,S-1;\\ 0,\ otherwise\quad .\end{cases}\qquad \text{where}$$

A_0 *is given by*,

$$\begin{pmatrix}-(\lambda+\beta) & \lambda & 0 & 0 & 0 & 0\cdots & 0 & 0\\ 0 & -(\beta+\lambda) & \lambda & 0 & \vdots & \vdots & \vdots & 0\\ 0 & \alpha & -(\alpha+\beta+\lambda) & \lambda & 0 & \vdots & \vdots & 0\\ 0 & 0 & \alpha & \ddots & \ddots & \ddots & \vdots & \vdots\\ \vdots & \vdots & 0 & \ddots & \ddots & \lambda & 0 & 0\\ 0 & \vdots & \vdots & \ddots & \alpha & -(\alpha+\beta+\lambda) & \lambda & 0\\ 0 & \vdots & \vdots & \vdots & 0 & \alpha & -(\alpha+\beta+\lambda) & \lambda\\ 0 & 0 & \cdots & 0 & 0 & 0 & \alpha & -(\alpha+\beta+\lambda)\end{pmatrix};$$

A_i *for* $i=1,2,\ldots,S-1$ *is given by*,

$$\begin{pmatrix}-(\lambda+\beta+\gamma) & \lambda & 0 & 0 & \cdots & 0 & 0\\ 0 & -(\lambda+\beta+\mu+\gamma) & \lambda & 0 & \vdots & \vdots & 0\\ 0 & \alpha & \ddots & \ddots & \ddots & \vdots & \vdots\\ 0 & 0 & \ddots & \ddots & \ddots & 0 & 0\\ \vdots & \vdots & \ddots & \alpha & -(\lambda+\beta+\alpha+\mu+\gamma) & \lambda & 0\\ 0 & \vdots & \ddots & 0 & \ddots & \ddots & \lambda\\ 0 & 0 & \vdots & 0 & 0 & \alpha & -(\lambda+\beta+\alpha+\mu+\gamma)\end{pmatrix};$$

A_S *is given by*,

$$\begin{pmatrix}-(\lambda+\gamma) & \lambda & 0 & 0 & \cdots & 0 & 0\\ 0 & -(\lambda+\mu+\gamma) & \lambda & 0 & \vdots & \vdots & 0\\ 0 & \alpha & \ddots & \ddots & \ddots & \vdots & \vdots\\ 0 & 0 & \ddots & \ddots & \ddots & 0 & 0\\ \vdots & \vdots & \ddots & \alpha & -(\lambda+\alpha+\mu+\gamma) & \lambda & 0\\ 0 & \vdots & \ddots & 0 & \alpha & -(\lambda+\alpha+\mu+\gamma) & \lambda\\ 0 & 0 & \vdots & 0 & 0 & \alpha & -(\lambda+\alpha+\mu+\gamma)\end{pmatrix};$$

$A_{ij}=B_{S-i+1}$ *for* $i=1,2,\ldots,S-1,$ *and* $j=i+1$ *is given by*,

$$\begin{pmatrix}\beta & 0 & 0 & 0 & 0 & \ldots & 0 & 0\\ 0 & \beta & 0 & 0 & 0 & \ldots & 0 & 0\\ 0 & 0 & \beta & 0 & 0 & \ldots & 0 & 0\\ 0 & 0 & 0 & \ddots & \ddots & \ddots & \vdots & 0\\ 0 & 0 & \ddots & \ddots & \ddots & 0 & 0 & 0\\ \vdots & \vdots & \ddots & 0 & 0 & \beta & 0 & 0\\ 0 & 0 & \ldots & \ldots & 0 & 0 & \beta & 0\\ 0 & 0 & 0 & \ldots & 0 & 0 & 0 & \beta\end{pmatrix};$$

$A_{ij}=C_{S-j-1}$ *for* $j=0,1,2,\ldots,S-1,$ *and* $i=j+1$ *is given by*,

$$\begin{pmatrix}\gamma & 0 & 0 & 0 & 0 & \ldots & 0 & 0\\ \mu & \gamma & 0 & 0 & 0 & \ldots & 0 & 0\\ 0 & \mu & \gamma & 0 & 0 & \ldots & 0 & 0\\ 0 & 0 & \mu & \ddots & \ddots & \ddots & \vdots & 0\\ 0 & 0 & \ddots & \ddots & \ddots & 0 & 0 & 0\\ \vdots & \vdots & \ddots & 0 & \mu & \gamma & 0 & 0\\ 0 & 0 & \ldots & \ldots & 0 & \mu & \gamma & 0\\ 0 & 0 & 0 & \ldots & 0 & 0 & \mu & \gamma\end{pmatrix}.$$

3. EQUILIBRIUM DISTRIBUTION

The equilibrium distribution, π is the unique solution to the equations π Q=0 and π e=1, where Q is the transition rate matrix obtained in the steady state analysis and e is the column vector with all the components equal to 1.

From π Q =0 we have, the following steady state equations using which we can find out the steady state probabilities $\left(\pi^{(i,q)}\right)$ where i = 0, 1, …, S and q = 0, 1, …, N respectively.

$$For \quad i=0;\; q=0\,;$$
$$-(\lambda+\beta)\,\pi^{(i,\,q)}+\gamma\,\pi^{(i+1,\,q)}+\mu\,\pi^{(i+1,\,q+1)}=0.$$

$$For \quad i=0;\; q=1;$$
$$\lambda\,\pi^{(i,\,q-1)}-(\lambda+\beta)\,\pi^{(i,\,q)}+\alpha\,\pi^{(i,\,q+1)}+\gamma\,\pi^{(i+1,\,q)}+\mu\,\pi^{(i+1,\,q+1)}=0.$$

$$For \quad i=0,\quad q=2,3,\cdots,N-1;$$
$$\lambda\,\pi^{(i,\,q-1)}-(\lambda+\alpha+\beta)\,\pi^{(i,\,q)}+\alpha\,\pi^{(i,\,q+1)}+\gamma\,\pi^{(i+1,\,q)}+\mu\,\pi^{(i+1,\,q+1)}=0.$$

$$For \quad i=0;\; q=N;$$
$$\lambda\,\pi^{(i,\,q-1)}-(\beta+\alpha)\,\pi^{(i,\,q)}+\gamma\,\pi^{(i+1,\,q)}=0.$$

$$For \quad i=1,\ldots S-1;\; q=0;$$
$$\beta\,\pi^{(i-1,\,q)}-(\lambda+\beta+\gamma)\,\pi^{(i,\,q)}+\gamma\,\pi^{(i+1,\,q)}+\mu\,\pi^{(i+1,\,q)}=0.$$

$$For \quad i=1,\ldots S-1;\; q=1;$$
$$\beta\,\pi^{(i-1,\,q)}+\lambda\,\pi^{(i,\,q-1)}-(\beta+\gamma+\lambda+\mu)\,\pi^{(i,\,q)}+\alpha\,\pi^{(i,\,q+1)}+\gamma\,\pi^{(i+1,\,q)}+\mu\,\pi^{(i+1,\,q+1)}=0.$$

$$For \quad i=1,\ldots S-1;\; q=2,\ldots,N;$$
$$\beta\,\pi^{(i-1,\,q)}+\lambda\,\pi^{(i,\,q-1)}-(\alpha+\beta+\gamma+\mu+\lambda)\,\pi^{(i,\,q)}+\alpha\,\pi^{(i,\;q+1)}+\gamma\,\pi^{(i+1,\,q)}+\mu\,\pi^{(i+1,\,q+1)}=0.$$

$$For \quad i=1,\ldots,S-1;\; q=N;$$
$$\beta\,\pi^{(i-1,\,q)}+\lambda\,\pi^{(i,\,q-1)}-(\alpha+\beta+\gamma+\mu\quad)\,\pi^{(i,\,q)}+\gamma\,\pi^{(i,\,q)}=0.$$

$$For \quad i=S,\quad q=0\,;$$
$$\beta\,\pi^{(i-1,\,q)}-(\lambda+\gamma)\,\pi^{(i,\,q)}=0.$$

$$For \quad i=S;\; q=1;$$
$$\beta\,\pi^{(i-1,\,q)}+\lambda\,\pi^{(i,\,q-1)}-(\lambda+\gamma+\mu)\,\pi^{(i,\;q)}+\alpha\,\pi^{(i,\;q+1)}=0.$$

$$For \quad i=S;\; q=2,\ldots,N-1;$$
$$\beta\,\pi^{(i-1,\,q)}+\lambda\,\pi^{(i,\,q-1)}-(\lambda+\alpha+\gamma+\mu)\,\pi^{(i,\,q)}+\alpha\,\pi^{(i,\,q+1)}=0.$$

$$For \quad i=S;\, q=N;$$
$$\beta\,\pi^{(i-1,\,q)}+\lambda\,\pi^{(i,\,q-1)}-(\alpha+\gamma+\mu)\,\pi^{(i,\,q)}=0.$$

Solving π e=1, and all the above equations from π Q =0, the steady state probabilities of the service facility system can be obtained. The equilibrium distribution of the state space (i, q) where i = 0, 1, 2 …, S; q=0, 1, 2... N is given by

$$\left(\pi^{(i,q)}\right)_{i=0,1,\ldots,S\,;\,q=0,1,\ldots,N}\,.$$

4. NUMERICAL EXAMPLE

Consider a service facility system with S=2, N=3 whose arrival and service rate are λ & μ respectively. The replenishment rate, the perishable rate and the service rate are considered as $\beta,\gamma,$ *and* μ respectively.

4.1 Transition matrix

The transition matrix for a service facility system whose maximum inventory level is 2 and the maximum capacity is 3 is as follows. The infinitesimal generator A can be written in terms of sub matrices A_{ij} , namely A= $\left(\left(A_{ij}\right)\right)$ and is given by

$$A_{ij}=\begin{cases} A_{2-i+1},\; j=i;\, i=0,1,2; \\ B_{2-i+1},\; i=0,1;\; j=i+1; \\ C_{2-j-1},\; i=j+1;\; j=0,1; \\ 0,\; otherwise \quad . \end{cases}$$

$$Hence\;\; A=\begin{pmatrix} A_0 & B_1 & O \\ C_1 & A_1 & B_0 \\ O & C_0 & A_2 \end{pmatrix},\quad where$$

$$A_0=\begin{pmatrix} -(\lambda+\beta) & \lambda & 0 & 0 \\ 0 & -(\lambda+\beta) & \lambda & 0 \\ 0 & \alpha & -(\alpha+\lambda+\beta) & \lambda \\ 0 & 0 & \alpha & -(\alpha+\beta) \end{pmatrix};$$

$$A_1=\begin{pmatrix} -(\gamma+\lambda+\beta) & \lambda & 0 & 0 \\ \alpha & -(\mu+\gamma+\lambda+\beta) & \lambda & 0 \\ 0 & \alpha & -(\alpha+\lambda+\beta) & \lambda \\ 0 & 0 & \alpha & -(\gamma+\alpha+\mu+\beta) \end{pmatrix};$$

$$A_2=\begin{pmatrix} -(\gamma+\lambda & \lambda & 0 & 0 \\ \alpha & -(\gamma+\lambda+\mu) & \lambda & 0 \\ 0 & \alpha & -(\mu+\lambda+\alpha+\gamma) & \lambda \\ 0 & 0 & \alpha & -(\alpha+\gamma+\mu) \end{pmatrix};$$

$$B_0=\begin{pmatrix} \beta & 0 & 0 & 0 \\ 0 & \beta & 0 & 0 \\ 0 & 0 & \beta & 0 \\ 0 & 0 & 0 & \beta \end{pmatrix};$$

$$B_1=\begin{pmatrix} \beta & 0 & 0 & 0 \\ 0 & \beta & 0 & 0 \\ 0 & 0 & \beta & 0 \\ 0 & 0 & 0 & \beta \end{pmatrix};$$

$$C_0=\begin{pmatrix} \gamma & 0 & 0 & 0 \\ \mu & \gamma & 0 & 0 \\ 0 & \mu & \gamma & 0 \\ 0 & 0 & \mu & \gamma \end{pmatrix};$$

$$C_1 = \begin{pmatrix} \gamma & 0 & 0 & 0 \\ \mu & \gamma & 0 & 0 \\ 0 & \mu & \gamma & 0 \\ 0 & 0 & \mu & \gamma \end{pmatrix}; \quad \&$$

$$O = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} .$$

4.2 Equilibrium distribution

The equilibrium distribution, π is obtained using the equations π Q=0 and π e=1, where Q is the block transition rate matrix given by A and e is the column vector with all the components equal to 1.

From π Q =0 we have,

$$-(\lambda+\beta)\,\pi^{(0,0)} + \gamma\,\pi^{(1,0)} + \mu\,\pi^{(1,1)} = 0;$$

$$\lambda\,\pi^{(0,0)} - (\lambda+\beta)\,\pi^{(0,1)} + \alpha\,\pi^{(0,2)} + \gamma\,\pi^{(1,1)} + \mu\,\pi^{(1,2)} = 0;$$

$$\lambda\,\pi^{(0,1)} - (\lambda+\alpha+\beta)\,\pi^{(0,2)} + \alpha\,\pi^{(0,3)} + \gamma\,\pi^{(1,2)} + \mu\,\pi^{(1,3)} = 0;$$

$$\lambda\,\pi^{(0,2)} - (\beta+\alpha)\,\pi^{(0,3)} + \gamma\,\pi^{(1,3)} = 0;$$

$$\beta\,\pi^{(0,0)} - (\lambda+\beta+\gamma)\,\pi^{(1,0)} + \gamma\,\pi^{(2,0)} + \mu\,\pi^{(2,1)} = 0;$$

$$\beta\,\pi^{(0,1)} + \lambda\,\pi^{(1,0)} - (\beta+\gamma+\lambda+\mu)\,\pi^{(1,1)} + \alpha\,\pi^{(1,2)} + \gamma\,\pi^{(2,1)} + \mu\,\pi^{(2,2)} = 0;$$

$$\beta\,\pi^{(0,2)} + \lambda\,\pi^{(1,1)} - (\alpha+\beta+\gamma+\mu+\lambda)\,\pi^{(1,2)} + \alpha\,\pi^{(1,3)} + \gamma\,\pi^{(2,2)} + \mu\,\pi^{(2,3)} = 0;$$

$$\beta\,\pi^{(0,3)} + \lambda\,\pi^{(0,2)} - (\alpha+\beta+\gamma+\mu)\,\pi^{(0,3)} + \gamma\,\pi^{(1,3)} = 0;$$

$$\beta\,\pi^{(1,0)} - (\lambda+\gamma)\,\pi^{(2,0)} = 0;$$

$$\beta\,\pi^{(1,1)} + \lambda\,\pi^{(2,0)} - (\lambda+\gamma+\mu)\,\pi^{(2,1)} + \alpha\,\pi^{(2,2)} = 0;$$

$$\beta\,\pi^{(1,2)} + \lambda\,\pi^{(2,1)} - (\lambda+\alpha+\gamma+\mu)\,\pi^{(2,2)} + \alpha\,\pi^{(2,3)} = 0;$$

$$\beta\,\pi^{(1,3)} + \lambda\,\pi^{(2,2)} - (\alpha+\gamma+\mu)\,\pi^{(2,2)} = 0.$$

The stationary distribution vector can be found by the phase of the Markov Chain Reduction Principle (MCRP). The above equations were solved to find the equilibrium distribution.

5. CONCLUSIONS

In this work we analyzed a new single server queue with reneging accomplished by a service facility system having a perishable inventory. The proposed method for the steady state analysis and an equilibrium distribution employs the Markov Decision process with Markov Chain reduction principle using block transition matrix. In this work the replenishment is done using ((S-1, S) policy). Also as the stock level does not increase beyond the warehouse capacity, the need for costly emergency storage is eliminated and facilitates capacity planning at the system centre. This simplifies the system and reduces it logically equivalent to a queuing system. Future directions for research are wide open. However an exact optimal cost analysis of the service facility system is challenging. Another direction of research is the analysis and optimization of systems with general distributions of inter-arrival and processing times.

6. ACKNOWLEDGEMENTS

We have considered a service facility system with perishable inventory and analysis of a single server queue with reneging as a stochastic model. Markov Process is the procedure used to get the transition rate matrix followed by steady state equations in order to get the steady state probabilities of the service facility system under consideration. To the best of our knowledge the Procedure of Markov process is applied everywhere to control the perishable inventory in service facility and reneging under queuing. Our thanks to the experts who have contributed towards Service facility system, Markov process, Supply chain management and single server queuing model for the development of this article.

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