Optimal Ordering policies of a Two Echelon Supply Chain for Deteriorating Items

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ABSTRACT
Supply chain management is a crucial task of managing large organizations. In a decentralized supply chain each member focuses on maximizing his own profit. As a result of it, the conflict between the manufacturer and the retailers will arise. To avoid this sort of situations, coordination model strike a balancing between the profit of manufacturers and retailers. This paper investigates a two echelon supply chain system which consisting of one manufacturer and multiple retailers. Using the mathematical modeling a coordination model which maximizes the total profit is developed and analyzed for deteriorating items. The optimal pricing and ordering policies of the model are also derived. A sensitivity analysis with respect to the parameters and costs is also presented. This model lower down the total cost of supply chain and increases the general profit. It also improves cooperation for both manufacturer and retailer.

Key words

Supply chain, Coordination Model, EOQ, EPQ.

1. INTRODUCTION

In this present competitive business environment, Supply Chain Management plays a dominant role due to its ready applicability in many practical situations arising at places like production processes, ware houses, market yards etc.,

In manufacturing and production processes the inventory control is very important. Hence several inventory models have been developed and analyzed independently for manufactures and retailers through EPQ and EOQ models respectively. These models are widely used for several inventory systems if we consider the retailer's inventory is independent of manufacturers inventory Goyal & Giri (2001), Abdullah Eroglu (2007), J. Gutierrez (2008).

In developing these EOQ or EPQ models the life time of the commodity is considered to be very important. Different inventory models for deteriorating items for single echelon are developed by various researchers. However, in a decentralized supply chain if each member focuses on maximizing his own profit it will conflict the efficiency of the supply chain since the inventory levels of retailers and the producers have interdependence. Taking this into consideration, coordination models of supply chain are developed. Recently to utilize the resources more effectively Cachon G.P. (2002) has reviewed on setting supply chain coordinating contracts.

In coordination models for supply chain vendor managed inventory strategy, quantity flexibility scheme, discount scheme, return policies etc., became popular (Xuxia Zou,Shaligram pokharel (2008),Cachon.G.P (2004), Weng.Z. K,Wong. R.T (1993), Zhao Quanwu (2005),Chung-Chi Hsieh (2008) recently Liao Li,Wu Yaohua(2008) developed a coordination model of supply chain for deteriorating items using price discount policy and Yu, Y., Huang, G.Q., & Liang, L. (2009), Yao, Y., Dong, Y., & Dresner, M. (2010) developed supply chain under vendor managed inventory. They assumed that the rate of deterioration is constant and demand is also constant for retailers. However, in many practical situations dealing with food processing industries the demand is a function of time at retailer's level. The influence of time on demand can be characterized through a power pattern.

The power pattern demand includes several types of demand including constant rate, increasing and decreasing rates depending up on the pattern index. Hence, in this paper a two echelon supply chain model is developed with the assumption that there is one manufacturer and multiple retailers. By maximizing the system total profit under coordination of manufacturer and retailers through the price discount model proposed by Abdullah Eroglu (2007). Here it is assumed that the demand rate at retailer's level is a function of time and follows a power pattern. It is also further assumed that the life time of commodity is random and having exponential distribution. The optimal operating policies of the Supply Chain are derived and analyzed.

2. ASSUMPTIONS AND NOTATIONS

The mathematical model is developed based on the following assumptions:

i) One manufacturer and n^1 retailer are considered for a single product, and the retailers have the same characteristics.

ii) The manufacture production is a typical Make to order production mode.

iii) The demand rate at any time "t" is

$$\frac{rt^{\frac{1}{n}-1}}{nT^{\frac{1}{n}}}.$$

Where 'r' is the fixed quantity, n is the parameter of power demand pattern, the value of n may be any positive number. T is the planning horizon.

iv) Shortages are allowed for the retailers, and the unsatisfied demand (due to shortages) is completely backlogged.

v) Replenishments are instantaneous, and the lead-time is assumed to be negligible.

vi) In the retailer's on-hand inventory, deterioration occurs once the item is bought. Deterioration rate is a known constant, and the deteriorated units are not replaced.

vii) The manufacture production rate is finite and constant. It is unaffected by the lot size.

viii) Shortage is not allowed for the manufacture. Therefore manufacturer's production rate is greater than the demand rate and the number of units deteriorated per unit time.

ix) For the retailers, the inventory holding cost per unit per unit time, the ordering cost per replenishment, the disposal cost of amelioration per unit, the shortage cost per unit, the purchase cost per unit are known and constant. For the manufacture, the holding cost per unit per unit time, the setup cost per replenishment, and the item cost per unit are known and constant.

We use the following notation throughout the paper:

n^1 number of retails

θ The constant deterioration rate, $0 < \theta < 1$

k Price discounting coefficient

Manufacturer's production rate,

$P_s > n^1\left(\frac{rt^{\frac{1}{n}-1}}{nT^{\frac{1}{n}}}\right)$

C_m Manufacturer's production cost per unit

B Transaction price per unit between Retailer and Consumers

A Transaction price per unit between manufacturer and retailers without cooperation, $C_m \leq A \leq B$

P Transaction price per unit between manufacturer and retailers under coordination, $C_m \leq P \leq B$

h_r Retailer's inventory holding cost per unit per unit time

h_s Manufacturer's inventory holding cost per unit per unit time

C_s Manufacturer's setup cost per order Cycle

C_{rl} Retailer's backlog cost per unit

C_{rd} Retailer's disposal cost for Deteriorating items per unit

C_{ro} Retailer's ordering cost per order cycle

T Length of the retailer's replenishment cycle

T_s Length of the manufacturer's production cycle

I(t) Retailer's finished goods inventory level at any time t

t_1 Time of the inventory level decreased to zero, $t_1 \in [0,T]$

t_s Length of the manufacturer's production period

n_s The manufacturer's ordering times during a production period

S Retailer's inventory shortage quantity during an ordering cycle

Q_r Retail's Order quantity

Q_s Manufacturer's production quantity

Q_{max} Retailer's maximum inventory Level

3. COORDINATION MODEL

3.1 Retailers Inventory Model

In this section we develop the coordination model of the two echelon supply chain based on price discounts. For obtaining the coordination model, we first derive the retailers profit in a replenishment cycle utilizing the retailer's inventory model. In the retailers inventory model we assume that there is an initial replenishment and the inventory level reaches max. Level at t=0.

During the period 0 to t_1 the inventory decreases due to deterioration and demand and reaches to zero at time $t=t_1$. During the period t_1 to T there is a '-ve' inventory due to shortages since shortages are allowed and fully back logged. The schematic diagram representing the Inventory level I(t) is shown in figure 1.

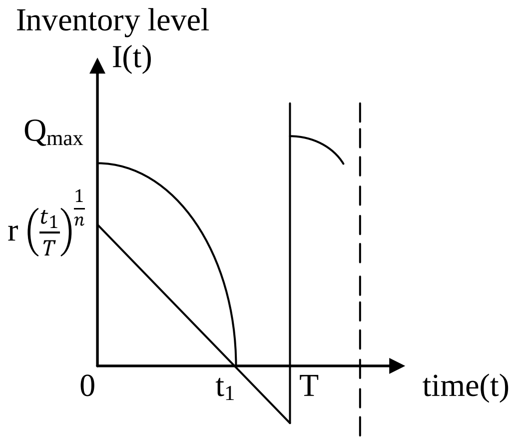

Fig.1 The retailer's inventory level

With these considerations the differential equations governing the inventory level at time t are.

$$\frac{dI(t)}{dt} = -\theta I(t) - \frac{rt^{\frac{1}{n}-1}}{nT^{\frac{1}{n}}}\ for\ t\epsilon[0,t_1] \quad (1)$$

$$\frac{dI(t)}{dt} = -\frac{rt^{\frac{1}{n}-1}}{nT^{\frac{1}{n}}} for\ t\epsilon[t_1,T] \quad (2)$$

With the boundary condition $I(t_1)=0$ solving the differential equations(1) and (2),

We get

$$I(t) = \frac{r}{e^{\theta t}T^{1/n}}\left[\left(t_1^{\frac{1}{n}} - t^{\frac{1}{n}}\right) + \frac{\theta}{n+1}\left(t_1^{\frac{1}{n}+1} - t^{\frac{1}{n}+1}\right)\right], t\epsilon[0,t_1] \quad (3)$$

$$I(t) = -\frac{r}{T^{\frac{1}{n}}}\left[t^{\frac{1}{n}} - t_1^{\frac{1}{n}}\right],\ t\epsilon[t_1,T] \quad (4)$$

From figure 1. $Q_{max} = I(0)$, the maximum inventory level is

$$Q_{max} = I(t)_{t=0} = \frac{r}{T^{\frac{1}{n}}}\left[t_1^{\frac{1}{n}} + \frac{\theta}{n+1}\left(t_1^{\frac{1}{n}+1}\right)\right] \quad (5)$$

The shortage quantity at interval $[t_1, T]$ is

$$S = \frac{r}{T^{\frac{1}{n}}}\left[T^{\frac{1}{n}} - t_1^{\frac{1}{n}}\right] \quad (6)$$

From Equations(5) and (6) we have the order quantity in the replenishment cycle as

$$Q_r = Q_{max} + S = \frac{r}{T^{\frac{1}{n}}}\left[\frac{\theta}{n+1}\left(t_1^{\frac{1}{n}+1}\right) + T^{\frac{1}{n}}\right] \quad (7)$$

The total cost for the retail during the replenishment cycle consists of the ordering cost (C_o), the backlog cost (C_s), the holding cost (C_h), the purchase cost (CP) and the disposal cost (C_d). The retails revenue is denoted by C_g.

1. There is an initial replenishment at the start of the cycle then the ordering cost is

$$C_o = C_{ro} \quad (8)$$

2. The backlog cost during the replenishment cycle is

$$C_s = C_{rl}\int_{t_1}^{T} I(t)dt$$

$$= \frac{C_{rl}\, r}{T^{\frac{1}{n}}}\left[\frac{n}{n+1}\left(T^{\frac{1}{n}+1} - t_1^{\frac{1}{n}+1}\right) + t_1^{\frac{1}{n}}(t_1 - T)\right] \quad (9)$$

3. From fig 1 inventory occurs at interval$[0,t_1]$.The holding cost is

$$C_h = h_r\int_0^{t_1} I(t)dt$$

$$= \frac{h_r r}{T^{\frac{1}{n}}}\left[\frac{t_1^{\frac{1}{n}+1}}{n+1} + \frac{\theta}{2(2n+1)}t_1^{\frac{1}{n}+2}\right] \quad (10)$$

4. The purchase cost during the replenishment cycle is

$$C_p = A\,(Q_{max}+S)$$

$$= \frac{A\, r}{T^{\frac{1}{n}}}\left[\frac{\theta}{n+1}t_1^{\frac{1}{n}+1} + T^{\frac{1}{n}}\right] \quad (11)$$

5. The disposal cost for deteriorating units

is

$$C_d = C_{rd}\left(Q_{max} - \int_0^{t_1}\frac{rt^{\frac{1}{n}-1}}{nT^{\frac{1}{n}}}dt\right)$$

$$= \frac{C_{rd}\, r}{T^{\frac{1}{n}}}\left[\frac{\theta}{n+1}t_1^{\frac{1}{n}+1}\right] \quad (12)$$

6. The retails revenue is

$$C_g = B\int_0^{T}\frac{rt^{\frac{1}{n}-1}}{nT^{\frac{1}{n}}}\,dt = Br \quad (13)$$

So, the retailers profit in a replenishment cycle is

$$TCR = C_g - (C_o + C_s + C_h + C_p + C_d)$$

$$TCR = Br - \left[C_{ro} + \frac{C_{r1} r}{T^{\frac{1}{n}}}\left\{\frac{n}{n+1}\left(T^{\frac{1}{n}+1} - t_1^{\frac{1}{n}+1}\right) + t_1^{\frac{1}{n}}(t_1 - T)\right\} + \frac{h_r r}{T^{\frac{1}{n}}}\left\{\frac{t_1^{\frac{1}{n}+1}}{n+1} + \frac{\theta}{2(2n+1)}t_1^{\frac{1}{n}+2}\right\} + \frac{A\, r}{T^{\frac{1}{n}}}\left\{\frac{\theta}{n+1}t_1^{\frac{1}{n}+1} + T^{\frac{1}{n}}\right\} + \frac{C_{rd}\, r}{T^{\frac{1}{n}}}\left\{\frac{\theta}{n+1}t_1^{\frac{1}{n}+1}\right\}\right] \quad (14)$$

The necessary conditions for optimality

$$\frac{\partial(TCR)}{\partial t_1} = 0, \frac{\partial(TCR)}{\partial T} = 0$$

$$\frac{\partial(TCR)}{\partial t_1} = C_{rl}\left[1 - \frac{T}{t_1}\right] + h_r\left[1 + \frac{\theta}{2}t_1\right] + \theta(A + C_{rd}) = 0 \quad (15)$$

$$\frac{\partial(TCR)}{\partial T} = C_{rl}\, r\left\{\frac{-n^2}{(n+1)}T^{\frac{1}{n}+1} + \frac{1}{n+1}t_1^{\frac{1}{n}+1} + t_1^{\frac{1}{n}}(n-1)T\right\} + h_r\left(\frac{t_1^{\frac{1}{n}+1}}{n+1} + \frac{\theta}{2(2n+1)}t_1^{\frac{1}{n}+2}\right) + (A + C_{rd})\frac{\theta}{n+1}t_1^{\frac{1}{n}+1} = 0 \quad (16)$$

By solving simultaneous equations (15) and (16) we get

t_1 and T.

The retailers economic order quantity for maximum profit is

$$Q_r^1 = \frac{r}{T^{\frac{1}{n}}}\left[\frac{\theta}{n+1}\left(t_1^{\frac{1}{n}+1}\right) + T^{\frac{1}{n}}\right] \quad (17)$$

Hence the retailer's maximum profit is

$$TCR^{*} = n^1\left\{Br - \left[C_{ro} + \frac{C_{rl}}{T^{1\frac{1}{n}}}\left\{\frac{n}{n+1}\left(T^{\frac{1}{n}+1} - t_1^{\frac{1}{n}+1}\right) + t_1^{\frac{1}{n}}(t_1 - T)\right\} + \frac{h_r r}{T^{\frac{1}{n}}}\left\{\frac{t_1^{\frac{1}{n}+1}}{n+1} + \frac{\theta}{2(2n+1)}t_1^{\frac{1}{n}+2}\right\} + \frac{Ar}{T^{\frac{1}{n}}}\left\{\frac{\theta}{n+1}t_1^{\frac{1}{n}+1} + T^{\frac{1}{n}}\right\} + \frac{C_{rd}\, r}{T^{\frac{1}{n}}}\left\{\frac{\theta}{n+1}t_1^{\frac{1}{n}+1}\right\}\right]\right\} \quad (18)$$

3.2 Manufacturer's Inventory Model

The manufacturer's inventory model is developed based on retailer's orders. Assuming that there is no deterioration at manufacturer's inventory level. The manufacturer's optimum profit is derived through the setup cost, holding cost and production cost.

The production lot-size per cycle is

$$T_s = \frac{TQ_s}{n^1 Q_r}$$

Production time during period T_s is

$$t_s = \frac{Q_s}{P_s}$$

Ordering times is

$$n_s = \frac{n^1 Q_r}{Q_s}$$

Average inventory level is

$$\frac{t_s}{2T_s}Q_s = \frac{n^1 Q_r Q_s}{2P_s}$$

Hence the manufacture's optimum profit is

$$TCM = (A - C_m)n^1 Q_r - h_s\frac{n^1 Q_r Q_s}{2TP_s} - C_s\frac{n^1 Q_r}{Q_s} \quad (19)$$

Let

$\frac{d(TCM)}{dQ_s} = 0$, we have the production lot-size for Maximum Avenue

$$Q_s^1 = \sqrt{\frac{2TC_sP_s}{h_s}} \quad (20)$$

So the optimum order times is

$$n_s = \frac{n^1 Q_r}{Q_s^1} = n^1 Q_r\sqrt{\frac{hs}{2TCsPs}} \quad (21)$$

Then the manufacture's maximum profit is

$$TCM^{*} = n^1 Q_r\left[(A - c_m) - \sqrt{\frac{2C_s h_s}{TP_s}}\right] \quad (22)$$

With the retailers inventory model and the manufacturers inventory models discussed in sections 3.1 and 3.2, we have the total revenue of the supply chain under decentralized decision is

$$\emptyset_1=TCR^*+TCM^*=n^1\left\{Br-\left[C_{ro}+\frac{C_{rl}}{T^{\frac{1}{n}}}\left\{\frac{n}{n+1}\left(T^{\frac{1}{n}+1}-t_1^{\frac{1}{n}+1}\right)+t_1^{\frac{1}{n}}(t_1-T)\right\}+\frac{h_r r}{T^{\frac{1}{n}}}\left\{\frac{t_1^{\frac{1}{n}+1}}{n+1}+\frac{\theta}{2(2n+1)}t_1^{\frac{1}{n}+2}\right\}+\frac{Ar}{T^{\frac{1}{n}}}\left\{\frac{\theta}{n+1}t_1^{\frac{1}{n}+1}+T^{\frac{1}{n}}\right\}+\frac{C_{rd}\,r}{T^{\frac{1}{n}}}\left\{\frac{\theta}{n+1}t_1^{\frac{1}{n}+1}\right\}\right]\right\}+n^1Q_r \qquad (23)$$

3.3 Lot sizing Coordination Model Based on Price Discount

In the equation (23) it is observed that this maximum revenue of the supply chain does not satisfy both retailer and manufactures, since there is no coordination between the two and both TCR and TCM are optimized separately.

But

$Q_s^1 \neq n^1Q_r^1$ in general as a result of it the manufacturers cost will greater and profit will less than that of the optimum lot size point due to order production mode. But in price discount strategy of the supply chain the coordination between the manufacturer and the retailer is done when the manufacturer stimulate the retailers to order more close to the Q_s^1 derived in production inventory model by offering price discounts. In this paper we propose to utilize the

$$P=A-k\,\frac{Q_r-Q_r^1}{Q_s^1-n^1Q_r^1},k>0 \qquad (24)$$

This model considers both quantity increased discount and quantity decreased one, and reduce the retailer's benefit when the retailer makes a false report of Q_r^1 on the condition of information dissymmetry.

Further, at the same time the manufacture obtains the optimum profit, it is necessary to make retailers cooperate with pleasure that the retailer's profit should not less than that of noncooperation condition, and the retailers cost does not increase.

The coordination objective here is to maximize the total profit of supply chain, by determining the discount coefficient k, order lot-size Q_r and production lot-size Q_s. Equation, (7) indicates that the value of Q_r is a function of variable t_1, and Eqn. (20) is a function of variable T. Hence the mathematical model for inventory-production system is presented below.

$$\emptyset_2=max\left\{n^1Br-n^1\left[C_{r0}+\frac{C_{rl}r}{T^{\frac{1}{n}}}\left\{\frac{n}{n+1}\left(T^{\frac{1}{n}+1}-t_1^{\frac{1}{n}+1}\right)+t_1^{\frac{1}{n}}(t_1-T)\right\}+\frac{h_r r}{T^{\frac{1}{n}}}\left\{\frac{t_1^{\frac{1}{n}+1}}{n+1}+\frac{\theta}{2(2n+1)}t_1^{\frac{1}{n}+2}\right\}+\frac{Pr}{T^{\frac{1}{n}}}\left\{\frac{\theta}{n+1}t_1^{\frac{1}{n}+1}+T^{\frac{1}{n}}\right\}+\frac{C_{rd}\,r}{T^{\frac{1}{n}}}\left\{\frac{\theta}{n+1}t_1^{\frac{1}{n}+1}\right\}\right]+n^1Q_r\left[(P-C_m)-\sqrt{\frac{2C_sh_s}{TP_s}}\right]\right\} \qquad (25)$$

$$Br-\left[C_{r0}+\frac{C_{rl}r}{T^{\frac{1}{n}}}\left\{\frac{n}{n+1}\left(T^{\frac{1}{n}+1}-t_1^{\frac{1}{n}+1}\right)+t_1^{\frac{1}{n}}(t_1-T)\right\}+\frac{h_r r}{T^{\frac{1}{n}}}\left\{\frac{t_1^{\frac{1}{n}+1}}{n+1}+\frac{\theta}{2(2n+1)}t_1^{\frac{1}{n}+2}\right\}+\frac{Pr}{T^{\frac{1}{n}}}\left\{\frac{\theta}{n+1}t_1^{\frac{1}{n}+1}+T^{\frac{1}{n}}\right\}+\frac{C_{rd}\,r}{T^{\frac{1}{n}}}\left\{\frac{\theta}{n+1}t_1^{\frac{1}{n}+1}\right\}\right]\geq TCR^* \qquad (26)$$

$$\left[(P-C_m)n^1Q_r-h_s\frac{n^1Q_rQ_s}{2TP_s}-C_s\frac{n^1Q_r}{Q_s}\right]\geq TCM^* \qquad (27)$$

$$P=A-k\,\frac{Q_r-Q_r^1}{Q_s^1-n^1Q_r^1},k>0 \qquad (28)$$

$$C_m\leq P\leq A \qquad (29)$$

$$Q_s^1-n^1*Q_r>0 \qquad (30)$$

$$Q_r=\frac{r}{T^{\frac{1}{n}}}\left[\frac{\theta}{n+1}\left(t_1^{\frac{1}{n}+1}\right)+T^{\frac{1}{n}}\right] \qquad (31)$$

$$0\leq t_1\leq T \qquad (32)$$

$$Q_s^1=\sqrt{\frac{2TC_sP_s}{h_s}} \qquad (33)$$

$$Q_r^1=\frac{r}{T^{1\frac{1}{n}}}\left[\frac{\theta}{n+1}\left(t_1^{\frac{1}{n}+1}\right)+T^{\frac{1}{n}}\right] \qquad (34)$$

The objective function seeks to maximize total profit. Constraints (26) and (27) ensure the Pareto improvement for both cooperation sides. Constraints (26) and (27) guarantee the rationality of price constraint (30) implies that the manufacture will not be out of stock. Restrictions (31-34) are from the previous section.

4. NUMERICAL ILLUSTRATION

To illustrate the developed model, we consider a system with one manufacture and three retailers, i.e. n^1=3.

The relevant parameter values are shown in Table 1.

TABLE.1

The Relevant Parameter Values

Parameter	R	θ	C_{rl}	h_r	C_{ro}	C_{rd}
Value	20	0.02	1	1	20	0.5
Parameter	A	B	C_m	h_s	C_s	P_s
Value	10	12	3	0.4	10	200

The optimal solution of the system is presented below.

1) On condition of decentralized decision, the retailers actual order quantity is the economic order quantity $Q_r^1=6.7401$, and the corresponding maximum profit TCR=17.2936. The corresponding manufactures production lot-size$Q_s^1=60.1087$, and the maximum profit TCM=134.8146.2) with coordination method, we obtain t_1=0.4042, k =0.18, T = 0.8945. $Q_r^1=20.0362$, and the retailers maximum profit TCR=51.8810. The corresponding manufactures production lot-size$Q_s^1=94.5801$, and the manufactures maximum profit TCM=404.4437. Table 2 shows the Optimal values of Q_r , Q_s based on different types of models.

TABLE. 2

Values of Q_r and Q_s

	Q_r	n^1Q_r	Q_s
Non-coordination	6.7401	36.735	60.1087
Coordination	20.0362	60.060	94.5801
	TCR	TCM	∅
Non-coordination	17.2936	134.8146	152.1083
Coordination	51.8810	404.4437	456.3248

From Table 2 It is observed that, based on decentralized decision $Q_s^1 - n^1 Q_r^1 > 0$, the manufacture adopts quantity increased discount policy. Both the retailers and the manufactures profit increased. Therefore, from the economical point of view, the coordination mechanism is effective.

5. SENSITIVITY ANALYSIS

The sensitivity analysis on the effects of changes in the model parameters such as the rate of deterioration, retailers inventory holding cost, manufacturer's inventory holding cost, manufacturer's setup cost, retailers backlog cost, retailers disposal cost for deteriorating items, retailers ordering cost, manufacturer's production rate, by changing each of the parameter by -15%,-10%,-5%,0%,+5%,+10%,+15% and keeping the other parameters unchanged is carried for the model under consideration. The results are presented in tables 3 and 4. The following observations are made from tables 3 and 4.

1. It is observed that as the deterioration rate increases the net profit is decreasing when other parameters remains fixed in both coordination and non coordination models whereas this decrease in coordination model is less compared to that of non coordination model. This is because there is cooperation between retailer and supplier.

2. Similarly regarding the holding costs of retailers and producers the profit for both the models is decreasing when the costs are increasing. This decrease in profit is small in coordination model compared to that of non coordination model.

3. With respect to the increase in other costs like manufacturers set up cost (Cs), disposal cost for deteriorating items (C_{rd}), the profits in both the models are decreasing when other parameters remain fixed. This rate of decrease in profit for coordination model is small compared to that of non coordination model.

4. With respect to increase in the retailers backlog cost (C_{rl}) the profit for both the models is increasing, when retailers ordering costs (C_{ro}) increases profit for both the models is not effected and when production rate (Ps) increases the profits for both the models are increasing.

With the sensitivity analysis one can understand that the supply chain profit and optimal ordering quantities of the manufacturer and retailers are tremendously influenced by deteriorating parameter and costs. This model is much useful for scheduling the supply chain of several products in industries dealing with deteriorated items. This model can also be extended for different types of demand at retailers and manufacturer levels which require further investigations.

Table 3. Sensitivity analysis of non-coordination model

Variation in parameters	**Optimal Policies**	**Change in Parameters**						
		-15%	**-10%**	**-5%**	**0%**	**+5%**	**+10%**	**+15%**
θ	Q_r	6.7399	6.7401	6.7401	6.7401	6.7401	6.7402	6.7402
	Q_s	60.0955	60.1001	60.1044	60.1087	60.1128	60.1209	60.1209
	TCR	17.3067	17.2022	17.2978	17.2936	17.2896	17.2820	17.2820
	TCM	134.8095	134.8114	134.8131	134.8146	134.8158	134.8178	134.8178
	∅	152.1162	152.0136	152.1109	152.1083	152.1054	152.0998	152.0998
h_r	Q_r	6.7476	6.7451	6.7426	6.7401	6.7376	6.7351	6.7325
	Q_s	60.1252	60.1197	60.1142	60.1087	60.1032	60.0981	60.0930
	TCR	17.3461	17.3286	17.3111	17.2936	17.2761	17.2617	17.2473
	TCM	134.9670	134.9162	134.8654	134.8146	134.7638	134.7144	134.6650
	∅	152.3131	152.2448	152.1765	152.1083	152.0399	151.9761	151.9123
h_s	Q_r	6.7480	6.7454	6.7427	6.7401	6.7376	6.7351	6.7328
	Q_s	60.1009	60.1035	60.1061	60.1087	60.1111	60.1136	60.1160
	TCR	17.5745	17.4801	17.3857	17.2936	17.2037	17.1157	17.0296
	TCM	134.9754	134.9210	134.8666	134.8146	134.7646	134.7166	134.6703
	∅	152.5499	152.4011	152.2523	152.1083	151.9683	151.8323	151.6999
C_s	Q_r	6.7125	6.7217	6.7309	6.7401	6.7493	6.7580	6.7672
	Q_s	60.1150	60.1129	60.1108	60.1087	60.1066	60.1040	60.1019
	TCR	17.0656	17.1416	17.2176	17.2936	17.3696	17.4627	17.5387
	TCM	135.2787	135.1240	134.9693	134.8146	134.6599	134.4980	134.3433
	∅	152.3443	152.2656	152.1869	152.1083	152.0295	151.9608	151.8820
C_{rl}	Q_r	6.7402	6.7402	6.7401	6.7401	6.7400	6.7399	6.7398
	Q_s	60.0974	60.1010	60.1046	60.1087	60.1123	60.1162	60.1198
	TCR	17.4766	17.4202	17.3638	17.2936	17.2372	17.1679	17.1115
	TCM	134.8185	134.8169	134.8153	134.8146	134.8139	134.8123	134.8107
	∅	152.2951	152.2371	152.1792	152.1083	152.0511	151.9802	151.9223
C_{rd}	Q_r	6.7401	6.7401	6.7401	6.7401	6.7400	6.7400	6.7400
	Q_s	60.1088	60.1088	60.1087	60.1087	60.1086	60.1085	60.1085
	TCR	17.2942	17.2940	17.2938	17.2936	17.2934	17.2932	17.2930
	TCM	134.8161	134.8156	134.8151	134.8146	134.8141	134.8136	134.8130

	∅	152.1103	152.1096	152.1089	152.1083	152.1075	152.1068	152.1060
C_{ro}	Q_r	6.7500	6.7466	6.7432	6.7401	6.7367	6.7333	6.7299
	Q_s	60.1178	60.1147	60.1116	60.1087	60.1056	60.1025	60.0994
	TCR	19.9692	19.0781	18.1870	17.2936	16.4025	15.5114	14.6203
	TCM	135.0118	134.9448	134.8778	134.8146	134.7476	134.6806	134.6136
	∅	154.9810	154.0229	153.0649	152.1083	151.1501	150.1920	149.2339
P_s	Q_r	6.7316	6.7343	6.7374	6.7401	6.7426	6.7449	6.7471
	Q_s	60.1172	60.1146	60.1113	60.1087	60.1062	60.1040	60.1019
	TCR	16.9848	17.0794	17.1990	17.2936	17.3813	17.4627	17.5387
	TCM	134.6465	134.6981	134.7620	134.8146	134.8641	134.9110	134.9554
	∅	151.6313	151.7775	151.9610	152.1083	152.2454	152.3737	152.4941

Table 4. Sensitivity analysis of co-ordination model

Variation in parameters	**Optimal Policies**	**Change in Parameters**						
		-15%	**-10%**	**-5%**	**0%**	**+5%**	**+10%**	**+15%**
θ	Q_r	20.0318	20.0333	20.0348	20.0362	20.0376	20.0389	20.0403
	Q_s	95.1300	94.9456	94.7623	94.5801	94.3991	94.2193	94.0406
	TCR	51.9203	51.9067	51.8936	51.8810	51.8689	51.8572	51.8461
	TCM	404.4286	404.4343	404.4393	404.4437	404.4475	404.4507	404.4533
	∅	456.3489	456.3410	456.3329	456.3248	456.3164	456.3079	456.2994
h_r	Q_r	20.0410	20.0396	20.0379	20.0362	20.0344	20.0327	20.0310
	Q_s	97.1361	96.2841	95.4321	94.5801	93.7281	92.9016	92.0496
	TCR	52.0391	51.9864	51.9337	51.8810	51.8283	51.7853	51.7326
	TCM	404.9006	404.7483	404.5960	404.4437	404.2914	404.1431	403.9908
	∅	456.9397	456.7347	456.5297	456.3248	456.1198	455.9284	455.7234
h_s	Q_r	20.0336	20.0345	20.0353	20.0362	20.0370	20.0378	20.0386
	Q_s	98.5986	97.2907	95.8880	94.5801	93.3561	92.2068	91.1245
	TCR	52.7165	52.4403	52.1572	51.8810	51.6111	51.3473	51.0890
	TCM	404.9193	404.7631	404.5999	404.4437	404.2939	404.1497	404.0109
	∅	457.6358	457.2034	456.7571	456.3248	455.9050	455.4970	455.0999
C_s	Q_r	20.0385	20.0378	20.0370	20.0362	20.0353	20.0346	20.0339
	Q_s	91.0752	92.2435	93.4118	94.5801	95.8597	97.0280	98.1963
	TCR	51.1973	51.4252	51.6531	51.8810	52.1604	52.3883	52.6162
	TCM	405.8360	405.3719	404.9078	404.4437	403.9581	403.4940	403.0299
	∅	457.0333	456.7971	456.5609	456.3248	456.1185	455.8823	455.6461
C_{rl}	Q_r	20.0324	20.0336	20.0348	20.0362	20.0375	20.0387	20.0399
	Q_s	95.2196	95.0093	94.7990	94.5801	94.3610	94.1507	93.9404
	TCR	52.4298	52.2607	52.0916	51.8810	51.6729	51.5038	51.3347
	TCM	404.4553	404.4506	404.4459	404.4437	404.4415	404.4368	404.4321
	∅	456.8851	456.7113	456.5375	456.3248	456.1144	455.9406	455.7669
C_{rd}	Q_r	20.0362	20.0362	20.0362	20.0362	20.0362	20.0362	20.0361
	Q_s	94.6059	94.5973	94.5887	94.5801	94.5715	94.5629	94.5543
	TCR	51.8827	51.8821	51.8816	51.8810	51.8804	51.8798	51.8792
	TCM	404.4484	404.4468	404.4453	404.4437	404.4422	404.4407	404.4391
	∅	456.3311	456.3289	456.3269	456.3248	456.3226	456.3205	456.3183
C_{ro}	Q_r	20.0392	20.0382	20.0372	20.0362	20.0352	20.0342	20.0332
	Q_s	98.4219	97.1413	95.8607	94.5801	93.2542	91.9283	90.6024
	TCR	59.9079	57.2345	54.5611	51.8810	49.2076	46.5342	43.8608
	TCM	405.0128	404.8231	404.6334	404.4437	404.2427	404.0530	403.8633
	∅	464.9207	462.0576	459.1945	456.3248	453.4504	450.5872	447.7241
P_s	Q_r	20.0390	20.0380	20.0371	20.0362	20.0354	20.0346	20.0339
	Q_s	90.5764	91.8627	93.2938	94.5801	95.8237	97.0280	98.1963
	TCR	50.9544	51.2383	51.5971	51.8810	52.1439	52.3883	52.6162
	TCM	403.9394	404.0970	404.2861	404.4437	404.5924	404.7329	404.8661
	∅	454.8938	455.3353	455.8832	456.3248	456.7363	457.1212	457.4823

6. CONCLUSION

Two-echelon supply chain scheduling is an important consideration for both manufacturer and retailers. In this paper a coordination model for a two echelon supply chain with price discounts is developed and analyzed under the coordination between manufacturers and retailers. It is also assumed that the item under consideration is subject to deterioration. By using the differential equations the instantaneous state of inventory at retailer's level is derived. With suitable cost considerations the total supply chain revenue (profit) with respect to decentralized decision and coordination with price discount are derived. By minimizing the total profit the optimal ordering policies of the supply chain are obtained. It is observed that the coordination model is more cost effective compared to the non coordination model.

A sensitivity analysis of the model with respect to the parameters and costs is also included to study the effect of change in input parameters. This model is much useful for scheduling the supply chain of several products in industries dealing with deteriorated items. This model can also be extended for different types of demand at retailer and manufacturer levels which require further investigations.

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