Anti Fuzzy T - Ideals Of TM- Algebras And Its Lower Level Cuts

T.Ramachandran
Department of Mathematics
Govt.Arts college,Karur.

T.Priya
Department of Mathematics
PSNACET, Dindigul – 624 622

M.Parimala
Department of Mathematics
BIT, Sathyamangalam – 638 401

ABSTRACT
In this paper, we introduce the concept of Anti fuzzy T-ideals of TM-algebras, lower level cuts of a fuzzy set, lower level T-ideal and prove some results . We show that a fuzzy subset of a TM-algebra is a T-ideal if and only if the complement of this fuzzy subset is an anti fuzzy T-ideal. Also we discussed few results of T-ideal of TM-algebra under homomorphism as well as anti homomorphism . Cartesian product of Anti fuzzy T-ideal also discussed.

Keywords
TM-algebra, Anti fuzzy subalgebra, fuzzy T- ideal, Anti fuzzy T-ideal, Anti homomorphism, Cartesian product, lower level cuts.
AMS Subject Classification (2000): 20N25, 03E72, 03F055 , 06F35, 03G25.

1. INTRODUCTION

Y.Imai and K.Iseki introduced two classes of abstract algebras : BCK-algebras and BCI –algebras. It is known that the class of BCK-algebras is a proper subclass of the class of BCI-algebras. Q.P.Hu and X .Li introduced a wide class of abstract BCH-algebras. They have shown that the class of BCI-algebras. J.Neggers, S.S.Ahn and H.S.Kim introduced Q-algebras which is generalization of BCK / BCI algebras and obtained several results. K.Megalai and A.Tamilarasi introduced a class of abstract algebras : TM-algebras , which is a generalisation of Q / BCK / BCI / BCH algebras. R.Biswas introduced the concept of Anti fuzzy subgroups of groups. Modifying his idea, in this paper we apply the idea of TM-algebras . We introduce the notion of Anti fuzzy T-ideals of TM-algebras and investigate how to deal with the homomorphism, Anti homomorphism and inverse image of Anti fuzzy T-ideals of TM-algebras.

2. PRELIMINARIES

In this section we site the fundamental definitions that will be used in the sequel.

2.1 Definition

A non empty set X with a constant 0 and a binary operation * is called a TM-algebra if it satisfies the following axioms.

1. $x * 0 = x$
2. $(x * y) * (x * z) = z * y$, for all $x, y, z \in X$

In X we can define a binary operation $\leq$ by $x \leq y$ if and only if $x * y = 0$.

In any TM-algebra , the following holds good for all $x, y, z \in X$

1) $x * x = 0$
2) $(x * y) * x = 0 * y$
3) $x * (x * y) = x$
4) $(x * z) * (y * z) \leq x * y$
5) $(x * y) * z = (x * z) * y$
6) $x * 0 = 0 \Rightarrow x = 0$
7) $x \leq y \Rightarrow x * z \leq y * z$ and $z * y \leq z * x$
8) $x * (x * (x * y)) = x * y$
9) $0 * (x * y) = y * x = (0 * x) * (0 * y)$
10) $(x * (x * y)) * y = 0$
11) $x * y = 0$ and $y * x = 0 \Rightarrow x = y$.

Example 2.1

Let X = { 0 , 1 , 2, 3 } be a set with a binary operation * defined by the following table. Then (X , * , 0) is a TM-algebra.

*	0	1	2	3
0	0	1	2	3
1	1	0	3	2
2	2	3	0	1
3	3	2	1	0

2.2 Definition

A nonempty subset S of a TM-algebra X is said to be TM-subalgebra of X , if $x, y \in S$ implies $x * y \in S$.

2.3 Definition

Let (X , * , 0) be a TM-algebra. A non empty subset I of X is called an ideal of X if it satisfies the following conditions

(1) $0 \in I$

(2) $x * y \in I$ and $y \in I \Rightarrow x \in I$ for all $x, y \in X$

2.4 Definition

Let (X , * , 0) be a TM-algebra. A non empty subset I of X is called a T-ideal of X if it satisfies the following conditions

(1) $0 \in I$

(2) $(x * y) * z \in I$ and $y \in I \Rightarrow x * z \in I$ for all $x, y, z \in X$

2.5 Definition

Let X be a non-empty set**.** A fuzzy subset μ of the set X is a mapping $\mu : X \to [0, 1]$.

Definition 2.6

Let X be a TM-algebra. A fuzzy subset μ in X is called a fuzzy ideal of X if

(i) $\mu(0) \geq \mu(x)$

(ii) $\mu(x) \geq \min\{\mu(x * y), \mu(y)\}$,for all x, y, z∈ X.

Definition 2.7

Let X be a TM-algebra. A fuzzy subset μ in X is called a fuzzy T-ideal of X if

(i) $\mu(0) \geq \mu(x)$

(ii) $\mu(x * z) \geq \min\{ \mu((x * y) * z), \mu(y)\}$,for all x,y,z∈ X.

Definition 2.8

A fuzzy subset μ of a TM-algebra X is called an anti fuzzy T-ideal of X, if

(i) $\mu(0) \leq \mu(x)$

(ii) $\mu(x * z) \leq \max\{ \mu((x * y) * z), \mu(y)\}$, for all x,y,z∈ X.

Definition 2.9

A fuzzy set μ in a TM-algebra X is called a fuzzy Sub algebra of X if $\mu(x * y) \geq \min\{\mu(x), \mu(y)\}$,for all x,y∈ X.

Definition 2.10

A fuzzy set μ in a TM-algebra X is called an Anti fuzzy sub algebra of X if $\mu(x * y) \leq \max\{\mu(x), \mu(y)\}$, for all x,y∈ X.

Theorem 2.1

If μ is an anti fuzzy sub algebra of a TM-algebra x , then $\mu(0) \leq \mu(x)$, for any $x \in X$.

Proof

Since $x * x = 0$ for any $x \in X$, then

$$\mu(0) = \mu(x * x)$$
$$\leq \max\{\mu(x), \mu(x)\}$$
$$= \mu(x)$$
$$\mu(0) \leq \mu(x).$$

Definition 2.11

Let μ be a fuzzy set of X. For a fixed $t \in [0, 1]$, the set $\mu^t = \{x \in X \mid \mu(x) \leq t\}$ is called the lower level subset of μ.

Clearly $\mu^t \cup \mu_t = X$ for $t \in [0,1]$ if $t_1 < t_2$, then $\mu^{t1} \subseteq \mu^{t2}$.

Theorem 2.2

A fuzzy set μ of a TM – algebra X is an anti fuzzy sub algebra if and only if for every $t \in [0,1]$, μ^t is either empty or a sub algebra of X.

Proof:

Assume that μ is an anti fuzzy sub algebra of X and $\mu^t \neq \phi$.Then for any $x,y \in \mu^t$, we have

$$\mu(x * y) \leq \max\{\mu(x), \mu(y)\} \leq t.$$

Therefore $x*y \in \mu^t$. Hence μ^t is a sub algebra of X.

Now Let x,y∈ X.

Take $t = \max\{\mu(x), \mu(y)\}$. Then by assumption μ^t is a sub algebra of X implies $x * y \in \mu^t$. Therefore $\mu(x * y) \leq t = \max\{\mu(x), \mu(y)\}$.

Hence μ is an Anti fuzzy sub algebra of X.

Theorem 2.3

Any sub algebra of a TM – algebra X can be realized as a level sub algebra of some Anti fuzzy sub algebra of X.

Proof:

Let μ be a sub algebra of a given TM – algebra X and let μ be a fuzzy set in X defined by

$$\mu(x) = \begin{cases} t, & \text{if } x \in A \\ 0, & \text{if } x \notin A. \end{cases}$$

Where $t \in [0,1]$ is fixed. It is clear that $\mu^t = A$.

Now we prove such defined μ is an anti fuzzy sub algebra of X.

Let x,y∈ X. If $x,y \in A$, then $x*y \in A$.

Hence μ (x) = μ (y) = μ (x*y) = t and $\mu(x * y) \leq \max \{\mu(x), \mu(y)\}$

If x,y $\notin$A, then μ (x) = μ (y) = 0 and $\mu(x * y) \leq \max \{\mu(x), \mu(y)\} = 0$.

If at most one of x,y $\in$ A, then at least one of μ (x) and μ (y) is equal to 0.

Therefore, $\max \{\mu(x), \mu(y)\} = 0$ so that $\mu(x * y) \leq 0$,which completes the proof.

Theorem 2.4

Two level sub algebras μ^s, μ^t (s < t) of an anti fuzzy sub algebra are equal iff there is no x$\in$X such that $s \leq \mu(x) < t$.

Proof

Let $\mu^s = \mu^t$ for some s < t. If there exist x$\in$X such that $s \leq \mu(x) < t$, then μ^t is a proper subset of μ_s, which is a contradiction.

Conversely, assume that there is no x$\in$X such that $s \leq \mu(x) < t$.

If x$\in$ μ^s, then $\mu(x) \leq s$ and $\mu(x) \leq t$,

Since $\mu(x)$ does not lie between s and t. Thus x $\in$ μ^t , which gives

$\mu^s \subseteq \mu^t$, Also $\mu^t \subseteq \mu^s$.

Therefore $\mu^s = \mu^t$.

Theorem 2.5

Every Anti fuzzy T-ideal μ of a TM-algebra X is order preserving. That is , If $x \leq y$, then $\mu(x) \leq \mu(y)$, for all x , y $\in$ X.

Proof

Let μ be an anti fuzzy T-ideal of a TM-algebra X and let x , y $\in$ X be such that

$x \leq y$, then $x * y = 0$

$$\begin{aligned} \text{Now } \mu(x) &= \mu(x * 0) \\ &\leq \max \{\mu((x * y) * 0), \mu(y)\} \\ &= \max \{\mu(0 * 0), \mu(y)\} \\ &= \mu(y) \end{aligned}$$

Hence $\mu(x) \leq \mu(y)$.

Theorem 2.6

A fuzzy set μ in TM – algebra X is an anti fuzzy T- ideal iff it is an anti fuzzy ideal of X.

Proof:

Let μ be an anti fuzzy T-ideal of X.

Then (i) $\mu(0) \leq \mu(x)$

(ii) $\mu(x * z) \leq \max \{\mu((x * y) * z), \mu(y)\}$ for all x,y,z $\in$ X.

By putting z = 0 in (ii) we get

$$\mu(x * 0) \leq \max \{\mu((x * y) * 0), \mu(y)\}$$

$$\Rightarrow \quad \mu(x) \leq \max \{\mu(x * y), \mu(y)\}$$

Hence μ is an anti fuzzy ideal of X.

Conversely, assume that μ is an anti fuzzy ideal of X.

$$\begin{aligned} \text{Then } \mu(x * z) &\leq \max \{\mu((x * z) * y), \mu(y)\} \\ &= \max \{\mu((x * y) * z), \mu(y)\} \end{aligned}$$

Hence μ is an anti fuzzy T-ideal of X.

Theorem 2.7

Let μ be a fuzzy set in a BCK – algebra X. Then μ is an anti fuzzy T – ideal iff μ is a anti fuzzy BCK – ideal.

Proof:

Since every BCK algebra is TM- algebra, every anti fuzzy T-ideal is an anti fuzzy ideal of a TM- algebra and hence an anti fuzzy BCK ideal (by Theorem 6 and definition).

Conversely, assume that μ be an anti fuzzy BCK- ideal of X.

Then,

$$\mu(x) \leq \max \{\mu(x * y), \mu(y)\}$$

$$\begin{aligned} \Rightarrow \quad \mu(x * z) &\leq \max \{\mu((x * z) * y), \mu(y)\} \\ &= \max \{\mu((x * y) * z), \mu(y)\} \end{aligned}$$

Hence μ is an anti fuzzy T- ideal of X.

Theorem 2.8

μ is a fuzzy T-ideal of a TM-algebra X if and only if

μ^c is an anti fuzzy T-ideal of X.

Proof

Let μ be a fuzzy T-ideal of X and let x,y,z $\in$ X.

$$\begin{aligned} \text{Then (i) } \mu^c(0) &= 1 - \mu(0) \leq 1 - \mu(x) \\ &= \mu^c(x) \end{aligned}$$

That is, $\mu^c(0) \leq \mu^c(x)$.

$$\begin{aligned} \text{(ii) } \mu^c(x * z) &= 1 - \mu(x * z) \\ &\leq 1 - \min \{\mu((x * y)*z), \mu(y)\} \end{aligned}$$

$= 1 - \min \{1 - \mu^c ((x * y)*z) , 1 - \mu^c (y)\}$

$= \max \{ \mu^c ((x * y)*z) , \mu^c (y) \}$

That is , $\mu^c(x * z) \leq \max \{ \mu^c ((x * y)*z) , \mu^c (y) \}$

Thus, μ^c is an anti fuzzy T – ideal of X. The converse also can be proved similarly

3. HOMOMORPHISM AND ANTI HOMOMORPHISM OF TM- ALGEBRAS

In this section, we discussed about T-ideals in TM-algebra under homomorphism and anti homomorphism and some of its properties.

Definition 3.1

Let (X,*,0) and (Y ,Δ ,0``) be TM – algebras. A mapping

f: X → Y is said to be a homomorphism if f(x * y) = f(x) Δ f(y) for all x,y ∈ X.

Definition 3.2

Let (X,*,0) and (Y ,Δ ,0``) be TM – algebras. A mapping

f: X → Y is said to be a anti homomorphism if f(x * y) = f(y) Δ f(x) for all x,y ∈ X.

Definition 3.3

Let f: X → X be an endomorphism and μ be a fuzzy set in X. We define a new fuzzy set in X by μ_f in X as $\mu_f (x) = \mu (f(x))$ for all x in X.

Definition 3.4

For any homomorphism f: X → Y, the set {x∈X / f(x) = 0'} is called the kernel of f, denoted by Ker(f) and the set { f(x) / x∈X} is called the image of f, denoted by Im(f).

Theorem 3.1

Let f be an endomorphism of a TM- algebra X. If μ is an anti fuzzy T – ideal of X, then so is μ_f .

Proof:

$\mu_f (0) = \mu (f (0))$

$\leq \mu (f(x)) = \mu_f (x)$, for all x ∈ X.

Let x,y,z ∈ X.

Then $\mu_f (x * z) = \mu (f (x * z))$

$= \mu (f(x) * f (z))$

$\leq \max \{ \mu (f(x) * f(y) * f(z)) , \mu (f(y) \}$

$= \max \{ \mu ((f(x * y)) * f(z)) , \mu (f(y) \}$

$= \max \{ \mu (f(x * y) * z) , \mu (f(y) \}$

$= \max \{ \mu_f ((x * y) * z) , \mu_f (y) \}$

Hence μ_f is an anti fuzzy T- ideal of X.

Theorem 3.2

Let (X ,*, 0) and (Y , Δ ,0′`) be TM – algebras. A mapping f: X → Y is an anti-homomorphism of TM-algebra, Then Ker(f) is a T-ideal.

Proof:

Let (x * y) * z ∈ ker (f) & y ∈ ker (f) .

Then

f ((x * y) * z) = 0′ & f (y) = 0′

Since

0′ = f ((x * y) * z)

= f ((x * z) * y)

= f (y) Δ f (x * z)

= 0′ Δ f (x * z)

= f (x * z)

⇒ x * z ∈ ker (f)

Hence ker f is a T-ideal.

4. CARTESIAN PRODUCT OF ANTI FUZZY T – IDEALS OF TM – ALGEBRAS

In this section, we introduce the concept of Cartesian product of anti fuzzy T-ideals of TM-algebra.

Definition 4.1

Let μ and δ be the fuzzy sets in X. The Cartesian product

μ x δ : X x X → [0,1] is defined by (μ x δ) (x, y) = min {μ(x),δ(y)}, for all x, y ∈ X.

Definition 4.2

Let μ and δ be the anti fuzzy T-ideals in X. The

Cartesian product μ x δ : X x X → [0,1] is defined by (μ x δ) (x, y) = max {μ(x), δ(y)} , for all x, y ∈ X.

Theorem 4.1

If μ and δ are anti fuzzy T- ideals in a TM – algebra X, then μ x δ is an anti fuzzy T- ideal in X x X .

Proof

For any $(x, y) \in X \times X$, we have

$(\mu \times \delta)(0, 0) = \max\{\mu(0), \delta(0)\}$

$\leq \max\{\mu(x), \delta(y)\}$

$= (\mu \times \delta)(x, y)$.

Let $(x_1, x_2), (y_1, y_2), (z_1, z_2) \in X \times X$.

$(\mu \times \delta)((x_1, x_2) * (z_1, z_2)) = (\mu \times \delta)(x_1 * z_1, x_2 * z_2)$

$= \max\{\mu(x_1 * z_1), \delta(x_2 * z_2)\}$

$\leq \max\{\max\{\mu((x_1 * y_1) * z_1), \mu(y_1)\}, \max\{\delta((x_2 * z_2) * z_2), \delta(y_2)\}\}$

$= \max\{\max\{\mu((x_1 * y_1) * z_1), \delta((x_2 * z_2) * z_2)\}, \max\{\mu(y_1), \delta(y_2)\}\}$

$= \max\{(\mu \times \delta)((x_1 * y_1) * z_1, (x_2 * y_2) * z_2), (\mu \times \delta)(y_1, y_2)\}$

Theorem 4.2:

Let μ & δ be fuzzy sets in a TM-algebra. X such that $\mu \times \delta$ is an Anti-fuzzy T-ideal of $X \times X$. Then

(i) Either $\mu(0) \leq \mu(x)$ (or) $\delta(0) \leq \delta(x)$ for all $x \in X$

(ii) If $\mu(0) \leq \mu(x)$ for all $x \in X$, then either $\delta(0) \leq \mu(x)$ (or) $\delta(0) \leq \delta(x)$

(iii) If $\delta(0) \leq \delta(x)$ for all $x \in X$, then either $\mu(0) \leq \mu(x)$ (or) $\mu(0) \leq \delta(x)$.

(iv) Either μ or δ is an anti fuzzy T-ideal of X.

Proof:

Let $\mu \times \delta$ be an anti fuzzy T-ideal of $X \times X$.

Therefore $(\mu \times \delta)(0,0) \leq (\mu \times \delta)(x, y)$ for all $(x, y) \in X \times X$ and

$(\mu \times \delta)((x_1, x_2) * (z_1, z_2)) \leq \max\{(\mu \times \delta)((x_1, x_2) * (y_1, y_2) * (z_1, z_2)), (\mu \times \delta)(y_1, y_2)\}$

for all $(x_1, x_2), (y_1, y_2)$ & $(z_1, z_2) \in X \times X$.

(i) Suppose that $\mu(0) > \mu(x)$ and $\delta(0) > \delta(x)$ for some $x, y \in X$

Then $(\mu \times \delta)(x, y) = \max\{\mu(x), \delta(y)\}$

$< \max\{\mu(0), \delta(0)\}$

$= (\mu \times \delta)(0,0)$, Which is a contradiction.

Therefore $\mu(0) \leq \mu(x)$ and $\delta(0) \leq \delta(x)$ for all $x \in X$.

(ii) Assume that there exist $x, y \in X$ such that $\delta(0) > \mu(x)$ and $\delta(0) > \delta(x)$.

Then $(\mu \times \delta)(0,0) = \max\{\mu(0), \delta(0)\} = \delta(0)$ and hence

$(\mu \times \delta)(x, y) = \max\{\mu(x), \delta(y)\} < \delta(0) = (\mu \times \delta)(0,0)$

Which is a contradiction.

Hence if $\mu(0) \leq \mu(x)$ for all $x \in X$, then either $\delta(0) \leq \mu(x)$ (or) $\delta(0) \leq \delta(x)$.

Similarly, we can prove that if $\delta(0) \leq \delta(x)$ for all $x \in X$, then either $\mu(0) \leq \mu(x)$ or $\mu(0) \leq \delta(y)$, which yields (iii).

(iv) First we prove that δ is an anti-fuzzy T-ideal of X

Since by (i) either $\mu(0) \leq \mu(x)$ (or) $\delta(0) \leq \delta(x)$ for all $x \in X$.

Assume that $\delta(0) \leq \delta(x)$, for all $x \in X$.

It follows from (iii) that either $\mu(0) \leq \mu(x)$ or $\mu(0) \leq \delta(x)$.

If $\mu(0) \leq \delta(x)$ for any $x \in X$, then

$\delta(x) = \max\{\mu(0), \delta(x)\} = (\mu \times \delta)(0, x)$

$\delta(x * z) = \max\{\mu(0), \delta(x * z)\}$

$= (\mu \times \delta)(0, x * z)$

$= (\mu \times \delta)(0 * 0, x * z)$

$= (\mu \times \delta)((0, x) * (0, z))$

$\leq \max\{(\mu \times \delta)(((0,x) * (0,y)) * (0,z)), (\mu \times \delta)(0,y)\}$

$= \max\{(\mu \times \delta)((0 * 0, x * y) * (0,z)), (\mu \times \delta)(0,y)\}$

$= \max\{(\mu \times \delta)((0 * 0) * 0, (x * y) * z), (\mu \times \delta)(0,y)\}$

$= \max\{(\mu \times \delta)(0, (x * y) * z), (\mu \times \delta)(0,y)\}$

$= \max\{\delta((x * y) * z), \delta(y)\}$

Hence δ is an Anti fuzzy T-ideal of X.

Next we will prove that μ is an anti fuzzy T-ideal of X.

Let $\mu(0) \leq \mu(x)$

Since by (ii), either $\delta(0) \leq \mu(x)$ or $\delta(0) \leq \delta(x)$.

Assume that $\delta(0) \leq \mu(x)$,then

$\mu(x) = \max\{\mu(x), \delta(0)\} = (\mu \times \delta)(x,0)$

$\mu(x * z) = \max \{\mu(x * z), \delta(0) \}$

$= (\mu \times \delta) (x * z ,0)$

$= (\mu \times \delta) (x * z ,0 * 0)$

$= (\mu \times \delta) ((x,0) * (z,0))$

$\leq \max \{(\mu \times \delta) ((x,0) * (y,0)) * (z,0)), (\mu \times \delta)(y,0)\}$

$= \max \{(\mu \times \delta) ((x * y , 0 * 0) * (z,0)), (\mu \times \delta)(y,0)\}$

$= \max \{(\mu \times \delta) ((x * y) * z ,0), (\mu \times \delta)(y,0)\}$

$= \max \{ \mu ((x * y)* z)), \mu(y)\}$

Hence μ is an anti fuzzy T- ideal of X.

5.CONCLUSION

In this article we have discussed anti fuzzy sub algebra and anti fuzzy T- ideal of TM-algebras. It has been observed that the TM-algebra satisfies the various conditions stated in the BCC / BCK / BCI / Q / BCH algebras and can be considered as the generalization of all these algebras. These concepts can further be generalized.

6.REFERENCES

[1] Ahn S.S and H.D.Lee , Fuzzy subalgebras of BG-algebras, Commun.korean math Soc. 19 (2004) 243-251.

[2] Biswas.R , Fuzzy subgroups and Anti Fuzzy subgroups , Fuzzy sets and systems , 35 (1990),121-124.

[3] Dudek W.A and Y.B.Jun, Fuzzification of ideals in BCC-algebras , Glasnik Matematicki,36 ,(2001) , 127-138.

[4] Hu Q.P. and X.Li , On BCH-algebras, Mathematics Seminar notes 11 (1983) , 313-320.

[5] Iseki .K and S.Tanaka , An introduction to the theory of BCK – algebras , Math Japonica 23 (1978), 1- 20 .

[6] Iseki . K , On BCI-algebras , Math.Seminar Notes 8 (1980), 125-130.

[7] Megalai .K and A.Tamilarasi , classification of TM-algebra , IJCA Special issue on "Computer Aided Soft Computing Techniques for Imaging and Biomedical Applications " CASCT, 2010.

[8] Megalai . K and A.Tamilarasi , Fuzzy Subalgebras and Fuzzy T- ideals in TM-algebra ,Journal of Mathematics and Statistics 7(2) , 2011, 107-111.

[9] Muthuraj .R , P.M.Sitharselvam , M.S.Muthuraman , Anti Q fuzzy group and its lower level subgroups, IJCA, 3 (2010) ,16-20.

[10] Muthuraj .R ,M.Sridharan, P.M.Sitharselvam, M.S.Muthuraman , Anti Q- Fuzzy BG-ideals in BG-algebra, IJCA, 4 , (2010), 27-31.

[11] Neggers. J , S.S.Ahn and H.S.Kim , On Q-algebras, IJMMS 27 (2001) , 749-757.

[12] Neggers. J and H.S.Kim , On B-algebras , math,vensik,54 (2002), 21-29.

[13] Neggers. J and H.S.Kim , On d-algebras , Math,slovaca, 49 (1999) , 19-26.

[14] Samy M.Mostafa ,Mokthar A.Abdel Naby and Osama R.Elgendy , Journal of American Sciences, 7(9) , (2011), 17-21.

[15] Zadeh.L.A. , Fuzzy sets , Inform.control,8 (1965) , 338 - 353.