Global Domination upon Edge Addition Stable Graphs

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ABSTRACT

Let G be a simple, finite and connected graph. A dominating set D of a graph G is a global dominating set if D is also a dominating set of $\overline{G}$. The global domination number $\gamma_g(G)$ is the minimum cardinality of a minimal global dominating set of G. A graph is global domination edge critical if addition of any arbitrary edge changes the global domination number. On the other hand, a graph is global domination edge stable if addition of any arbitrary edge has no effect on the global domination number. In this paper, we study the concepts of global domination and connected global domination upon edge addition stable property for cycle and path graphs. We determine sharp bounds on the global domination and connected global domination number of global domination, total global domination and connected global domination edge addition stable graphs.

Keywords

domination, global domination, connected global domination, edge addition stable. AMS Subject Classification: 05C69

1. PRELIMINARIES

In our study, we consider only simple, finite and undirected graphs. In this section, we review the notions of domination, global domination and connected global domination [2, 3, 4].

Definition 1.1

Let G = (V, E) be a graph. A subset D of V is called a *dominating set* of G if every vertex in V – D is adjacent to at least one vertex in D. A dominating set D is called a minimal dominating set if no proper subset of D is a dominating set. The *domination number* γ(G) of a graph G is the minimum cardinality of a minimal dominating set in G.

Definition 1.2

A dominating set D of a graph G is called a *global dominating set* if D is also a dominating set of $\overline{G}$. The minimum cardinality of a minimal global dominating set is denoted by γ_g and is called the global domination number of G.

Definition 1.3

A connected dominating set D of a graph G is called a *connected global dominating set* if D is also a connected dominating set of $\overline{G}$. The connected global domination number of G is the minimum cardinality of a minimal connected global dominating set is denoted by γ_{cg}.

Definition 1.4

Let P be a graph parameter. A graph G is said to be *P- edge addition critical* if P(G + e) ≠ P(G) for every edge e of $\overline{G}$, the complement of G. A graph G is said to be *P- edge addition stable* if P(G + e) = P(G) for every edge e of $\overline{G}$.

2. Global Domination upon Edge Addition Stable Graphs

In this section, we investigate 'global domination edge addition stable' property for cycles and paths [1].

Definition 2.1

A graph G is said to be *global domination edge addition stable* or γ_g^+ -stable for short, if addition of any edge to G does not change the global domination number. In other words, a graph G is γ_g^+ -stable if $\gamma_g(G + e) = \gamma_g(G)$ for every edge $e \in E(\overline{G})$.

Theorem 2.2

For n ≥ 5, n ≠ 3k+1, k ≥ 2, cycle graphs C_n are γ_g^+-stable.

Proof

Let C_n: $v_1, v_2, ..., v_n$ be a cycle of length n ≥ 5. Here n is either 5 or takes one of the two forms 3k or 3k+2, k ≥ 2 (as n ≠ 3k+1). A global dominating set of C_n can be obtained by taking $D = \{v_{3i-1} / i = 1, 2, \ldots, k\}$ if n = 3k, $D = \{ v_{3i+2} / i = 0, 1, 2, \ldots, k \}$ if n = 3k+2 or $D = \{v_1, v_2, v_4\}$ when n = 5. The different cases that arise when an edge is added between two non-adjacent vertices v_i and v_j in C_n, are such that $2 \le d(v_i, v_j) \le \lfloor n/2 \rfloor - 1$, where $d(v_i, v_j)$ denotes the distance between v_i and v_j, and in all these cases the global domination is not affected. Hence C_n is γ_g^+ - stable when n ≠ 3k+1, k ≥ 1.

Corollary 2.3

For n = 3k+1, k ≥ 2 , cycle graphs C_n are not γ_g^+-stable.

Proof

By using the labeling of C_n as in the above theorem, global dominating set of C_n, n = 3k+1 is given by $D = \{v_{3i+1} / i = 0, 1, 2, \ldots, k\}$. Up on adding an edge between non-adjacent vertices at distance 2, the global domination number is reduced by 1. Hence C_n is not γ_g^+ - stable, when n = 3k+1, k ≥ 2.

Proposition 2.4

For n = 5 or 3k+1, $k \geq 2$,
$\gamma_g(C_n) \geq \gamma_g(C_n + e) \geq \gamma_g(C_n) - 1$.

Theorem 2.5

For $n \geq 8$, $n \neq 3k + 1$, $k \geq 3$, path graphs P_n are γ_g^+-stable.

Proof

Let P_n: $v_1, v_2, ..., v_n$ be the path graph of order $n \geq 8$. Here n takes one of the two forms 3k+2, $k \geq 2$ or 3k, $k \geq 3$ (as $n \neq 3k+1$). A global dominating set of P_n can be obtained by taking $D = \{v_{3i+2} / i = 0, 1, 2, \dots k\}$ when n = 3k+2 and $D = \{v_{3i+2} / i = 0, 1, 2, \dots k\text{-}1\}$ when n = 3k. By adding an edge between non-adjacent vertices v_i and v_j, $1 \leq i < j \leq n$ in P_n, the global domination number is not changed. Hence P_n is γ_g^+ -stable when $n \neq 3k+1$, $k \geq 3$.

Corollary 2.6

For $n = 3k + 1$, $k \geq 2$, path graphs P_n are not γ_g^+-stable.

Proof

Let P_n: $v_1, v_2, ..., v_n$ be the path graph of order $n = 3k + 1$, $k \geq 2$. Addition of an edge between the pendant and support vertex v_1 and v_{n-1}, global domination number is decreased by one. Hence P_n, $n = 3k + 1$, $k \geq 2$ are not γ_g^+- stable.

Result 2.7

For n = 5 and 6, path graphs P_n are not γ_g^+-stable.

Example 2.8

Consider the path graph P_7: $v_1, v_2, \dots, v_7$.

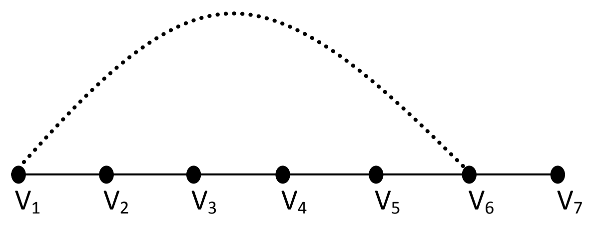

Figure 1

The minimal global dominating set is $\{v_2, v_5, v_7\}$. The minimal global dominating set for $P_7 + e$ ($= v_1v_6$) is $\{v_3, v_6,\}$, thereby decreasing the global domination number by one.

3. CONNECTED GLOBAL DOMINATION UPON EDGE ADDITION STABLE GRAPHS

In this section, we investigate 'connected global domination edge addition stable' property for cycles and paths.

Definition 3.1

A graph G is said to be a *connected global domination edge addition stable* or γ_{cg}^+ -stable for short, if addition of any edge to G does not change the connected global domination number. In other words, a graph G is γ_{cg}^+ -stable if $\gamma_{cg}(G + e) = \gamma_{cg}(G)$ for every edge $e \in E(\overline{G})$.

Theorem 3.2

For $n \geq 7$, cycle graphs C_n are γ_{cg}^+- critical.

Proof

Let C_n: $v_1, v_2, ..., v_n$ be the cycle graph of order n, $n \geq 7$. Global connected dominating set with minimum cardinality can be obtained by taking $D = \{v_1, v_2, \dots, v_{n-2}\}$ and therefore, $\gamma_{cg}(C_n) = n - 2$. By adding an edge between vertices at distance 2, global connected domination number is reduced by 1. But adding an edge e between vertices v_i and v_j, $i < j$ at distance k, $3 \leq k \leq \lfloor n/2 \rfloor$, results in a smaller global connected dominating set with vertices $\{v_i, v_j, v_{j-1}, v_{j-2}, \dots, v_{i+3}, v_{j+1}, v_{j+2}, \dots, v_{i-3}\}$, where the addition in the subscripts are such that, when they exceed n, subtract n; reduces the global connected domination number by 2. Therefore cycle graphs are γ_{cg}^+-critical.

Result 3.3

Cycle graphs C_5 and C_6 are γ_{cg}^+-stable.

Example 3.4

Consider the cycle graph C_7 with $V = \{v_1, v_2, v_3, v_4, v_5, v_6, v_7\}$.

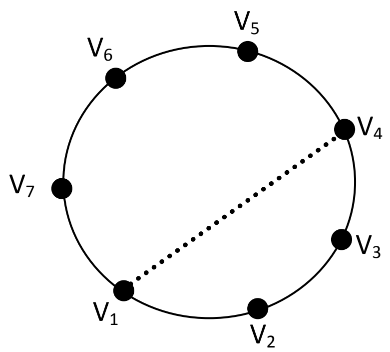

Figure 4

An global connected dominating set is $\{v_1, v_2, v_3, v_4, v_5\}$, implying $\gamma_{cg}(C_7) = 5$. After adding an edge e between vertices

v_1 and v_4, global connected dominating set for (C_7 + e) is {v_1, v_4, v_5}. Hence global connected domination number is reduced by 2.

Proposition 3.5

(i) For $n \geq 7$, $\gamma_{cg}(C_n + e) = \gamma_{cg}(C_n) - 1$ if an edge e is added between two vertices at distance two.

(ii) For $n \geq 8$, $\gamma_{cg}(C_n + e) = \gamma_{cg}(C_n) - 2$ if an edge e is added between two vertices at distance more than two.

Theorem 3.6

For $n \geq 7$, path graphs P_n are not γ_{cg}^{+}-stable.

Proof

Let P_n: v_1, v_2, ..., v_n be the path graph of order $n \geq 7$. The set of vertices {v_2, v_3, ..., v_{n-1}} forms the minimal global connected dominating set. If an edge is added between v_1 and v_{n-1}, a support vertex, the global connected domination number is decreased by one. On the other hand, if an edge is added between the pendant vertices the global connected domination number remains the same. Therefore, path graphs are not γ_{cg}^{+} - stable.

Result 3.7

For n = 5 and 6, path graphs P_n are γ_{cg}^{+}-stable.

Proposition 3.8

For $n \geq 7$, $\gamma_{cg}(P_n) \geq \gamma_{cg}(P_n + e) \geq \gamma_{cg}(P_n) - 2$.

4. CONCLUSION

In this present study, the concept of stability of global domination and connected global domination upon edge addition stable property for cycle and path graphs are investigated. In our future work, stability of different families of graphs for other kinds of domination upon edge addition is proposed.

5. REFERENCES

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