Effective Speed Control in 3-Phase BLDC Motor by Reaching Law based Sliding Mode Technique

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ABSTRACT
In this paper a Sliding mode controller (SMC) scheme based on the power rate reaching law approach has been proposed for the inner loop current control of the 3-Phase Brushless DC motor (BLDC) drive. A proportional- integral (PI) controller is used to control the outer loop speed performance. The developed PI-SMC control scheme is simulated on MATLAB/SIMULINK platform under a sequence of load torque disturbances incremented up to 100% to demonstrate efficient speed tracking. Experimental verification of the developed scheme is carried out under no load conditions using National Instruments Data Acquisition Card (NI DAQ) card 6229 as the interface with MATLAB environment. The experimental results validate the performance of the developed scheme.

General Terms
Sliding Mode Control

Keywords
Sliding Mode Control, Power rate reaching law approach, BLDC motors, NI DAQ 6229

1. INTRODUCTION
BLDC motors attain an extremely significant place as electric drives finding applications in wide range of areas such as automotives, industrial controls, automation and robotics, aviation etc. For a BLDC motor the controller plays a very vital role in influencing its performance for applications concerning load variations or in positioning applications. In constant load applications[1] of BLDC motor such as in air compressors, pumps, the controllers required are usually of low cost and the operation is basically in open loop. In case of varying load applications [1] of the BLDC motor such as in fuel pump control, electric vehicle control [2], pneumatic device control etc the controllers required are necessarily complex for smooth operation of the plant with high speed accuracy coupled with paramount dynamic responses. Similarly in case of positioning applications[1] in the industry like a simple belt driven system, due to high demands of efficient dynamic response of speed and a ripple free torque generation, the control algorithms of the controllers employed here are intricate in nature. The robustness of a BLDC drive is thus dogged by the performance of its controller. The BLDC motor consists of permanent magnets on the rotor for an efficient and condensed design. The stator windings back emf is trapezoidal or sinusoidal in shape depending upon the shape of the rotor permanent magnets. For efficient ripple free toque production, a rectangular current in phase with the back emf is fed to the windings. The windings of the stator are externally commutated by use of VSI (Voltage Source Inverter) or CSI (Current Source Inverter). The performance of the BLDC drive has been analyzed using various control algorithms like P-I (Proportional-Integral) [3], Fuzzy Logic Control [3], Neural Network Control [4], Adaptive Control [4], Optimal Control [4] amongst others. Neural Network control failed as its learning capabilities were hindered by the non-linearities of the BLDC drive system. Optimal Control and Fuzzy logic control failed too in the domain of load disturbance rejection. The simplest of controllers, P-I controller was able to handle load disturbance upto a specific limits but its performance was curtailed in case of varying loads leading to integral windup [3] .In was then in the early 1950's that Sliding Mode control (SMC) [4, 11] evolved as a simple control algorithm that was capable of giving a very robust system performance by rejection of disturbance and with invariance to bounded parametric uncertainties. With research progressing on sliding mode control, this technique was implemented for efficient control of drives like BLDC motor when the major work in this direction was reported in 1990's by Utkin [5]. The load disturbances rejection capabilities of this control scheme made it ideally suitable for a non-linear time varying systems such as the BLDC motor despite its major disadvantage which is chattering.

In this work, the performance of a 3-Phase BLDC motor is analyzed using power rate based reaching law sliding mode technique. The load disturbance rejection capabilities of the control scheme are suitably explored. The three phase modeling of the BLDC drive is developed on MATAB/SIMULINK platform and using the developed sliding control algorithm for the inner loop current control the results are obtained. These results are then verified on a hardware platform for no load condition using NI DAQ card 6229 as the interfacing device.

The paper is organized as follows:

- Section 2 introduces the concept of sliding mode and power rate reaching law algorithm.
- The mechanical and electrical model of the 3-phase BLDC motor is developed in Section 3.

- The design of the Sliding mode controllers is in Section 4.
- The results of the simulation and the hardware representation are shown in Section 5.
- The conclusion of the study is given in Section 6.

2. SLIDING MODE CONTROL

In the early 1950's, Sliding Mode control evolved as a non linear control technique to counter the effect of load disturbances and parametric uncertainties on the system. Sliding Mode control forces the state trajectory onto a stable manifold christened the Sliding manifold [6] by continuous switching of the control input. The control algorithm relies on Lyapunov's second theorem of stability to prove that by forcing the state trajectory on to the sliding manifold and causing it to stay there forever it causes the system to attain stability. The control designer has to design a control law that performs two very specific tasks, (i) The control law should force the trajectory onto the sliding surface in the minimum time possible in the phase named Reaching phase and (ii) Once on the sliding surface the trajectory should move towards the equilibrium point and lie there forever. Once on the sliding surface the motion equation of the trajectory is S=0 regardless of parametric variations or load conditions. Thus dynamics of the system in sliding phase is constant and disturbance rejection is ensured.

For a single input non linear system of the form

$$\dot{x} = f(x,\dot{x}) + G(x,\dot{x})u + D \quad (1)$$

Where, D= external disturbances
G= uncertainty in input
u =control input
Then the states of the system are,

$$X = [x_1, x_2, x_3, ..., x_n] \quad (2)$$

The sliding surface is thus, $S = C^T X$ [5].Where, $'C'$ is the sliding surface parameter.

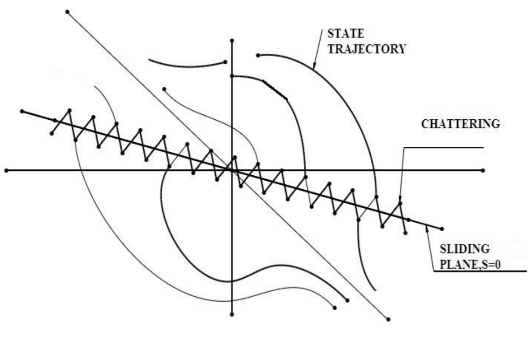

Fig 1: Depiction of Siding Mode and Chattering

The convergence of the state trajectory onto the sliding surface by controlling the dynamic characteristics is done by employing The Reaching Law [7] approach which directly specifies the dynamics of the switching surface during the reaching phase. The general representation of the reaching law approach is given as,

$$\dot{S} = -\varepsilon\, sgn(S) - K\, f(S) \quad (3)$$

Where the gains $\varepsilon > 0, K > 0$

Signum function $sgn(S)$ is defined as

$$sgn(S) = \begin{cases} 1 & S > 0 \\ -1 & S < 0 \end{cases} \quad (4)$$

Any reaching law developed must satisfy the reaching condition of sliding mode $S\dot{S} < 0$ given by Lyapunov's second theorem of stability. In the power rate reaching law approach, the speed at which the trajectory converges onto the sliding surface when it is far away from the surface is fast but when the state is nearer to the surface the rate of convergence reduces. The power rate reaching law is given as,

$$\dot{S} = -\varepsilon |S|^{\alpha}\, sgn(S) \quad (5)$$

Where the constants α, ε as chosen as per the reaching time criterion given as,

$$t_{reach} \le \frac{1}{(1-\alpha)\varepsilon} S_0^{(1-\alpha)} \quad (6)$$

3. BLDC MOTOR MODEL

The three phase BLDC motor has a construction similar to the DC motor but is electronically commutated by the use of a three phase VSI (Voltage Source Inverter) instead of a mechanical commutator. The gating sequence for these switches is provided by inner current loop control. The representation of a VSI fed BLDC motor is shown below:

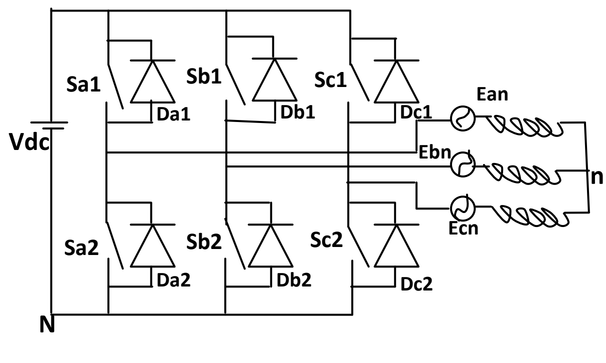

Fig 2: Schematic of the VSI fed BLDC motor

In the above figure 2, S_{xy} refers to the switches used while D_{xy} refers to the protection diodes where x= '*a*' or '*b*' or '*c*' while y='1' or '2' or '3'. From the Fig it can thus be summed

Using KVL,

$$V_{an} = V_{aN} + V_{nN} \quad (7)$$

$$V_{bn} = V_{bN} + V_{nN} \quad (8)$$

$$V_{cn} = V_{cN} + V_{nN} \quad (9)$$

The per phase equations of the BLDC motor are then developed as follows:

$$V_{an} = Ri_a + L\frac{di_a}{dt} + E_{an} \quad (10)$$

$$V_{bn} = Ri_b + L\frac{di_b}{dt} + E_{bn} \quad (11)$$

$$V_{cn} = Ri_c + L\frac{di_c}{dt} + E_{cn} \quad (12)$$

Or, $V_{aN} = i_a R + L\frac{di_a}{dt} + E_{aN} + V_{vN}$ (13)

$$V_{bN} = i_b R + L\frac{di_b}{dt} + E_{bN} + V_{vN} \quad (14)$$

$$V_{cN} = i_c R + L\frac{di_c}{dt} + E_{cN} + V_{vN} \quad (15)$$

Where,

V_{xn} = Per phase voltage across the windings.

V_{xN} = Per phase pole voltage applied.

V_{nN} = Pole to neutral voltage applied.

V_{dc} = DC voltage applied in Volts.

$L =$ Inductance of the windings in Henry.

$R =$ Resistance of the windings in Ohms.

E_{xn} = Per Phase Back emf of the motor.

x = Phase 'a' or 'b' or 'c'

There are basically two modes of conduction, two phase mode and three phase mode. During the two phase mode of conduction considering phases 'a' and 'b' are conducting while 'c' is floating, From equation (10) and (11),

$$V_{aN} + V_{bN} - 2V_{nN} = E_{an} + E_{bn}$$

But since

$$E_{an} = -E_{bn}$$

So, $V_{nN} = (V_{aN} + V_{bN})/2$ (16)

Similarly during the three phase mode of conduction,

$$V_{aN} + V_{bN} + V_{cN} - 3V_{nN} = E_{an} + E_{bn} + E_{cn} \quad (17)$$

Or, $V_{nN} = \frac{1}{3}\left[\Sigma V_{xN} - \Sigma E_{xn}\right]$ (18)

The above equations (7), (8) and (9) can be written as,

$$\begin{pmatrix} \frac{di_a}{dt} \\ \frac{di_b}{dt} \\ \frac{di_c}{dt} \end{pmatrix} = \begin{pmatrix} -R/L & 0 & 0 \\ 0 & -R/L & 0 \\ 0 & 0 & -R/L \end{pmatrix}\begin{pmatrix} i_a \\ i_b \\ i_c \end{pmatrix} + \frac{1}{L}\begin{pmatrix} V_{aN} \\ V_{bN} \\ V_{cN} \end{pmatrix} +$$

$$\frac{1}{L}\begin{pmatrix} -E_{an} - V_{nN} & 0 & 0 \\ 0 & -E_{bn} - V_{nN} & 0 \\ 0 & 0 & -E_{cn} - V_{nN} \end{pmatrix} \quad (19)$$

Where, $i_x =$ Current in Ampere.

$V_{xN} = V_{dc}$ =Voltage as input.

E_{xn}, V_{nN} = Disturbance input.

The relation between torque and speed can be obtained by the following differential equation as,

$$T = J\frac{d\omega}{dt} + B\omega + T_L \quad (20)$$

$T =$ Torque in Newton-meter

$J =$ Moment of inertia in $Kg.m^2$

$T_L =$ Disturbance input.

$B =$ Coefficient of friction in Kg/ms .

The above equation in (20) can also be written as

$$\frac{d\omega}{dt} = \frac{1}{J}\left(-B\omega + T - T_L\right) \quad (21)$$

4. PROPOSED CONTROL ALGORITHM

As per sliding surface concept form Slotine and Li, 1991[8] the switching surface for a tracking problem can be expressed as, $S = \left(\alpha + \frac{d}{dt}\right)^{n-1} e$ (22)

For that, $n =$ Order of the system.

$\alpha =$ Constant.

$e =$ Error signal (Reference output-actual output).

For current control of each phase the sliding surfaces are developed as follows:

$$S_1 = e_a = i_a^* - i_a \quad (23)$$

$$S_2 = e_b = i_b^* - i_b \quad (24)$$

$$S_3 = e_c = i_c^* - i_c \quad (25)$$

Where, i_x^* = Desired/Reference value of current

i_x = Obtained value of current.

Now by using the power rate reaching law approach, the sliding mode controllers can be designed.

$$\dot{S}_1 = -\varepsilon_1 |S_1|^{\alpha_1} sgn(S_1) = \frac{de_a}{dt} \quad (26)$$

Considering phase 'a' only, from equation (26),

$$\dot{S}_2 = -\varepsilon_2 |S_2|^{\alpha_2} sgn(S_2) = \frac{de_b}{dt} \quad (27)$$

$$\dot{S}_3 = -\varepsilon_3 |S_3|^{\alpha_3} sgn(S_3) = \frac{de_c}{dt} \quad (28)$$

$$\frac{de_a}{dt} = \frac{d(i_a^* - i_a)}{dt} = -\varepsilon_1 |S_1|^{\alpha_1} sgn(S_1) \quad (29)$$

$$\text{Or}, V_{aN} = L\left(\varepsilon_1 |S_1|^{\alpha_1} sgn(S_1) + \frac{d i_a^*}{dt}\right) + E_{an} + (i_a^* - e_a)R + V_{nN} \quad (30)$$

Similarly the control laws developed for phases 'b' and 'c' are,

$$V_{bN} = L\left(\varepsilon_2 |S_2|^{\alpha_2} sgn(S_2) + \frac{d i_b^*}{dt}\right) + E_{bn} + (i_b^* - e_b)R + V_{nN} \quad (31)$$

$$V_{cN} = L\left(\varepsilon_3 |S_3|^{\alpha_3} sgn(S_3) + \frac{d i_c^*}{dt}\right) + E_{cn} + (i_c^* - e_c)R + V_{nN} \quad (32)$$

The inner loop speed control of the three phase BLDC motor is achieved by the use of the control laws developed in equations (30), (31) and (32). For the outer speed loop we use Proportional- Integral control action. The PI controller is of the form $G_{PI} = (K_p + \frac{K_i}{s})$ to control the speed error.

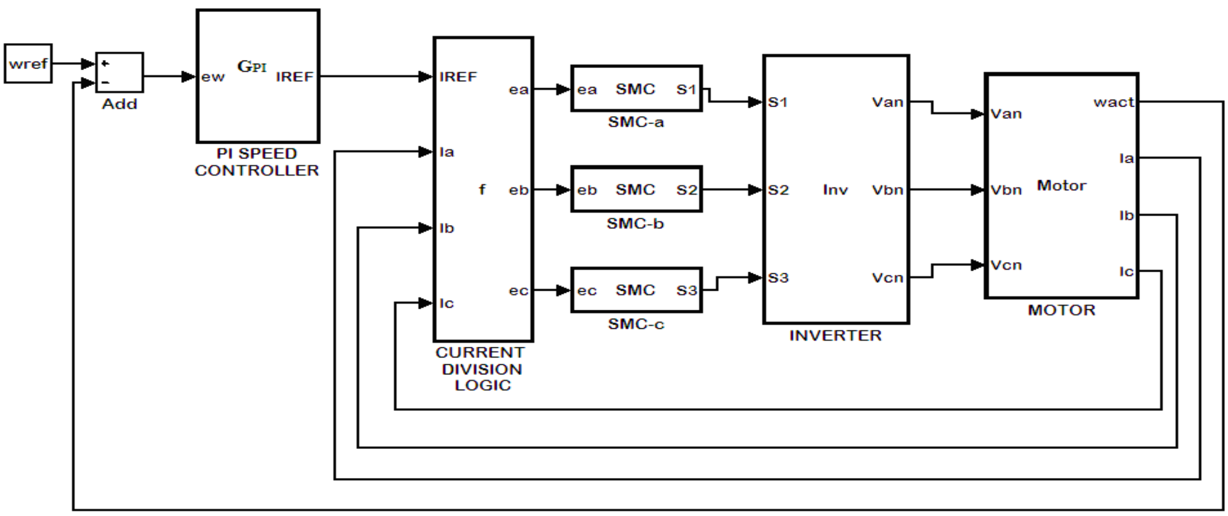

Fig 3: Schematic of the developed control scheme

5. RESULTS

5.1 Simulation

The simulation of the above designed controllers was carried out on MATLAB/SIMULINK platform tracking a reference speed of 100 rad/sec.

5.1.1 Under No Load condition

For simulation purposes in accordance with power rate reaching law approach, the constants are taken $\alpha \in \mathbb{R}^{+}, 0<\alpha<1, \varepsilon \in \mathbb{R}^{+}, 0<\varepsilon<100000$ for the inner current loop.

(a) With $\varepsilon \in \mathbb{R}^{+}, 0<\varepsilon<15000$ and $K_p = 1, K_i = 0.001, \alpha = 0.9$ all the performance like percentage speed and current error, settling time, percentage of overshoot are summarized is in Table 1.

TABLE 1: SIMULATION AT NO LOAD FOR ' ε '

Symbol	*Quantity*	$\varepsilon = 5000$	$\varepsilon = 10000$	$\varepsilon = 15000$
ew(%)	Error (Speed)	0.008	0.0083	0.0085
ec(%)	Error(Current)	1.2674	1.1038	1.104
tw	Speed Settling Time(Sec)	0.046	0.046	0.046
tc	Current Settling Time(sec)	0.00068	0.00068	0.00068
M_p(%)	Percentage of Overshoot	0.03667	0.026	0.023167

(b)With $\alpha \in \mathbb{R}^{+}, 0<\alpha<1$ and $K_p = 1, K_i = 0.001, \varepsilon = 5000$ all the performance like speed and current error, settling time, percentage of overshoot are summarized is in Table 2.

TABLE 2: SIMULATION AT NO LOAD FOR ' α '

Symbol	*Quantity*	$\alpha = 0.5$	$\alpha = 0.75$	$\alpha = 0.95$
ew(%)	Error (Speed)	0.008	0.0075	0.0073
ec(%)	Error(Current)	2.247	2.737	2.9008
tw	Speed Settling Time(Sec)	0.046	0.046	0.046
tc	Current Settling Time(sec)	0.00067	0.00067	0.00067
M_p(%)	Percentage of Overshoot	0.03667	0.03667	0.03667

(c)With $K_p \in \mathbb{R}^{+}, 0<K_p<10$ and $\alpha = 0.7, K_i = 0.001, \varepsilon = 5000$ all the performance like speed and current error, settling time, percentage of overshoot are summarized is in Table 3.

TABLE 3: SIMULATION AT NO LOAD FOR ' K_P '

Symbol	*Quantity*	$K_p = 0.1$	$K_p = 1$	$K_p = 5$
ew(%)	Error (Speed)	0.0867	0.008	0.00133
ec(%)	Error(Current)	2.4108	2.4281	2.574
tw	Speed Settling Time(Sec)	0.047	0.047	0.047
tc	Current Settling Time(sec)	0.00067	0.00067	0.00067
M_p(%)	Percentage of Overshoot	-0.08163	0.03767	0.14067

From the simulation results it was concluded that with the increase in values of ' ε ' the percentage error in current as well as speed increases along with a decrease in percentage overshoot in speed while settling time of current and speed is not affected. Increase in the values of ' α ' causes the percentage error in speed to decrease but that of current increases. At the same time the settling time of the speed loop or the current loop is not affected. Increase in values of proportional constant,' K_p ' causes the percentage speed error and of current to increase along with increase percentage overshoot in speed. However the settling time of both current and speed loop remains constant. There was no significant changes observer by the change in values of the integral constant K_i . With the parameters chosen $\alpha = 0.9, \beta = 10000, K_p = 0.5, K_i = 0.01$ as

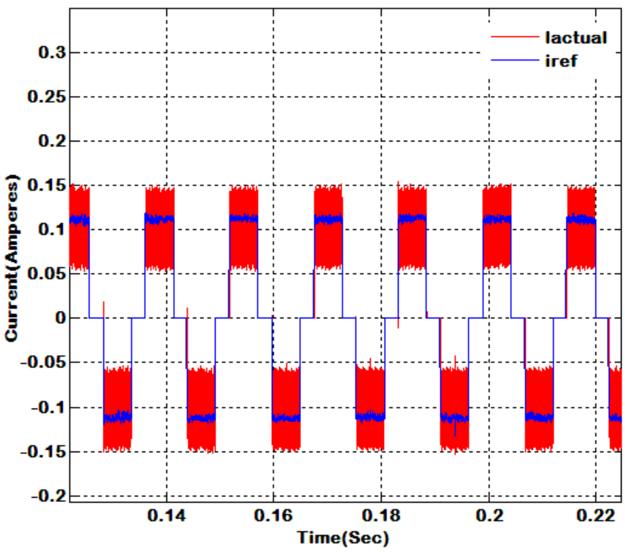

Fig 4: Phase 'a' Current response

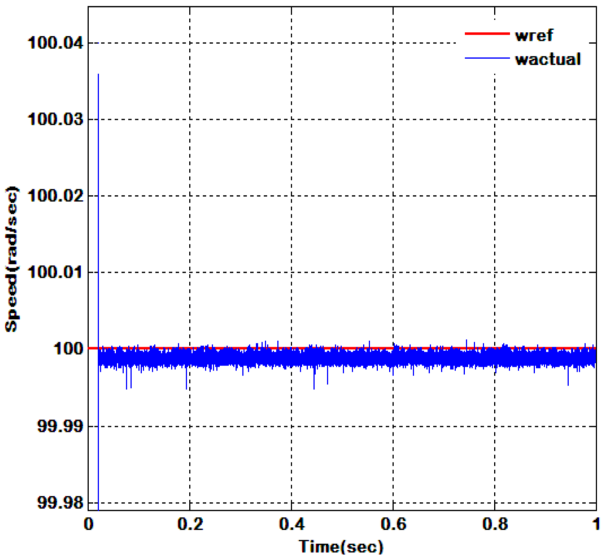

Fig 5: Speed response

In accordance with the power rate reaching law approach the parameters chosen as per equation (6) make the current error to reach the sliding mode in 68 μ s where the calculated value is 1.19ms. The system thus reaches the sliding mode in time.

5.1.2 Under Load condition

With the parameters of the control law for the inner current loop and the outer speed loop chosen as $\alpha = 0.9, \beta = 5000, K_p = 1, K_i = 0.001$ the system was subjected to a sequence of load disturbances and the speed response was analyzed tracking a speed of 100 rad/sec. To enhance the performance of the P-I based speed controller a gain $K^{'} = K / K_t$ was introduced into the system where K_t is the torque constant of the motor.

Table 4: Simulation On Load

Symbol	*Quantity*	$T_l = 10\%$	$T_l = 20\%$	$T_l = 30\%$
ew(%)	Error (Speed)	0.011	0.034	0.065
ec(%)	Error(Current)	0.489	0.7186	2.248
tw	Speed Settling Time(Sec)	0.036	0.027	0.011
tc	Current Settling Time(sec)	0.00316	0.00215	0.00288
M_p(%)	Percentage of Overshoot	9.11	0.1167	0.1029

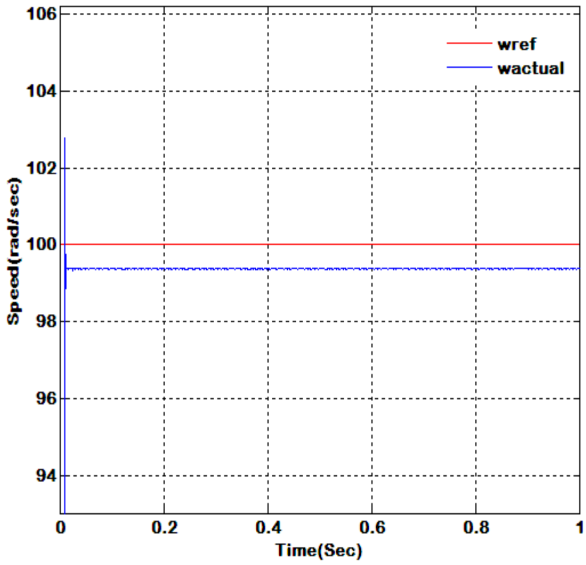

Fig 6: Speed response of Phase 'a' at load of 30%

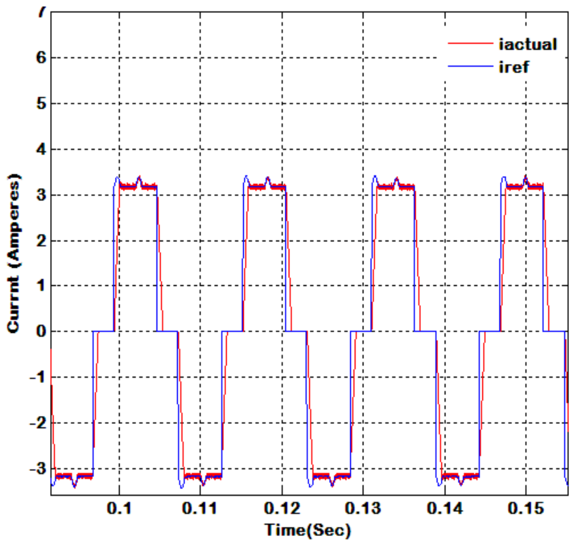

Fig 7: Current response of Phase 'a' at load of 30%

5.2 Experimental Verification

The developed control scheme was then realized on hardware and the results were obtained experimentally. A 36 V, 440W, 1000 rpm 3-Phase Brushless DC motor with specifications as R (Resistance) =0.198 ohms, L (Inductance) =0.0011 Henry, K_t (Torque constant) =0.098, K_b (Back emf Constant)=0.105 was used for experimental purposes. NI DAQ card 6229[9] was used as the interfacing device and the controller developed on MATLAB/SIMULINK platform was then used to control the speed of the motor under no load conditions. The Schematic of the developed real time MATLAB model is shown in Fig 8. The experiment was carried out with a sampling time of 0.0001seconds.

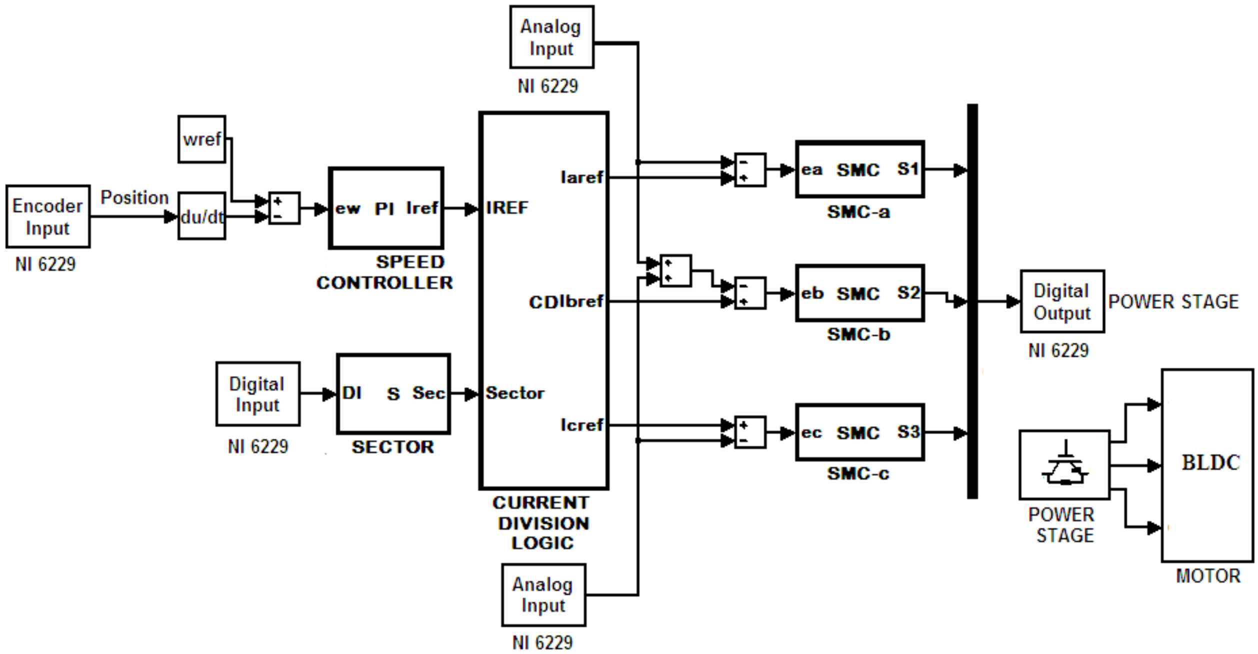

Fig 8: Schematic of MATLAB-NI DAQ Card interface

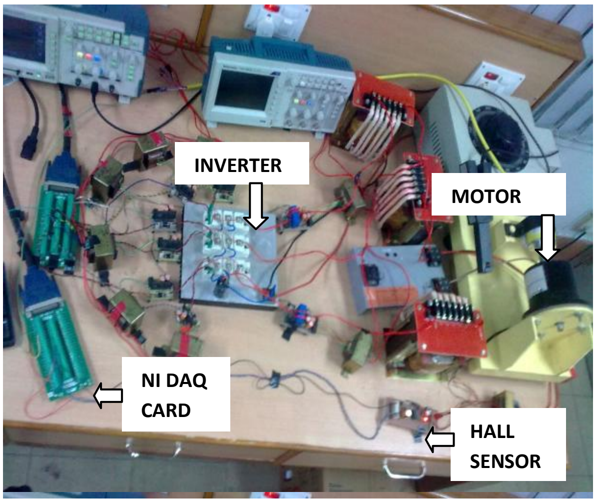

Fig 9: Hardware Setup of the 3- Phase BLDC

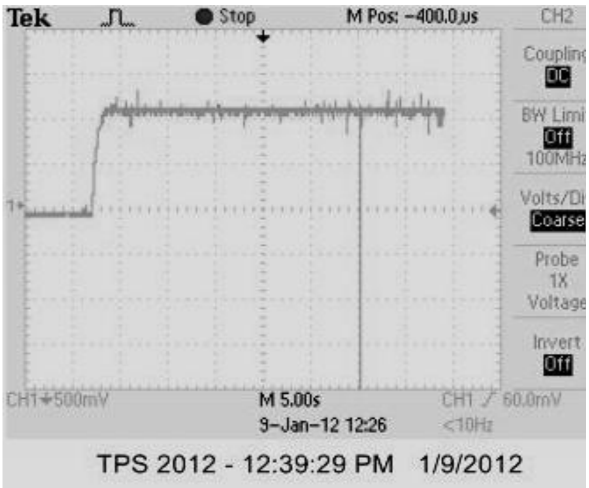

Fig 11: Speed response of the motor at 700 rpm

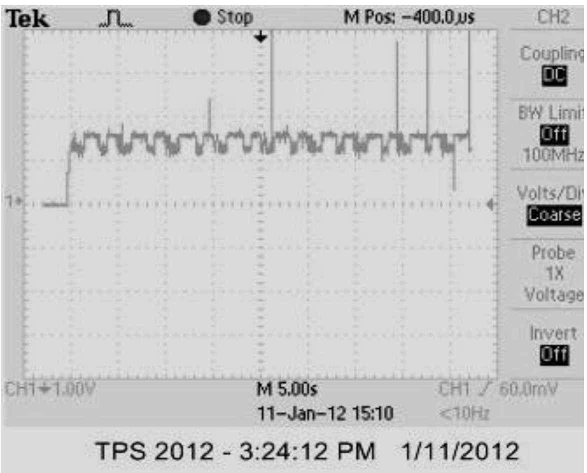

Fig 10: Speed Response tracking speed of 1000rpm

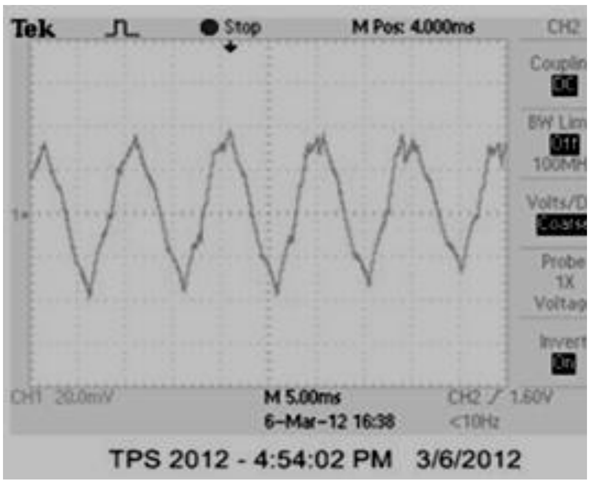

Fig 12: Current Response tracking speed of 1000rpm

However due to high spikes which resulted due to chattering the experiment was then carried out to track a desired speed of 700 rpm at no load conditions. To filter the ripples in the current an inductance of 5mH was added to each phase as shown in Fig 9. The output of the SMC controllers was fed to the gates of the 3-phase VSI based on IGBT, SKM75GB063D after proper PWM modulation. The frequency of the PWM was kept at 1 kHz. The speed of the motor was detected using PG encoder EN 801. In the inner current loop a sliding mode based back emf observer [10] was designed to obtain the values of phase to phase back emf.

6. CONCLUSION

In this work a PI-SMC based control scheme was effectively designed for a 3-Phase Brushless DC motor. The designed controller satisfied the reaching time criterion and was able to obtain efficient speed tracking under load disturbances up to rated load conditions. The controller was then verified experimentally on MATLAB platform using NI DAQ card 6229 interface on a 48 V, 440W motor under no load condition and the results were analyzed. The real time performance of the drive was limited however because of the limitations of MATLAB and thus the work can be further extended to use faster processors like DSP and analyze the control scheme under load condition.

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