Identification of MIMO Hammerstein models using Singular Value Decomposition approach

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ABSTRACT

In this paper, we present a new approach to identify multivariable Hammerstein systems based on the Singular Value Decomposition (***SVD***) method. The technique allows for the determination of the memoryless static nonlinearity as well as the estimation of the model parameters of the dynamic Auto-Regressive model with eXogenous input (***ARX***) part. First of all, an iteration procedure is proposed to identify the parameters of Multi-Input Multi-Output (***MIMO***) Hammerstein models by using the Recursive Least Squares (***RLS***) algorithm. Secondly, the obtained parameter estimates of the identification model include the product terms of the parameters of the original systems. So, to separate these parameters of the original parameters from the product terms, the singular value decomposition method is discussed. Finally, a simulation study is performed to demonstrate the effectiveness of the proposed method compared with the existing approaches.

General Terms

Hammerstein systems, System identification.

Keywords

MIMO Hammerstein systems, Parameter estimation, Singular value decomposition method, Recursive least squares algorithm, Non linear systems.

1. INTRODUCTION

Transfer function models are used for design of control systems for mildly nonlinear systems. However, for highly nonlinear systems, the controller design based on the linear model may not be adequate. For such cases, the linear model based on a fixed controller will not give a satisfactory response. Indeed, a suitable nonlinear model representation will be desirable [16]. Two of the most frequently studied nonlinear systems are the Hammerstein and Wiener models where the nonlinear block is static and follows or followed by a linear system. A Hammerstein model consists of static nonlinear block followed by a linear dynamic block, and a Wiener model consists of linear dynamic block followed by a static nonlinear function.

To identify the Hammerstein model, a various system identification methods have been proposed in the literature. However, most of the methods focus on Single-Input Single-Output (***SISO***) processes [4 – 10, 32]. The first work which developed an iterative identification procedure for Hammerstein model is presented by Narendra et al. [26]. Recently, to guarantee the global convergence of the model parameters in an iterative manner [14] developed an updating algorithm based on the Lyapunov approach. In addition, several approaches have been proposed to identify Hammerstein models in a non-iterative fashion. For examples Pottmann et al. in [27] proposed a two-stage identification algorithm to extract the model parameters. To separate the identification of the linear dynamic part from that of the static nonlinear part, Sung in [29] used a special test signal. Laksminarayanan et al. in [19] proposed multivariate statistical tools to identify the Hammerstein models. Al-Duwaish et al. in [2] used an hybrid model consisting of a neural network to identify the static nonlinear part in series with Auto-Regressive Moving Average (***ARMA***) model for identification of ***SISO*** and Multi-Input Multi-Output (***MIMO***) Hammerstein models. Several other identification and controller design methods for Hammerstein models were developed by [1, 20, 30].

Although there are some methods that can handle ***MIMO*** processes [22], many of them assume the structure of the nonlinearity to be separate [28], i.e., the i^{th} output of the nonlinear function is only affected by the i^{th} input. neural networks or fuzzy logic [1, 2, 14, 15], and polynomial with cross-terms have often been used to deal with more general nonlinearities. Recently, several approaches have been proposed to identify ***MIMO*** Hammerstein models. For examples, Liu et al. in [22] extended Sung's [29] decoupling method to ***MIMO*** systems, which however requires nonlinear optimization. Kwong et al. in [18] extended Ramos's [28] method to ***MIMO*** models. They proposed an approach based on multivariable cardinal cubic spline functions to model the static nonlinearities. The proposed method is effective in modeling processes with hard and/or coupled nonlinearities.

The authors in [12, 13] presented a method for the identification of Hammerstein models based on Least Squares Support Vector Machines (***LS-SVMs***). It will be shown that the linear model parameters and the static nonlinearity can be obtained by solving a set of linear equations with size in the order of the number of observations. Hlaing et al. in [11] proposed a generalized Hammerstein model consisting of a static polynomial function in series with a time-varying linear model in order to model the Hammerstein-like multivariable processes whose linear dynamics vary over the operating space. An iteration procedure is proposed to identify the generalized Hammerstein model by using the Just-In-Time Learning (***JITL***) technique.

For Hammerstein systems, the parameters from the identification model include the products of the original system parameters [3, 25], so separating the original parameters from the obtained parameter estimates of the product terms is required. The authors in [6 – 9] proposed a simple average method of separating parameters for Hammerstein models. Another separating parameter method is the singular value decomposition done by Bai in [3].

This paper presents an algorithm for identification of ***MIMO*** Hammerstein nonlinear systems by using the Recursive Least Squares (***RLS***) with an exponential forgetting factor and uses the singular value decomposition method to separate the system estimated parameters.

The paper is organized as follows. Section 2 describes the system formulation related to the ***MIMO*** Hammerstein models. Section 3 presents an identification algorithm to estimate the parameters of the system. Section 4 introduces a separating parameter method. The main results are given in section 5. Section 6 gives some conclusions.

2. PROBLEM FORMULATION

Two possible structures as depicted in figures 1 and 2 can be used to describe a ***MIMO*** Hammerstein model depending on whether the nonlinearities are separate or combined [2, 19]. The combined nonlinearity case is more general, but it can cause a very challenging parameter estimation problem because of the large number of parameters to be estimated [11]. Therefore, the ***MIMO*** Hammerstein model with separate nonlinearities will be considered in this paper (figure 2). As mentioned previously, the system consists of a static part which contains all the nonlinearity followed by a Linear Time Invariant (***LTI***) model $H(q^{-1})$ which contains all the dynamics of the process.

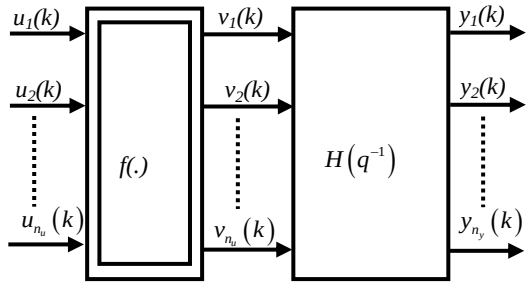

Fig 1 : *MIMO Hammerstein model with combined nonlinearities.*

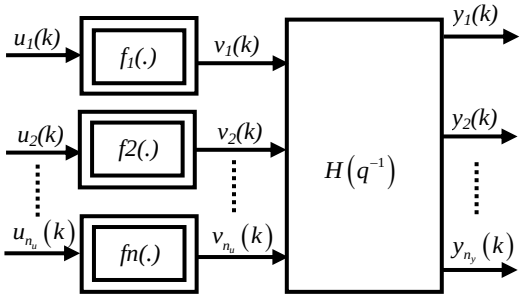

Fig 2 : *MIMO Hammerstein model with separate nonlinearities.*

where

$U(k)=\begin{bmatrix} u_1(k) & \cdots & u_{n_u}(k) \end{bmatrix}^T$, $Y(k)=\begin{bmatrix} y_1(k) & \cdots & y_{n_y}(k) \end{bmatrix}^T$ and $E(k)=\begin{bmatrix} e_1(k) & \cdots & e_{n_y}(k) \end{bmatrix}^T$ are the system input, output and white noise with zero mean at time ***k*** respectively. $f_i(.)$ are polynomials of a known order in the input as follows :

$$f_i(k)=\gamma_i^1 u_i(k)+\gamma_i^2 u_i^2(k)+\cdots+\gamma_i^{m_i} u_i^{m_i}(k) \quad (1)$$

where γ_i^1, $\gamma_i^2,\cdots,$ $\gamma_i^{m_i}$ present the nonlinear system parameters.

Assume that the ***LTI*** system has the Auto-Regressive with eXogenous input (***ARX***) structure (but other structure like Auto-Regressive Moving Average with eXogenous input (***ARMAX***), Auto-Regressive Integrated Moving Average with eXogenous input (***ARIMAX***) etc. are also possible). The input– output relationship is then given by :

$$Y(k)=\sum_{i=1}^{n_A} A_i\, Y(k-i)+\sum_{j=1}^{n_B} B_j\, V(k-j)+E(k) \quad (2)$$

$$V(k)=\begin{bmatrix} v_1(k) \\ \vdots \\ v_i(k) \\ \vdots \\ v_{n_u}(k) \end{bmatrix}=\begin{cases} v_1(k)=f_1(u_1(k))=\sum_{r=1}^{m_1}\gamma_1^r\, u_1^r(k) \\ \vdots \\ v_i(k)=f_i(u_i(k))=\sum_{r=1}^{m_i}\gamma_i^r\, u_i^r(k) \\ \vdots \\ v_{n_u}(k)=f_{n_u}(u_{n_u}(k))=\sum_{r=1}^{m_{n_u}}\gamma_{n_u}^r u_{n_u}^r(k) \end{cases} \quad (3)$$

where

$A_i=\begin{bmatrix} a_{11}^i & \cdots & a_{1n_y}^i \\ \vdots & \cdots & \vdots \\ a_{n_y1}^i & \cdots & a_{n_yn_y}^i \end{bmatrix}\in \mathbb{R}^{n_y \times n_y}$ and $B_j=\begin{bmatrix} b_{11}^j & \cdots & b_{1n_u}^j \\ \vdots & \cdots & \vdots \\ b_{n_y1}^j & \cdots & b_{n_yn_u}^j \end{bmatrix}\in \mathbb{R}^{n_y \times n_u}$

denote the linear system parameter matrices.

Assume that for $k \le 0$, $u_j(k) = 0$, $y_j(k) = 0$ and $e_j(k) = 0$ for $j \in \{1, \ldots, n_y\}$ and n_A, n_B, m_j for $j \in \{1, \ldots, n_u\}$, n_y and n_u represent the order of the output, the order of the input, the order of the nonlinearity, the number of outputs and the number of inputs respectively.

This paper presents an identification algorithm to estimate the parameters a_{kj}^i, b_{kj}^i and γ_i^r of the system in (2) from given input – output data $\{u_i(k), y_i(k)\}$ and to evaluate the accuracy of the estimated parameters by simulation on computers.

The ***i*th** output equation from system (2) can be written as follows

$$\begin{aligned}
&y_i(k) = A_i^1 Y(k-1) + \cdots + A_i^{n_A} Y(k-n_A) + b_{i1}^1 \gamma_1^1 u_1(k-1) + \cdots + \\
&b_{i1}^1 \gamma_1^{m_1} \left(u_1(k-1)\right)^{m_1} + b_{i2}^1 \gamma_2^1 u_2(k-1) + \cdots + b_{i2}^1 \gamma_2^{m_2} \left(u_2(k-1)\right)^{m_2} + \\
&+ \cdots + b_{i\,n_u}^1 \gamma_{n_u}^1 u_{n_u}(k-1) + \cdots + b_{i\,n_u}^1 \gamma_{n_u}^{m_{n_u}} \left(u_{n_u}(k-1)\right)^{m_{n_u}} + \\
&b_{i1}^2 \gamma_1^1 u_1(k-2) + \cdots + b_{i1}^2 \gamma_1^{m_1} \left(u_1(k-2)\right)^{m_1} + b_{i2}^2 \gamma_2^1 u_2(k-2) + \\
&: + \cdots + b_{i2}^2 \gamma_2^{m_2} \left(u_2(k-2)\right)^{m_2} + \cdots + b_{i\,n_u}^2 \gamma_{n_u}^1 u_{n_u}(k-2) + \cdots + \qquad (4) \\
&b_{i\,n_u}^2 \gamma_{n_u}^{m_{n_u}} \left(u_{n_u}(k-2)\right)^{m_{n_u}} + \cdots + b_{i1}^{n_B} \gamma_1^1 u_1(k-n_B) + \cdots + \\
&b_{i1}^{n_B} \gamma_1^{m_1} \left(u_1(k-n_B)\right)^{m_1} + b_{i2}^{n_B} \gamma_2^1 u_2(k-n_B) + \cdots + \\
&b_{i2}^{n_B} \gamma_2^{m_2} \left(u_2(k-n_B)\right)^{m_2} + \cdots + b_{i\,n_u}^{n_B} \gamma_{n_u}^1 u_{n_u}(k-n_B) + \cdots + \\
&b_{i\,n_u}^{n_B} \gamma_{n_u}^{m_{n_u}} \left(u_{n_u}(k-n_B)\right)^{m_{n_u}}
\end{aligned}$$

where $A_i^j = \left[a_{i1}^j \quad a_{i2}^j \quad \cdots \quad a_{i\,n_y}^j \right]$, for $j \in \{1, 2, \ldots, n_A\}$ and for $i \in \{1, 2, \ldots, n_y\}$.

Then, we can write all the parameters of the system in (4) as a vector form :

$$\theta_i = \left[A_i^1 \quad \cdots \quad A_i^{n_A} \quad B_i^1 \quad \cdots \quad B_i^{n_B} \quad \gamma_1 \quad \cdots \quad \gamma_{n_u} \right]^T \qquad (5)$$

where $B_i^j = \left[b_{i1}^j \quad b_{i2}^j \quad \cdots \quad b_{i\,n_u}^j \right]$, for $j \in \{1, 2, \ldots, n_B\}$ and for $i \in \{1, 2, \ldots, n_y\}$.

Before closing this section, we observe that the parameterization of the ***MIMO*** Hammerstein model is actually not unique. For instance, any pair $\left(\delta_i B_i^j, \delta_i^{-1} \gamma_i^r\right)$ for some nonzero and finite constants δ_i provides an identical system as the one in (4). In the other words, any identification experiment cannot distinguish between the parameter vector sets $\left(B_i^j, \gamma_i^r\right)$ and $\left(\delta_i B_i^j, \delta_i^{-1} \gamma_i^r\right)$. Therefore, to obtain a unique parameterization, without loss of generality, one of the elements of $\left(B_i^j, \gamma_i^r\right)$ has to be fixed. So, we adopt the following assumption :

Assumption : For system (4), assume that $B_i^j \left(\gamma_i^r\right)^T$ are not zero and $\|\gamma_r\|_2 = 1$ or $\|B_i^j\|_2 = 1$ ($\|\ldots\|_2$ stands for the 2-norm) where $r = 1, 2, \ldots, n_u$, $i = 1, 2, \ldots, n_y$ and $j = 1, 2, \ldots, n_B$ [1, 26].

3. IDENTIFICATION OF THE MIMO HAMMERSTEIN MODEL

Define the parameter vector ϑ_i and information vector $\Psi(k)$ of the system in (4) as :

$$\begin{aligned}
\vartheta_i = \Big[& A_i^1 \quad \cdots \quad A_i^{n_A} \quad b_{i1}^1 \gamma_1 \quad \cdots \quad b_{i\,n_u}^1 \gamma_{n_u} \quad b_{i1}^2 \gamma_1 \quad \cdots \quad b_{i\,n_u}^2 \gamma_{n_u} \\
& \cdots \quad b_{i1}^{n_B} \gamma_1 \quad \cdots \quad b_{i\,n_u}^{n_B} \gamma_{n_u} \Big]^T \in \mathbb{R}^{1 \times n_0}
\end{aligned} \qquad (6)$$

$$\Psi(k) = \begin{bmatrix} \psi(k) \\ \phi(k) \end{bmatrix} \in \mathbb{R}^{n_0}, \; n_0 = n_A n_y + n_B \sum_{i=1}^{n_u} m_i \qquad (7)$$

$$\psi(k) = \left[\psi_1(k) \quad \psi_2(k) \quad \cdots \quad \psi_i(k) \quad \cdots \quad \psi_{n_A}(k) \right]^T \in \mathbb{R}^{1 \times n_A n_y}$$

$$\psi_i(k) = \left[y_1(k-i)\, y_2(k-i) \cdots y_{n_y}(k-i) \right]^T \in \mathbb{R}^{1 \times n_y}, \; i = 1, 2, \ldots, n_A$$

$$\begin{aligned}
\phi(k) = \Big[& f_1\left(u_1(k-1)\right) \quad \cdots \quad f_{n_u}\left(u_{n_u}(k-1)\right) \\
& \cdots \quad f_1\left(u_1(k-n_B)\right) \quad \cdots \quad f_{n_u}\left(u_{n_u}(k-n_B)\right) \Big]^T \in \mathbb{R}^{1 \times m_i n_B n_u}
\end{aligned}$$

$$f_i\left(u_i(k)\right) = \left[u_i(k) \left(u_i(k)\right)^2 \cdots \left(u_i(k)\right)^{m_i} \right]^T \in \mathbb{R}^{1 \times m_i}, \; i = 1, 2, \ldots, n_u$$

Then, equation (4) is rewritten as :

$$y_i(k) = \Psi^T(k) \vartheta_i + e_i(k) \qquad (8)$$

Equation (7) is formulated in a standard state space format for the nonlinear ***MIMO*** Hammerstein ***ARX*** system in (4). Note that $\psi(k)$ and $\phi(k)$ in the information vector $\Psi(k)$ are available.

We define the prediction error $\varepsilon_i(k)$ by the following expression :

$$\varepsilon_i(k) = y_i(k) - \Psi^T(k) \hat{\vartheta}_i(k-1) \qquad (9)$$

The ***RLS*** method is an effective approach in online identification.

This technique is to discount old measurements so that the model adapts to the changing situation dynamically. The complete algorithm [17, 19, 32] of the ***RLS*** method for ***ARX*** modeling is given as follows [24] :

$$\begin{cases}
\hat{\vartheta}_i(k) = \hat{\vartheta}_i(k-1) + P(k) \Psi(k) \varepsilon_i(k) \\
P(k) = \dfrac{1}{\lambda(k)} \left[P(k-1) - \dfrac{P(k-1) \Psi(k) \Psi^T(k) P(k-1)}{\lambda(k) + \Psi^T(k) P(k-1) \Psi(k)} \right] \\
\varepsilon_i(k) = y_i(k) - \Psi^T(k) \hat{\vartheta}_i(k-1)
\end{cases} \qquad (10)$$

where $P(k) \in \mathbb{R}^{n_0 \times n_0}$ is the parameter estimation error covariance matrix with $P(0) = \alpha * I_{n_0 \times n_0}$, where $\boldsymbol{\alpha}$ is a positive scalar. Also, $\boldsymbol{\lambda(k)}$ is an exponential forgetting function to discount old measurements and can be determined by the following first-order difference equation :

$$\lambda(k) = \lambda_0 \lambda(k-1) + \lambda^0 (1 - \lambda_0) \qquad (11)$$

where $0 < \lambda_0 < 1$, $0 < \lambda^0 < 1$ and $\lim_{k \to \infty} \lambda(k) = \lambda^0$.

For the purposes of comparison, three different performance criteria have been computed. Namely, the Mean Square Error (***MSE***), the Variance Accounted For (***VAF***) and the best ***FIT*** criterion who are given by the following expressions, respectively :

$$MSE_i = \frac{1}{M} \sum_{k=1}^{M} \left(y_i(k) - \hat{y}_i(k) \right)^2 \qquad (12)$$

$$VAF_i = \max \left\{ 1 - \frac{Var\left(y_i(k) - \hat{y}_i(k) \right)}{Var\left(y_i(k) \right)}, 0 \right\} \text{x} 100\% \qquad (13)$$

where $y_i(k)$ denotes the real output, $\hat{y}_i(k)$ denotes the output of the model, ***M*** is the number of validation data and

$Var\{.\}$ denotes the variance of a quasi-stationary signal [24, 25].

$$FIT_i = \left(1 - \frac{\| Y_i - Y_{i\,v} \|}{\| Y_i - y_{i\,mean} \|}\right) \text{x} 100 \qquad (14)$$

where $\boldsymbol{Y_i}$ is a vector containing the output of the model when it is simulated with the validation input data, $\boldsymbol{Y_{i\,v}}$ is a vector with the validation output data and $\boldsymbol{y_{i\,mean}}$ is the mean value of the output $\boldsymbol{y_i}$ [16].

4. SEPARTING PARAMETERS : SINGULAR VALUE DECOMPOSITION METHOD (SVD METHOD)

After getting the estimates of the parameter vector $\hat{\vartheta}_i(k)$ (for $i \in \{1, ..., n_y\}$) by the above ***RLS*** algorithm, the following step is to obtain the estimated parameter vector $\hat{\theta}_i(k)$ of θ_i from the parameter vector $\hat{\vartheta}_i(k)$.

Firstly, the estimates $\hat{A}_i^j(k)$ of A_i^j (for $j \in \{1, ..., n_A\}$) can be read from the first n_A . n_y entries of the parameter vector $\hat{\vartheta}_i(k)$. Secondly, to obtain the other parameters which include the estimates of the elements products of the parameter vector $\hat{\theta}_i(k)$, the singular value decomposition method is discussed.

Under assumption 1 with $\|\gamma_i\|_2 = 1$ (for $i \in \{1, ..., n_u\}$), the singular value decomposition method [25] is applied to decompose the parameter vector $\hat{\vartheta}_i(k)$ of the $\boldsymbol{i^{th}}$ Hammerstein system.

To simplify the matrix expression, we omit *(k)* and denote $\hat{\gamma_r^j}\,\hat{b_{ir}^j}(k)$ by $\hat{\gamma_r^j}\,\hat{b_{ir}^j}$, and rearrange the n_A .n_y + 1 to n_0 entries of $\hat{\vartheta}_i(k)$ into

$$\hat{\Theta}_i^j = \hat{\gamma}_j . \hat{\beta}_j^T = \begin{bmatrix} \hat{\gamma_j^1}\,\hat{b_{i\,j}^1} & \hat{\gamma_1^1}\,\hat{b_{i\,j}^2} & \cdots & \hat{\gamma_j^1}\,\hat{b_{i\,j}^{n_B}} \\ \hat{\gamma_j^2}\,\hat{b_{i\,j}^1} & \hat{\gamma_1^2}\,\hat{b_{i\,j}^2} & \cdots & \hat{\gamma_j^2}\,\hat{b_{i\,j}^{n_B}} \\ \vdots & \vdots & \cdots & \vdots \\ \hat{\gamma_j^{m_j}}\,\hat{b_{i\,j}^1} & \hat{\gamma_2^{m_j}}\,\hat{b_{i\,j}^2} & \cdots & \hat{\gamma_j^{m_j}}\,\hat{b_{i\,j}^{n_B}} \end{bmatrix} \in \mathbb{R}^{\,m_j \text{ x } n_B} \qquad (15)$$

where $\hat{\beta}_j = \begin{bmatrix} \hat{b}_{i\,j}^1 & \hat{b}_{i\,j}^2 & \cdots & \hat{b}_{i\,j}^{n_B} \end{bmatrix}$ and for $j \in \{1, ..., n_u\}$.

The problem is how to estimate the parameter vectors β_j and γ_j from the estimate $\hat{\Theta}_i^j$. It is clear that the closest estimates $\hat{\beta}_j$ and $\hat{\gamma}_j$, in the 2-norm sense, are those that solve the following optimization problem :

$$\left(\hat{\gamma}_j, \hat{\beta}_j\right) = \arg\min_{\gamma_j, \beta_j} \left\{ \left\| \hat{\Theta}_i^j - \gamma_j \beta_j^T \right\|_2^2 \right\} \qquad (16)$$

The solution to this optimization problem is provided by the ***SVD*** of the matrix $\hat{\Theta}_i^j$. The result is summarized in the following theorem [4].

Theorem 1 : Let $\hat{\Theta} \in \mathbb{R}^{\,n \text{ x } m}$ have rank $k > p$, and let the economy-size ***SVD*** of Θ_i^j be given by :

$$\hat{\Theta} = R_k \sum_k \left(W_k\right)^T = \sum_{i=1}^{k} \sigma_i r_i \left(w_i\right)^T \qquad (17)$$

where $\sum_k$ is a diagonal matrix containing the k nonzero singular values $\left(\sigma_i,\ i = 1, ..., k\right)$ of $\hat{\Theta}$ in nonincreasing order, the matrices $R_k = \begin{bmatrix} r_1 & \cdots & r_k \end{bmatrix} \in \mathbb{R}^{\,n \text{ x } k}$ and $W_k = \begin{bmatrix} w_1 & \cdots & w_k \end{bmatrix} \in \mathbb{R}^{\,m \text{ x } k}$ contain only the first k columns of the unitary matrices $R \in \mathbb{R}^{\,n \text{ x } n}$ and $W \in \mathbb{R}^{\,m \text{ x } m}$ is provided by the full ***SVD*** of $\hat{\Theta}$, $\hat{\Theta} = R \sum W^T$, respectively. Then, the matrices $\hat{a} \in \mathbb{R}^{\,n \text{ x } p}$ and $\hat{b} \in \mathbb{R}^{\,m \text{ x } p}$ that minimize the norm $\left\| \hat{\Theta} - a\, b^T \right\|_2^2$, are given by :

$$\left(\hat{a}, \hat{b}\right) = \arg\min_{a,b} \left\{ \left\| \hat{\Theta} - a\, b^T \right\|_2^2 \right\} = \left(R_1, W_1 \Sigma_1\right) \qquad (18)$$

where $R_1 \in \mathbb{R}^{\,n \text{ x } p}$, $W_1 \in \mathbb{R}^{\,m \text{ x } p}$ and $\Sigma_1 = diag\left(\sigma_1, \sigma_2, ..., \sigma_p\right)$ are given by the following partition of the economy-size ***SVD*** in (18),

$$\hat{\Theta} = \begin{bmatrix} R_1 & R_2 \end{bmatrix} \begin{bmatrix} \Sigma_1 & 0 \\ 0 & \Sigma_2 \end{bmatrix} \begin{bmatrix} W_1^T \\ W_2^T \end{bmatrix} \qquad (19)$$

and the approximation error is given by :

$$\left\| \hat{\Theta} - \hat{a}\, \hat{b}^T \right\|_2^2 = {\sigma_{p+1}}^2 \qquad (20)$$

Using this input, we can improve the linear part of the model. So, this algorithm can be summarized as follows:

Algorithm :

Step 1: Compute the least squares estimate $\hat{\vartheta}_i(k)$ as in (10), and the matrix $\hat{\Theta}_i^j$ such that :

$$\hat{\vartheta}_i = \text{blockvec}\left(\hat{\Theta}_i^j\right) \qquad (21)$$

Step 2: Compute the economy-size ***SVD*** of $\hat{\Theta}_i^j$ as in Theorem 1, and the partition of this decomposition as in Eq. (19).

Step 3: Compute the estimates of the parameter matrices β_j and γ_j as respectively :

$$\begin{aligned} \hat{\gamma}_j &= R_1 \\ \hat{\beta}_j &= W_1 \Sigma_1 \end{aligned} \qquad (22)$$

5. SIMULATION EXAMPLES AND DISCUSSIONS

To illustrate the proposed identification approach, we introduce two simulation examples. Without loss of generality, a multivariable process with two inputs and two outputs will be utilized to detail the proposed identification procedure.

5.1 Example 1

Consider a process with two inputs and two outputs described by the following equations :

$$\begin{bmatrix} y_1(k) \\ y_2(k) \end{bmatrix} = \begin{bmatrix} 0.5 & -0.1 \\ 0.8 & -0.7 \end{bmatrix} \begin{bmatrix} y_1(k-1) \\ y_2(k-1) \end{bmatrix} + \begin{bmatrix} -0.3 & 0.2 \\ 0.9 & -0.5 \end{bmatrix} \begin{bmatrix} y_1(k-2) \\ y_2(k-2) \end{bmatrix} + \begin{bmatrix} 0.7 & -1.5 \\ 0.85 & 0.65 \end{bmatrix} \begin{bmatrix} v_1(k-1) \\ v_2(k-1) \end{bmatrix} + \begin{bmatrix} e_1(k) \\ e_2(k) \end{bmatrix} \quad (23)$$

where the nonlinearities are given by :

$$v_1(k) = 0.8585\, u_1(k) + 0.0149\, u_1(k)^2 - 0.5113\, u_1(k)^3 - 0.0263\, u_1(k)^4 \quad (24)$$
$$v_2(k) = 0.9\, u_2(k) + 0.4\, u_2(k)^2 + 0.1721\, u_2(k)^3$$

The inputs $\{u_1(k), u_2(k)\}$ are a multi-step signal sequence which shown in Fig. 3, and $\{e_1(k), e_2(k)\}$ are a white noise sequence with zero mean and variance $\sigma^2 = 0.1^2$. The output responses $\{y_1(k), y_2(k)\}$ and their estimates are shown in Fig. 4 and Fig. 5 respectively. True nonlinearity and mean estimated nonlinearity are shown in Fig. 6 and Fig. 7 respectively.

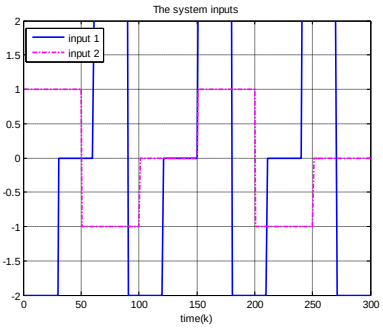

Fig 3. ***The training input sequence for*** $u_1(k)$ ***and*** $u_2(k)$.

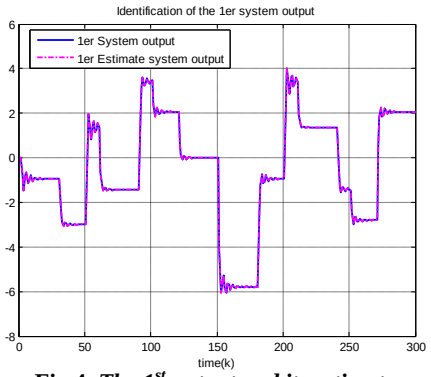

Fig 4. ***The 1st output and its estimate***

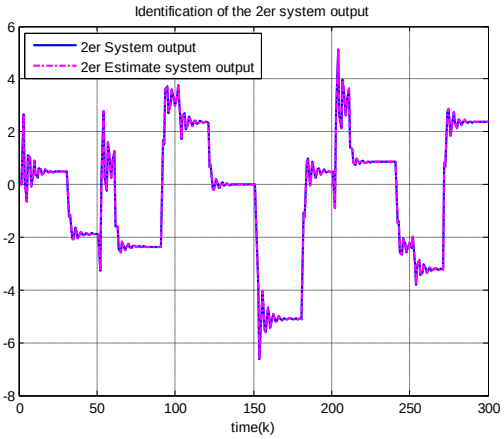

Fig 5. ***The 2nd output and its estimate.***

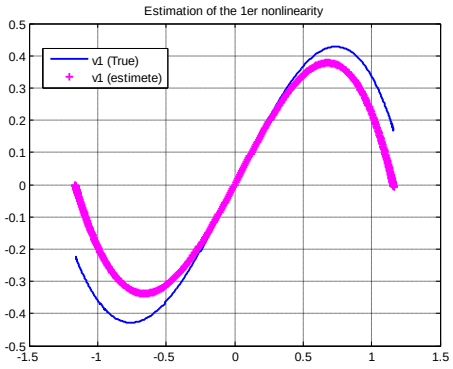

Fig 6. ***The 1st nonlinearity and its estimate.***

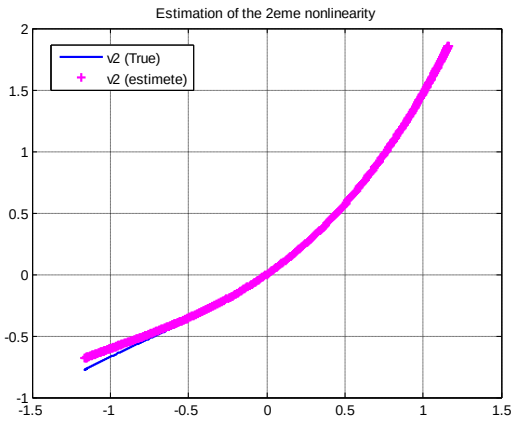

Fig 7. ***The 2nd nonlinearity and its estimate.***

By using the proposed method, the matrices in the ***ARX*** model and the vectors of the nonlinearities are converged to the following :

$$\hat{A}_1 = \begin{bmatrix} 0.4824 & -0.1029 \\ 0.7932 & -0.6926 \end{bmatrix}; \hat{A}_2 = \begin{bmatrix} -0.2886 & 0.1936 \\ 0.8836 & -0.4931 \end{bmatrix};$$

$$\hat{B}_1 = \begin{bmatrix} 0.4554 & -1.4731 \\ 0.7245 & 0.6293 \end{bmatrix}$$

$$\hat{\gamma}_1 = [0.8002 \quad 0.0685 \quad -0.5934 \quad -0.0538];$$

$$\hat{\gamma}_2 = [0.8880 \quad 0.4346 \quad 0.1501]$$

It is clear that the matrices $\hat{A}_1(k)$ and $\hat{A}_2(k)$ are closed to the same matrices in the ***ARX*** plant model. But the matrix $\hat{B}_1(k)$ and the vectors $\hat{\gamma}_1$ and $\hat{\gamma}_2$ are different from the model. This uncertainty is related to the ***SVD*** approach. However, the multiplication and the output of the system converged to the nonlinearities at the inputs which are shown in Figs. 4, 5, 6 and 7. The different performance criteria with the ***SVD*** method between the actual and identified nonlinearities are shown in Table 1.

	1st system	***2nd system***
MSE	4.3804e-007	6.7237e-007
VAF (%)	100.0000	100.0000
FIT (%)	99.9734	99.9660

Table 1. ***The different performance criteria***

5.2 Example 2

Consider a nonlinear process described by the Hammerstein system as follows :

$$\begin{bmatrix} y_1(k) \\ y_2(k) \end{bmatrix} = \begin{bmatrix} 0.7 & 0.2 \\ 0.1 & -0.5 \end{bmatrix} \begin{bmatrix} y_1(k-1) \\ y_2(k-1) \end{bmatrix} + \begin{bmatrix} 0.18 & 0.1 \\ 0.15 & 0.14 \end{bmatrix} \begin{bmatrix} y_1(k-2) \\ y_2(k-2) \end{bmatrix} + \begin{bmatrix} 0.3 & 0.1 \\ 0.5 & 1.6 \end{bmatrix} \begin{bmatrix} v_1(k-1) \\ v_2(k-1) \end{bmatrix} + \begin{bmatrix} -0.06 & 0.01 \\ 0.1 & -0.16 \end{bmatrix} \begin{bmatrix} v_1(k-2) \\ v_2(k-2) \end{bmatrix} + \begin{bmatrix} e_1(k) \\ e_2(k) \end{bmatrix} \quad (25)$$

where the nonlinearities are given by :

$$v_1(k) = u_1(k) + 0.50\, u_1(k)^2$$
$$v_2(k) = 0.941\, u_2(k) + 0.028\, u_2(k)^3 \quad (26)$$

The inputs $\{u_1(k), u_2(k)\}$ are a multi-step signal sequence which shown in Fig. 8, and $\{e_1(k), e_2(k)\}$ are a white noise sequence with zero mean and variance $\sigma^2 = 0.1^2$. The output responses $\{y_1(k), y_2(k)\}$ and their estimates are shown in Fig. 9 and 10 respectively. True nonlinearity and mean estimated nonlinearity are shown in Fig. 11 and Fig. 12 respectively.

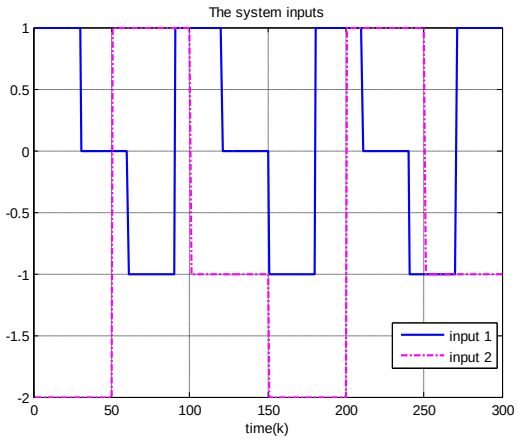

Fig. 8. ***The training input sequence for*** $u_1(k)$ ***and*** $u_2(k)$**.**

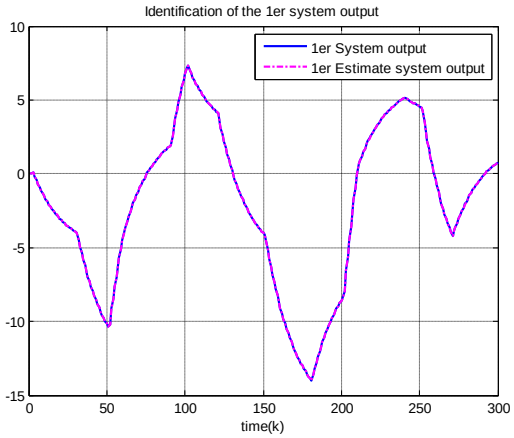

Fig. 9. ***The 1st output and its estimate.***

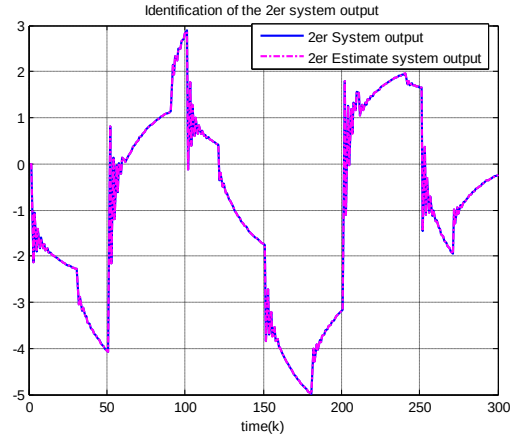

Fig. 10. ***The 2nd output and its estimate***.

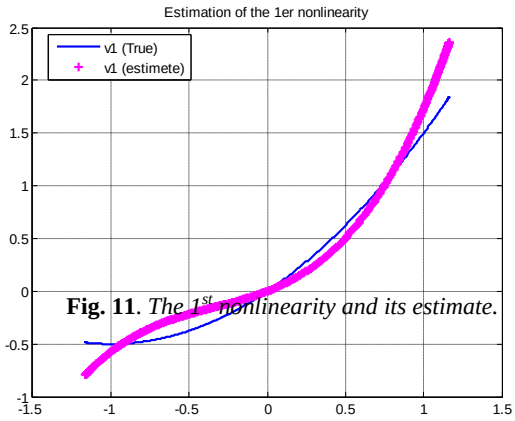

Fig. 11. *The 1st nonlinearity and its estimate.*

Fig. 11. The 1st nonlinearity and its estimate

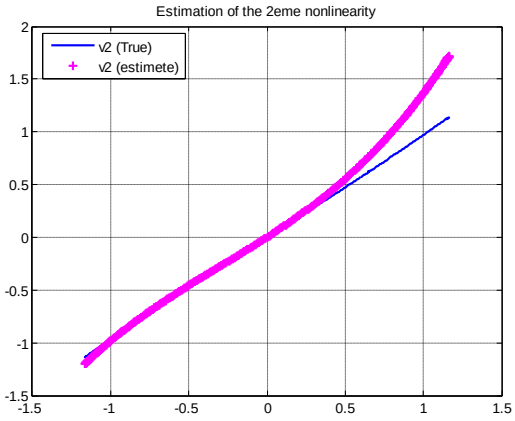

Fig. 12. ***The 2nd nonlinearity and its estimate.***

By using the proposed method, the matrices in the ***ARX*** model and the vectors of the nonlinearity are converged to the following :

$$\hat{A}_1 = \begin{bmatrix} 0.6861 & 0.1968 \\ 0.0995 & -0.4958 \end{bmatrix}; \hat{A}_2 = \begin{bmatrix} 0.1796 & 0.0987 \\ 0.1468 & 0.1373 \end{bmatrix};$$

$$\hat{B}_1 = \begin{bmatrix} 0.2329 & 0.0637 \\ 0.3861 & 1.0093 \end{bmatrix}; \hat{B}_2 = \begin{bmatrix} -0.0455 & 0.0117 \\ 0.0086 & -0.0243 \end{bmatrix}$$

$$\hat{\gamma}_1 = [0.5794 \quad 0.5782]; \hat{\gamma}_2 = [0.9568 \quad 0.2183]$$

The results were obtained using the same algorithm that the matrices $\hat{A}_1(k)$, $\hat{A}_2(k)$, $\hat{B}_1(k)$ and $\hat{B}_2(k)$ are closed to the same matrices in the ***ARX*** plant model. In the same way, the output of the system converged to the nonlinearities at the inputs which are shown in Figs. 9, 10, 11 and 12. The different performance criteria with the ***SVD*** method between the actual and identified nonlinearities are shown in Table 2.

	1st system	***2nd system***
MSE	1.1988 e-005	7.5143 e-007
VAF (%)	100.0000	100.0000
FIT (%)	99.9357	99.9578

Table 2. *The different performance criteria*

5.3 DISCUSSIONS

In this paper, ***MIMO*** Hammerstein model identification based on ***RLS*** algorithm and ***SVD*** method for decomposition of the linear and nonlinear parameters have been demonstrated for two simulation examples. This approach offers the advantage of being more general than the other approaches to present in other papers such as the ***LS-SVM*** approach presented by [13] and ***JITL*** approach presented by [11] of made that these two approaches consider that the matrices are selected equal to the identities, on the other hand in our approach we consider the case more general where ***B*** is unspecified.

It is important to know the conditions under which the ***RLS/SVD*** algorithm will converge. The ***RLS/SVD*** is a combination of the ***RLS*** and ***SVD*** algorithms. Hence, the convergence properties of the ***RLS/SVD*** algorithm are directly associated to the convergence properties of the ***RLS*** and ***SVD*** algorithms. For deterministic systems, it is well known that the ***RLS*** produces unbiased estimates of the parameters provided that the process order is known and the input is persistently exciting [23]. On the other hand, the linear model parameters and the static nonlinearity can be obtained simultaneously by solving a set of linear equations followed by the singular value decomposition (***SVD***). Then, by recurring to ***SVD*** and rank reduction, optimal estimates of the parameter matrices characterizing the linear and nonlinear parts can be obtained.

Comparing the nonlinearities in Figs. 6 and 7 in the first example and Figs. 11 and 12 in the second example, we can consider that the approach is satisfactory by identifying the nonlinearities. The inputs of the first example are shown in Fig. 3 and that of the second example are shown in Fig. 8. The comparison between real and estimate output curves of the system are given in Figs. 4 and 5 for the first example and Figs. 9 and 10 for the second example.

From Tables 1 and 2 and Figs. 3 to 12, we can infer the following conclusions:

- The parameter estimates provided by the identification algorithm converge to their true values.
- The responses of the original system and the identified system are very similar.
- It is clear that the errors are becoming smaller as ***k*** increases. This remark confirms the proposed algorithm.
- For the same data length, the recursive algorithm gives good estimated parameters.
- The convergence of the estimated parameters to their true values.
- The ***MIMO*** Hammerstein model outperforms the estimates model for each of the three considered criteria. This state shows that the proposed algorithm is effective.

6. CONCLUSION

Exhaustive simulations have been used to study the convergence properties of the ***RLS/SVD*** algorithm and to study the effects of the type of static nonlinearity, and noise on the behavior of the ***RLS/SVD*** algorithm. In all cases, strong convergence is observed provided that some guide lines are followed.

In this paper, we have proposed a new technique for the identification of ***MIMO*** Hammerstein ***ARX*** systems. The method is based on recursive least squares (***RLS***) approximation and allows to determine the memoryless static nonlinearity as well as the linear model parameters from a linear set of equations. The obtained estimated parameters of the identification model include the products of the original system parameters. To separate the estimated parameters into the original parameters, the singular value decomposition (***SVD***) method is discussed. Moreover, the proposed method is applicable to ***MIMO*** systems with separate or combined nonlinearities. The recursive algorithm is a novel combination of ***RLS*** and ***SVD*** algorithms. Simulation results reveal the robustness and effectiveness of the proposed method.

7. ACKNOWLEDGMENTS

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