ON (1, 2)*-πgb-CLOSED SETS

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ABSTRACT
In this paper we introduce a new class of sets called $(1, 2)^*$-πgb-closed sets and a new class of generalized functions called $(1, 2)^*$-πgb- continuous maps and $(1, 2)^*$-πgb- irresolute maps in bitopological spaces. Also we obtain basic properties of $(1, 2)^*$-πgb-closed sets. Further, we introduce a new space called $(1, 2)^*$- πgb-space.

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1. INTRODUCTION
Andrijevic [3] introduced an innovative class of generalized open sets in a topological space, the so-called b-open sets. Ekici and Caldas [9] already discussed this type of sets under the name of γ-open sets. The class of b-open sets is contained in the class of semi-pre-open sets and contains all semi-open sets and pre-open sets.It is noteworthy to decipher that there is a unique feature in the class of b-open sets. In subsequent to the findings of these concepts, varied research papers (see [1, 3, 6, 9, 10, 19, 20, and 21]) unleashed new results in various respects. Levine [13] introduced the notion of generalized closed sets in topological space and a class of topological spaces called $T_{\frac{1}{2}}$ spaces. In recent years, research on a large scale was carried out in generalizing closedness as the notions of a generalized closed, generalized semi-closed, α-generalized closed, generalized semi-pre-open closed sets [2,7,13,16,17]. Thivagar et al [22] have introduced the concepts of $(1, 2)^*$-semi open sets, $(1, 2)^*$-α-open sets and $(1, 2)^*$- semi-generalized closed sets, and $(1, 2)^*$- α-generalized closed sets.

This paper attempts to highlight a new type of generalized sets called $(1, 2)^*$-πgb-closed sets and a new class of generalized functions called $(1, 2)^*$-πgb- continuous maps and $(1, 2)^*$-πgb-irresolute maps. These findings results in procuring several characterizations of this class. It analyses its bitopological properties. The application of this set leads to the introduction of a new space called $(1, 2)^*$- πgb-space.

2. PRELIMINARIES
Throughout the present paper (X, τ_1, τ_2), (Y, σ_1, σ_2) and $(Z, \eta1, \eta2)$ (or simply X, Y, Z) denote bitopological spaces.

Definition 2.1: A subset A of a space (X, τ) is called

(1) a pre-open set [15] if $A \subset$ int(cl (A)) and a pre-closed set if cl (int (A))$\subset$ A;

(2) a semi-open set[12] if $A \subset$ cl(int(A)) and a semi-closed set if int (cl(A))$\subset$A;

(3) a α -open set[18] if $A \subset$int (cl(int (A))) and a α -closed set if cl (int(cl (A)))$\subset$ A;

(4) a semi-preopen set[1] if $A \subset$ cl (intcl(A)) and a semi-pre-closed set if int (cl (int(A))) $\subset$ A;

(5) a regular open set if A=int (cl(A)) and a regular closed set if A=cl(int (A));

(6) b-open [3] or sp-open [8], γ −open [9] if A $\subset$ cl(int(A))∪int (cl(A)).

(7) πgα-closed [11] if αCl(A) $\subset$U whenever A $\subset$U and U is π-open in X.

(8) πgb-closed [24] if bCl(A) $\subset$U whenever A $\subset$U and U is π-open in X.

The complement of a b-open set is said to be b-closed [3]. The intersection of all b-closed sets of X containing A is called the b-closure of A and is denoted by bCl(A). The union of all b-open sets of X contained in A is called b-interior of A and is denoted by bInt(A).

Definition 2.2: A function f: $(X, \tau) \rightarrow (Y, \sigma)$ is called

1) π- irresolute: [4] if $f^{-1}(V)$ is π- closed in (X, τ) for every π-closed of (Y, σ).

2) b-irresolute**: [9]** if for each b-open set V in Y,$f^{-1}(V)$ is b-open in X

3) b-continuous**: [9]** if for each open set V in Y,$f^{-1}(V)$ is b-open in X

Definition 2.3: [22] A subset A of X is called $\tau_1\tau_2$ -open if S∈ $\tau_1 ∪ \tau_2$ and the complement of $\tau_1\tau_2$ -open set is $\tau_1\tau_2$ -closed.

Definition 2.4: [22] Let A be a subset of X

(i)   The $\tau_1\tau_2$ -closure of A, denoted by $\tau_1\tau_2$ -cl(S) is defined by ∩{F/S⊂F and F is $\tau_1\tau$-closed}

(ii)   The $\tau_1\tau_2$ -interior of A, denoted by $\tau_1\tau_2$ -int(S) is defined by $\cup\{F/F\subset S$ and F is $\tau_1\tau$-open$\}$

Remark 2.5: [23]

(i) $\tau_1\tau_2$ -int(S) is $\tau_{1,2}$-open for each $S\subset X$ and $\tau_1\tau_2$ -cl(S) is

$\tau_{1,2}$-closed for each $S\subset X$.

(ii)A subset $S\subset X$ is $\tau_{1,2}$-open iff S= $\tau_1\tau_2$ -int(S) and $\tau_{1,2}$-closed iff S= $\tau_1\tau_2$ -cl(S)

(iii) $\tau_1\tau_2$ -int(S)=int $_{\tau1}$(S)$\cup$ int $_{\tau2}$(S) and $\tau_1\tau_2$ -cl(S)=cl

$_{\tau1}$(S)$\cup$ cl $_{\tau2}$(S) for any $S\subset X$.

(iv) for any family $\{S_i/i\in I\}$ of subsets of X we have

(a) $\underset{i}{\cup}$ $\tau_1\tau_2$ -int(S_i)$\subset$ $\tau_1\tau_2$ -int($\underset{i}{\cup}$ S_i)

(b) $\underset{i}{\cup}$ $\tau_1\tau_2$ -cl(S_i)$\subset$ $\tau_1\tau_2$ -cl($\underset{i}{\cup}$ S_i)

(c) $\tau_1\tau_2$ -int($\underset{i}{\cap}$ S_i)$\subset$ $\underset{i}{\cap}$ $\tau_1\tau_2$ -int(S_i)

(d) $\tau_1\tau_2$ -cl($\underset{i}{\cap}$ S_i)$\subset$ $\underset{i}{\cap}$ $\tau_1\tau_2$ -cl(S_i)

(v) $\tau_{1,2}$-open sets need not form a topology.

Definition 2.6: The finite union of (1, 2)*-regular open sets [5] is said to be $\tau_{1,2}$ - π- open. The complement of $\tau_{1,2}$ - π- open is said to be $\tau_{1,2}$ -π- closed.

Definition 2.7: A subset A of a bitopological space (X, $\tau1$, $\tau2$) is called

(i)   (1, 2)*- semi-open set[22] if A$\subset$ $\tau_1\tau_2$- cl($\tau_1\tau_2$ -int(A))

(ii)   (1, 2)*-preopen set [22] if A$\subset$ $\tau_1\tau_2$ -int ($\tau_1\tau_2$ -cl(A))

(iii)   (1, 2)*-α -open set[22] if A$\subset$ $\tau_1\tau_2$ - int ($\tau_1\tau_2$ - cl($\tau_1\tau_2$ - int (A)))

(iv)   (1, 2)*-$\pi g\alpha$-closed [5] if (1, 2)*-αCl(A) $\subset$U whenever A $\subset$U and U is (1, 2)*-π-open in X.

(v)   (1, 2)*-regular open [22] if $A = \tau_{1,2}$-int($\tau_{1,2}$ -cl(A)).
     Complement of the (1, 2)*-regular open set is called (1, 2)*-regular closed set.

(vi)   (1, 2)-b-open [14] if A $\subset$ $\tau_1\tau_2$- cl(τ_1- int(A))$\cup$ τ_1- int ($\tau_1\tau_2$- cl(A)).

(vii)   (1, 2)*- generalized closed (briefly (1, 2)*-g-closed)[22] if $\tau_1\tau_2$ - cl(A) $\subset$ U whenever A $\subset$ U and U is $\tau_{1,2}$- open in X.

(viii)   (1, 2)*- α-closure [22] (resp (1, 2)*-semi-closure) of a subset A of X,denoted by (1, 2)*-αCl(A)   (resp.   (1, 2)*-sCl(A))   is defined to be the intersection of all (1, 2)*- α-closed (resp. (1, 2)*-semi-closed) sets containing A.

(ix)   (1, 2)*- α-interior [22] (resp (1, 2)*-semi-interior) of a subset A of X denoted by (1, 2)*-α-Int(A) (resp. (1, 2)*-sInt(A)) is defined to be the union of all (1, 2)*- α-open(resp. (1, 2)*-semi-open) sets containing A.

(x)   (1, 2)*- semi-generalized closed (briefly (1, 2)*-sg-closed)[22] if (1, 2)*-sCl(A) $\subset$ U whenever A $\subset$ U and U is (1,2)* semi-open in X.

(xi)   (1,2)* -generalized semi closed (briefly (1, 2)*-gs-closed) [22] if (1, 2)*-sCl(A) $\subset$ U whenever A $\subset$ U and U is (1,2)* $\tau_{1,2}$ open in X.

(xii)   (1, 2)*- α-generalized closed (briefly (1, 2)*-αg-closed) [22] if (1,2)* - αCl(A) $\subset$ U whenever A $\subset$ U and U is $\tau_{1,2}$- open in X.

(xiii)   (1,2)* -generalized α-closed (briefly (1, 2)*-gα-closed)[22] if (1,2)* - αCl(A) $\subset$ U whenever A $\subset$ U and U is $\tau_{1,2}$-α- open in X.

3. (1, 2)*-πgb-closed sets

Definition 3.1: A subset A of a bitopological space (X, $\tau1$, $\tau2$) is called (1, 2)*-π generalized b-closed (briefly (1, 2)*-πgb-closed) if $\tau_1\tau_2$-bcl(A)$\subset$U whenever A$\subset$U and U is $\tau_1\tau_2$ -π-open in X.

Definition 3.2: A subset A of a bitopological space (X, $\tau1$, $\tau2$) is called (1, 2)*-b-open [14] if A $\subset$ $\tau_1\tau_2$- cl($\tau_1\tau_2$-int(A))$\cup$ $\tau_1\tau_2$- int ($\tau_1\tau_2$- cl(A)).

Complement of (1, 2)*-b-open is called (1, 2)*-b-closed.

Definition 3.3: A subset A of a bitopological space (X, $\tau1$, $\tau2$) is called (1, 2)*-generalized b-closed (briefly (1, 2)*-gb-closed) if $\tau_1\tau_2$-bcl(A)$\subset$U whenever A$\subset$U and U is $\tau_1\tau_2$-open in X.

The complement of a (1, 2)*-πgb-closed set is called (1, 2)*-πgb-open.

Theorem 3.4: Every (1, 2)*-α-closed set is (1, 2)*-πgb-closed set.

Proof: Let A be a (1, 2)*-α-closed set and U be any $\tau_1\tau_2$ -π-open set containing A. Since A is (1, 2)*-α-closed set, (1, 2)*-

αcl(A) =A⊂U .Every (1, 2)*-α-closed set is (1, 2)*-b-closed, (1, 2)*-bcl(A) ⊂U.

Theorem 3.5: Every $\tau_1\tau_2$ -closed set is (1, 2)*-πgb-closed set.

Proof: Let A⊂U whenever U is $\tau_1\tau_2$ -π-open. Since A is $\tau_1\tau_2$ closed, $\tau_1\tau_2$-cl(A)=A⊂U. Every $\tau_1\tau_2$-closed is (1, 2)*-b-closed. Hence $\tau_1\tau_2$-closed set is (1, 2)*-πgb-closed.

Theorem 3.6: Every (1, 2)*-g-closed set is (1, 2)*-πgb-closed set.

Proof: Let A⊂X be (1, 2)* -g-closed and A⊂U where U is $\tau_1\tau_2$ -π-open.This implies $\tau_1\tau_2$ -cl(A)⊂U whenever A ⊂U and U is $\tau_1\tau_2$ -open in X. Every (1, 2)* -g-closed is (1, 2)*-b-closed. Hence (1, 2)* -bcl(A)⊂U and U is $\tau_1\tau_2$ -π-open in X. Hence (1, 2)*-g-closed set is (1, 2)*-πgb-closed.

Theorem 3.7: Every (1, 2)*-b-closed set is (1, 2)*-πgb-closed set.

Theorem 3.8: Every (1, 2)*-πgα-closed set is (1, 2)*-πgb-closed set.

Theorem 3.9: Every (1, 2)*-pre-closed set is (1, 2)*-πgb-closed set.

Theorem 3.10: Every (1, 2)*-gb-closed set is (1, 2)*-πgb-closed set.

Theorem 3.11: Every (1, 2)* -πgp-closed set is (1, 2)*-πgb-closed set.

Converse of the above statements need not be true as shown in the following examples.

Example 3.12: Consider X={a,b,c,d}, τ_1 ={Φ, {a}, {d}, {a,d}, X} and τ_2={Φ, c,d},{a,c,d} ,X}.Let A={c}.$\tau_1\tau_2$-opensets={Φ,{a},{d},{a,d},{c,d}.{a,c,d},X};
$\tau_1\tau_2$-closed sets={Φ,{b,c,d},{a,b,c},{b,c},{a,b},{b},X};
(1, 2)*-bcl(A)={c}. Hence A is (1, 2)*-πgb-closed set but not $\tau_1\tau_2$-closed, (1, 2)*-g-closed, (1, 2)*-gb-closed.

Example 3.13: Consider X ={a,b,c,d}, τ_1 = {Φ,{a},{b},{a,b},{a,b,c}X} and τ_2 = {Φ,{a,b,d}X}.$\tau_1\tau_2$–open set= {Φ, {a}, {b}, {a,b}, {a,b,c}, {a,b,d},X} $\tau_1\tau_2$–closed set= {Φ,{a,c,d},{b,c,d},{c,d},{d},{c},X}.Let A={a,b,c}.Here A is (1, 2)*-πgb-closed but not (1, 2)*-b-closed

Example 3.14: Let X={a,b,c,d,e} and τ_1 = {Φ,{a,b},a,b,c,d},X}, τ_2 ={Φ,{c,d},{a,b,c,d},X}, A={a}. $\tau_1\tau_2$-open sets= {Φ,{ a,b}, {c,d}, {a,b,c,d},X};
$\tau_1\tau_2$-closed sets={Φ,{c,d,e},{a,b,e},{e},X};
$\tau_1\tau_2$-π-opensets={Φ,{a,b},{c,d},{a,b,c,d},X};
A is (1, 2)*-πgb- closed but not, (1, 2)*-α- closed, (1, 2)*-πgα-closed.

Example 3.15: Let X={a,b,c,d,e} and τ_1 = {Φ,{a,b},a,b,c,d},X}, τ_2 = {Φ,{c,d},{a,b,c,d},X}, A={a,b}. $\tau_1\tau_2$-opensets={Φ,{a,b},{c,d},{a,b,c,d},X};

$\tau_1\tau_2$-closed sets={Φ,{c,d,e},{a,b,e},{e},X};
$\tau_1\tau_2$-π-opensets={Φ,{a,b},{c,d},{a,b,c,d},X};
A is (1, 2)*-gb- closed but not, (1, 2)*-pre- closed, (1, 2)*-πgp- closed.

Remark 3.16: Finite union of (1, 2)*-πgb- closed sets need not be (1, 2)*-πgb -closed.

Example 3.17: Consider X={a,b,c}, τ_1={ Φ,{a},{a,b},X} and τ_2={ Φ,{b},{a,b},X}. $\tau_1\tau_2$ –open set = {Φ ,{a},{ b},{a,b} ,X},$\tau_1\tau_2$–closed set= {Φ,{a,c},{b,c}{c},X}. Let A={a},B={b}.Here A and B are (1, 2)*-πgb- closed but A ∪ B ={a,b} is not (1, 2)*-πgb-closed since {a,b}is $\tau_1\tau_2$ –π-open and bcl({a,b}) = X.

Remark 3.18: Finite intersection of (1, 2)*-πgb -closed sets need not be (1, 2)*-πgb- closed.

Example 3.19: Consider X ={a,b,c,d}, τ_1 = {Φ,{a},{b},{a,b},{a,b,c}X} and τ_2 = {Φ,{a,b,d}X}.$\tau_1\tau_2$–open set= {Φ, {a}, {b}, {a,b}, {a,b,c}, {a,b,d},X} $\tau_1\tau_2$–closed set= {Φ,{a,c,d},{b,c,d},{c,d},{d},{c},X}.Let A={a,b,c},B={a,b.d}.Here A and B are (1, 2)*-πgb-closed but A ∩ B ={a,b} is not (1, 2)*- πgb- closed, since {a,b}is $\tau_1\tau_2$ -π-open and (1, 2)*-bcl({a,b})=X.

Theorem 3.20: If a set A is (1, 2)*-πgb-closed in (X, τ_1, τ_2), then (1, 2)* bcl(A) –A does not contain any non empty $\tau_1\tau_2$-π-closed set.

Proof: Let A be (1, 2)*-πgb-closed and F be a non empty $\tau_1\tau_2$ -π-closed set such that F⊂ (1, 2)* bcl(A)-A. Since A is (1, 2)*-πgb-closed, A ⊂ X-F where X-F is $\tau_1\tau_2$ -π-open implies (1, 2)* -bcl (A) ⊂ X-F. Hence F⊂ X-(1, 2)* bcl(A).Now F⊂(1, 2)*- bcl (A)∩(X-(1, 2)*-bcl (A)) =Φ which is a contradiction. Therefore (1, 2)*-bcl(A) does not contain any non empty $\tau_1\tau_2$ -π- closed set.

Corollary 3.21: Let A be (1, 2)* πgb -closed in (X, τ_1, τ_2) Then A is (1, 2)* -b-closed iff (1, 2)*- bcl(A)-A is $\tau_1\tau_2$ -π -closed.

Proof: Let A be (1, 2)* - b-closed. Then (1, 2)* - bcl(A) = A. This implies (1, 2)* -bcl(A)-A= Φ which is $\tau_1\tau_2$ -π- closed. Assume (1, 2)*-bcl(A)-A is $\tau_1\tau_2$ -π- closed. Then (1, 2)*-bcl(A)-A= Φ. Hence, (1, 2)*-bcl (A) = A.

Theorem 3.22: If A is $\tau_1\tau_2$ -π-open and (1, 2)*-πgb-closed subset of (X, τ_1, τ_2), then A is (1, 2)* -b-closed of (X, τ_1, τ_2).

Proof: Since A⊂A and A is $\tau_1\tau_2$ -π-open, (1, 2)*-πgb-closed then (1, 2)*-bcl(A)⊂A.Hence A=(1, 2)*-bcl(A).Therefore A is (1, 2)*-b-closed.

Theorem 3.23: Let A is (1, 2)*-πgb-closed subset of (X, τ_1, τ_2) and B is any set such that A ⊂B⊂ (1, 2)*-bcl (A), then B is (1, 2)*-πgb-closed subset of (X, τ_1, τ_2).

Proof: Let B ⊂ U and U be (1, 2)*-π-open of (X, τ_1, τ_2). Given A ⊂ B. Then A ⊂ U.Since A is (1, 2)*-πgb- closed,

A⊂ U implies $(1, 2)^*$-bcl(A) ⊂ U. Also since B⊂ $(1, 2)$*-bcl (A) it follows that $(1, 2)$*-bcl(B) ⊂ $(1, 2)$*-b cl $((1, 2)$*-bcl (A)) =$(1, 2)$*-bcl (B) ⊂ U. Hence B is a $(1, 2)^*$-πgb-closed subset of (X, τ_1, τ_2).

Theorem 3.24: Let B ⊆ A ⊆ X where A is $(1, 2)^*$-πgb-closed and $\tau_1\tau_2$ -π- open set. Then B is $(1, 2)$*-πgb- closed relative to A iff B is $(1, 2)$*-πgb-closed in X.

Proof: Let B⊆A⊆X where A is $(1, 2)^*$-πgb- closed and $\tau_1\tau_2$ -π-open set. Let B be $(1, 2)^*$-πgb-closed in A. Let B⊂U where U is $\tau_1\tau_2$-π-open in X.Since B⊂A, B = B∩A⊂U∩A, this implies $(1, 2)$*-bcl(B) = $(1, 2)$*- bcl$_A$(B)⊂U∩A⊂U .Hence is $(1, 2)$*-πgb-closed in X.

Let B be $(1, 2)^*$-πgb-closed in X. Let B⊂ O where O is $\tau_1\tau_2$ -π-open in A. Then O=U∩A where U is $\tau_1\tau_2$ -π-open in X. This implies B⊂O=U∩A⊂U. Since B is $(1, 2)$*-πgb-closed in X, $(1, 2)$*-bcl(B)⊂U, $(1, 2)$*-bcl$_y$(B)⊂A∩ $(1, 2)$*-bcl(B)⊂U∩A=O that is bcl$_A$(B)⊂O.Hence,B is $(1, 2)$*-πgb-closed relative to A.

4. APPLICATIONS

Definition 4.1: A subset A of (X, τ_1, τ_2) is called $(1, 2)$*-πgb- open if and only if its compliment is $(1, 2)$*-πgb- closed in (X, τ_1, τ_2).

Remark 4.2: $(1, 2)^*$-bcl (X-A) = X- $(1, 2)^*$-bint (A)

Theorem 4.3: Every $\tau_1\tau_2$ –open set is $(1, 2)^*$- πgb-open.

Converse of the above theorem need not be true as shown in the following example.

Example 4.4: Let X= {a,b,c}. τ_1={Φ,{a,b},X}; τ_2 = {Φ ,{a,c} ,X}.$\tau_1\tau_2$–open set= {Φ,{a,c},{a,b},X} ; {a} is $(1, 2)$*-πgb-open but not $\tau_1\tau_2$ –open.

Theorem 4.5: A subset A of (X, τ_1, τ_2) is $(1, 2)^*$-πgb-open iff F ⊂ $(1, 2)^*$-bint (A) whenever F is $\tau_1\tau_2$ -π-closed and F⊂A

Proof: Necessity: Let A be $(1, 2)^*$-πgb-open in (X, τ_1, τ_2). Let F be $\tau_1\tau_2$ -π-closed and F ⊂ A. Then X-A ⊂ X-F where X-F is $\tau_1\tau_2$ -π-open. By assumption $(1, 2)^*$- bcl (X-A) ⊂ X-F (by theorem 3.18).By remark 4.2, X- $(1, 2)^*$-bint (A) ⊂ X-F. Hence F ⊂ $(1, 2)^*$- bint (A).

Sufficiency: Suppose F is $\tau_1\tau_2$ -π- closed and F ⊂ A such that F⊂ $(1, 2)^*$-bint(A).Let X-A ⊂ U where U is $\tau_1\tau_2$ -π-open. Then X-U⊂A where X-U is $\tau_1\tau_2$ - π-closed. By hypothesis X-U⊂ $(1, 2)^*$-bint (A)⇒X- $(1, 2)^*$-bint (A) ⊂ U⇒$(1, 2)^*$-bcl (X-A) ⊂ U. Thus X-A is $(1, 2)^*$-πgb-closed and A is $(1, 2)^*$-πgb-open.

Theorem 4.6: If $(1, 2)$*- bint (A) ⊂ B ⊂ A and A is $(1, 2)$*-πgb-open then B is $(1, 2)$*-πgb-open.

Proof: Let $(1, 2)^*$-bint (A) ⊂B⊂A.Thus X-A⊂ X-B ⊂ X-$(1, 2)^*$-bint A.(ie) X-A⊂X-B ⊂ $(1, 2)$*- bcl (X-A) by remark 4.2. Since A is $(1, 2)^*$-πgb- open, X-A is $(1, 2)$*-πgb- closed. Since X-A is $(1, 2)$*-πgb- closed, by theorem 3.23, (X-A) ⊂ (X-B) ⊂ $(1, 2)^*$- bcl (X-A)⇒(X-B) is $(1, 2)^*$-πgb- closed. Hence B is $(1, 2)^*$-πgb-open.

Definition 4.7: A space (X, τ_1, τ_2) is called a $(1, 2)^*$-πgb-$T_{1/2}$ space if every $(1, 2)^*$-πgb- closed set is b-closed.

Theorem 4.8: For a bitopological space (X, τ_1, τ_2) the following are equivalent

(i) (X, τ_1, τ_2) is $(1, 2)^*$- πgb-$T_{1/2}$ space.

(ii)Every singleton set is τ1τ2 -π-closed or $(1, 2)^*$-b-open.

Proof: To prove (i) implies (ii).Let X be a $(1, 2)^*$-πgb-$T_{1/2}$ space .Let x ∈ X and assuming that {x} is not $\tau_1\tau_2$ -π- closed of (X, τ_1, τ_2). Then clearly X-{x} is not $\tau_1\tau_2$ -π- open. So X is the only $\tau_1\tau_2$ -π- open set containing X-{x}. Hence X-{x} is trivially a $(1, 2)^*$-πgb-closed set. Since (X, τ_1, τ_2) is $(1, 2)^*$-πgb- $T_{1/2}$ space, X-{x} is $(1, 2)^*$- b-closed set of (X, τ_1, τ_2) or equivalently, {x} is $(1, 2)^*$-b-open set of (X, τ_1, τ_2).

(ii) implies (i) Assume every singleton of (X, τ_1, τ_2) is either $(1, 2)$*-π-closed or $(1, 2)$*-b-open.Let A be a $(1, 2)^*$-πgb-closed set of (X, τ_1, τ_2). Obviously A⊂ $(1, 2)^*$-bcl(A). Let {x} be $(1, 2)^*$-bcl (A) .To prove $(1, 2)^*$-bcl(A) ⊂ A.

Case (i): Let {x} be $\tau_1\tau_2$ - π- closed. Suppose {x} does not belong to A. Then {x} ∈ bcl(A)-A ,by theorem 3.18,which is a contradiction. Therefore {x} ∈A. Hence $(1, 2)^*$-bcl(A) ⊂ A.

Case(ii): Let {x} be $(1, 2)$*-b-open .Since {x}∈ $(1, 2)$*-bcl(A),we have {x}∩ A≠Φ,This implies {x} ∈ A. Therefore $(1, 2)^*$-bcl (A) ⊂A. Hence Therefore $(1, 2)^*$-bcl (A) =A or equivalently A is $(1, 2)^*$-b-closed. Hence (X, τ_1, τ_2) is $(1, 2)$*-πgb-$T_{1/2}$ space.

Definition 4.9: A topological space X is a $(1,2)^*$-πgb-space if every $(1, 2)^*$-πgb- closed set is $\tau_1\tau_2$ -closed.

Theorem 4.10: Every $(1, 2)$*-πgb-space is $(1, 2)$*-πgb-$T_{1/2}$ space

Proof: Suppose A is $(1, 2)$*-πgb-closed in (X, τ_1, τ_2). By assumption A is $\tau_1\tau_2$ -closed. Every $\tau_1\tau_2$ -closed set is $\tau_1\tau_2$ -b-closed. Therefore A is $(1, 2)$*-πgb- $T_{1/2}$ space.

Converse of the above theorem need not be true as shown in the following example.

Example 4.11: Let X={a,b,c}; τ_1={Φ,{a},X}; {Φ,{b},X}; $\tau_1\tau_2$ –open sets= {Φ,{a},{b},X}.(X, τ_1, τ_2) is $(1, 2)$*-πgb-$T_{1/2}$ space but not $(1, 2)$*-πgb-space since {b} is $(1, 2)$*-πgb-closed but not $\tau_1\tau_2$-closed.

5. (1, 2)*-πgb- continuous and (1, 2)*-πgb-irresolute functions

Definition 5.1: A function f: $(X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is called (1, 2)*-πgb- continuous if every $f^{-1}(V)$ is (1, 2)* (1, 2)*-πgb- closed in (X, τ_1, τ_2) for every $\sigma_1\sigma_2$ -closed set V of (Y, σ_1, σ_2).

Definition 5.2: A function f: $(X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is called (1, 2)*-πgb- irresolute if $f^{-1}(V)$ is (1, 2)*-πgb- closed in (X, τ_1, τ_2) for every (1, 2)*-πgb-closed set V in (Y, σ_1, σ_2).

Remark 5.3: A map f: $(X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is (1, 2)*-πgb- irresolute iff the inverse image of every (1, 2)*-πgb-open in Y is (1, 2)*-πgb-open in X.

Remark 5.4: Every (1, 2)*-πgb- irresolute function is (1, 2)*-πgb- continuous.

Proof: Let f be (1, 2)*-πgb-irresolute. Let V be a $\tau_1\tau_2$ -closed set in (Y, σ_1, σ_2).Since every $\tau_1\tau_2$ -closed set is(1, 2)*-πgb-closed ,V is (1, 2)*-πgb-closed in (Y, σ_1, σ_2).Since f is (1, 2)*-πgb- irresolute, $f^{-1}(V)$ is (1, 2)*-πgb-closed in(X, τ_1, τ_2) Therefore f is (1, 2)*-πgb- continuous.

Converse need not be true as seen in the following example.

Example 5.5: Consider X=Y={a,b,c}, τ_1 = {Φ, {a}, {c}, {a,c}, X}, τ_2={Φ,{a},X}.So the sets in{Φ,{a},{c}{a,c},X} are $\tau_1\tau_2$ –open sets in{Φ,{b,c}{a,b},{b}X} are $\tau_1\tau_2$ –closed.Let σ_1 ={Φ,{a},Y} and σ_2 ={Φ,Y}. So the sets in {Φ, {a},Y} are $\sigma_1\sigma_2$ –open and the sets in {Φ,{b,c},Y} are σ_1 σ_2 –closed. Define f: $(X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ by f(a)= a ,f(b)= b,f(c) = c. Then f is (1, 2)*- πgb- continuous map but not (1, 2)*- πgb-irresolute since the inverse image of (1, 2)*-πgb- closed sets {a,c} in Y is not (1, 2)*-πgb-closed set in X.

Theorem 5.6: Every (1, 2)*-continuous map is (1, 2)*-πgb- continuous map.

Converse of the above theorem need not be true as shown in the following example.

Let X=Y={a,b,c}, τ_1={Φ,{a},X} and τ_2={Φ,X} So the sets in{Φ,{a},X} are $\tau_1\tau_2$ –open and sets in{Φ,{b,c},X} are $\tau_1\tau_2$ –closed. Let σ_1 = {Φ,{a},{a,b},Y} and σ_2 ={Φ,{a,b},Y}. So the sets in {Φ,{a},{a,b},Y} are $\sigma_1\sigma_2$ –open and the sets in {Φ,{b,c},{c},Y} are $\sigma_1\sigma_2$ –closed. Let f: X→Y be an identity mapping.Then f is (1, 2)*- πgb- continuous map but not (1, 2)*-continuous since f^{-1}(a,b)={a,b}which is not $\tau_1\tau_2$ –open.

Composition of two (1, 2)*-πgb-continuous functions need not be (1, 2)*-πgb-continuous.

Example 5.7: Let X = Y = Z , {a,b,c,d} , τ_1= {Φ,{b}, {c}, {b,c},X},τ_2={Φ,X}, So the sets in {Φ,{b},{c},{b,c},X } are $\tau_1\tau_2$ –open and the sets in {Φ,{a,c},{a,b},{a},X } are $\tau_1\tau_2$ –closed. Let σ_1= {Φ, {a,b,d},Y} and σ_2={Φ,Y}. So the sets in {Φ,{a,b,d},Y} are $\sigma_1\sigma_2$ –open and the sets in{Φ,{c},Y} are $\sigma_1\sigma_2$–closed.Let η_1={ Φ,{a,d},X} and η_2={ Φ,Z}.So $\eta_1\eta_2$ –open={ Φ,{a,d},X} and $\eta_1\eta_2$ –closed={ Φ,{b,c},X}.Let f: $(X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ by f(a)= a,f(b) = c,f(c) = b.f(d)=d. Define g: $(Y, \sigma_1, \sigma_2) \to (Z, \eta_1, \eta_2)$ by g(a)= d, g(b) = c, g(c) = b, g(d)=a. Then f and g are (1, 2)*-πgb-continuous but gof is not (1, 2)*-

πgb-continuous since {b,c} is $\eta_1\eta_2$ -closed in (Z, η) but (gof)$^{-1}$({b,c})=f^{-1}(g^{-1}({b,c})={b,c} which is not (1, 2)*- πgb-closed in (X, τ_1, τ_2).Hence gof is not (1, 2)*-πgb-continuous.

Definition 5.8: A function f: $(X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is $\tau_1\tau_2$ -π-open map if f (F) is $\tau_1\tau_2$ -π-open map in Y for every $\tau_1\tau_2$ -π-open in X.

Definition 5.9: A function f: $(X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is (1, 2)*- b-irresolute **if** for each b-open set V in Y,f^{-1}(V) is b-open in X.

Theorem 5.10: If the bijective f : $(X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is (1, 2)*-b-irresolute and $\tau_1\tau_2$ - π-open map ,then f is (1, 2)*-πgb- irresolute.

Proof: Let V be (1, 2)*-πgb-closed in Y.Let f^{-1}(V)⊂U where U is $\tau_1\tau_2$-π-open in X. Then V⊂f(U) where f(U) is $\sigma_1\sigma_2$-π-open implies (1, 2)*-bcl(V)⊂f(U) f^{-1} ((1, 2)*-(bcl(V)))⊂U. Since f is (1, 2)*-b-irresolute, f^{-1}((1, 2)*-(bcl(V))) is (1, 2)*-b-closed in (X, τ_1, τ_2). Hence (1, 2)*-bcl(f^{-1}(V))⊂ (1, 2)*-bcl(f^{-1} ((1, 2)*- (bcl(V))) = f^{-1} ((1, 2)*-(bcl(V)))⊂ U. Hence f^{-1}(V) is (1, 2)*-πgb-closed in (X, τ_1, τ_2). Therefore, f is (1, 2)*-πgb- irresolute.

Theorem 5.11: For any irresolute (1, 2)*-irresolute map f: X→Y and any (1, 2)*-πgb-continuous map g:Y→Z, the composition g∘ f :X→Z is (1, 2)*-πgb-continuous map.

Proof: Let V be $\eta_1\eta_2$-closed set in Z. Since g:Y→Z is (1, 2)*-πgb-continuous map,g^{-1}(V) is (1, 2)*-πgb-closed in Y.Since f:X→Y is (1,2)*-πgb-irresolute map,f^{-1}(g^{-1}(V))=(g∘f)$^{-1}$(V) is (1,2)*-πgb-closed in X.Hence g∘f: X→Z is (1, 2)*-πgb-continuous map.

From this paper we conclude the following results.
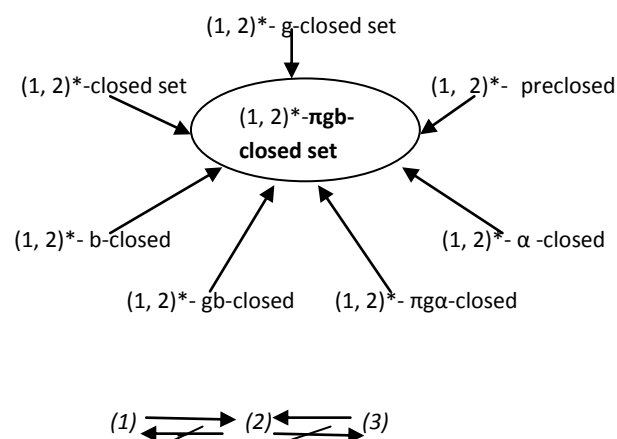
Where (1)= (1, 2)*-continuity, (2)= (1, 2)*- πgb-continuity and (3)= (1, 2)*- πgb-irresolute.

CONCLUSION

This paper has attempted to compare (1, 2)*-πgb-closed sets with the other closed sets in bitopological spaces.It also aims to state that the several definitions and results in this paper will result in obtaining several characterizations and also enable to study various properties. It brings to limelight that the weaker form of continuity in bitopological settings is the future scope of study.

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