Acyclic Coloring on Double Star Graph Families

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ABSTRACT
The purpose of this study is to find the acyclic chromatic number for central graph, middle graph, total graph and line graph of Double star graph $K_{1,n,n}$, denoted by C($K_{1,n,n}$), M($K_{1,n,n}$), T($K_{1,n,n}$) and L($K_{1,n,n}$) respectively. Vernold vivin (et.al.,2011) [9], have proved that the star chromatic number and equitable chromatic number of these graphs are same. We discuss the relationship between the acyclic and star chromatic number of these graphs.

Keywords
Central graph, middle graph, total graph, line graph and acyclic coloring.

1. INTRODUCTION
Let G be a finite, undirected graph with no loops and multiple edges. Let G be a graph with vertex set V(G) and edge set E(G). The middle graph(Michalak,1981) of G ,denoted by M(G), is defined as follows. The vertex set of M(G) is V(G)UE(G). Two vertices v,w in the vertex set of M(G) are adjacent in M(G) if one of the following holds.

1. v,w are in V(G) and v is adjacent to w in G.
2. v is in V(G) and w is in E(G) and v,w are incident in G.

The central graph (Vernold et al., 2009 a,b) of G, denoted by C(G), is the graph obtained from G by subdividing each edge exactly once and joining all the non-adjacent vertices of G.

The total graph (Michalak,1981; Harary 1969) of G,denoted by T(G), is defined as follows. The vertex set of T(G) is V(G)UE(G) and any two vertices v,w in this set are adjacent in T(G) if one of the following holds.

1. v,w are in V(G) and v is adjacent to w in G.
2. v,w are in E(G) and v,w are adjacent in G.
3. v is in V(G), w is in E(G) and v,w are incident in G.

The line graph (Harary,1969) of G, denoted by L(G), is the graph with vertices are the edges of G and two vertices of L(G) are adjacent whenever, the corresponding edges of G are adjacent. Double star K $_{1,n,n}$, is a tree obtained from the star $K_{1,n}$ by adding a new pendent edge of the existing n pendent vertices. It has 2n+1 vertices and 2n edges.

The notion of acyclic chromatic number and star chromatic number were introduced by Grunbaum(1973). An acyclic coloring of a graph G is a proper coloring such that the sub graph induced by two colors α,β is a forest. The minimum number of colors necessary to acyclically color G is called the acyclic chromatic number and is denoted by a(G). A star coloring of a graph G is a proper vertex coloring in which every path on four vertices uses atleast three distinct colors. The star chromatic number, X_S(G), of G, is the least number of colors needed to star color G. Vernold vivin(et al.,2011) have derived the following results[9] for the Double star graph families.

1. X_S (M[$K_{1,n,n}$]) = n+1.
2. X_S (C[$K_{1,n,n}$]) =2n+1.
3. X_S (T[$K_{1,n,n}$]) =n+1.
4. X_S (L[$K_{1,n,n}$]) = n.

In the following chapters, we have derived the acyclic chromatic number for the Double star graph.

Let v be the root vertex of $K_{1,n,n}$ and $v_1,v_2,\ldots,v_n$ be the vertices joined to the vertex v and $w_1,w_2,\ldots,w_n$ be the n pendent vertices of $K_{1,n,n}$. Let $e_1,e_2,\ldots,e_n$ be the newly introduced vertices on the edge joining v and v_i (i= 1 to n) and $s_1,s_2,\ldots,s_n$ be the newly added vertices on the edge joining v_i and w_i.

2. ACYCLIC COLORING OF M($K_{1,n,n}$)

2.1 Theorem
For any Double star graph $K_{1,n,n}$, the acyclic chromatic number of M($K_{1,n,n}$) is given by

a (M[$K_{1,n,n}$]) = n+1, $n \geq 3$.

Proof:

By the definition of middle graph, M[$K_{1,n,n}$] has the following structural properties.

1. < v,e_k ; k = 1 to n > form a clique of order n+1.
2. e_i and s_i are adjacent.

Now, consider the coloring C of M($K_{1,n,n}$) as follows. Assign C_i to e_i, i=1 to n and c_{n+1} to v. Assign c_{i+1} to s_i ,i=1 to n-1 and c_1 to s_n. Now, we show that C is acyclic.

Case(i). As < v,e_i ;i=1 to n > form a clique of order n+1, it has no bicolored cycle. Consider c_{n+1} and c_i ($1 \leq i \leq n$). The induced sub graph of these color classes contains only bicolored path of length atmost 2.(eg.$e_iv_iv_{i-1}s_{i-1}w_{i-1}$) and hence M($K_{1,n,n}$) has no bicolored ($c_i$-$c_{n+1}$) cycle in the coloring C.

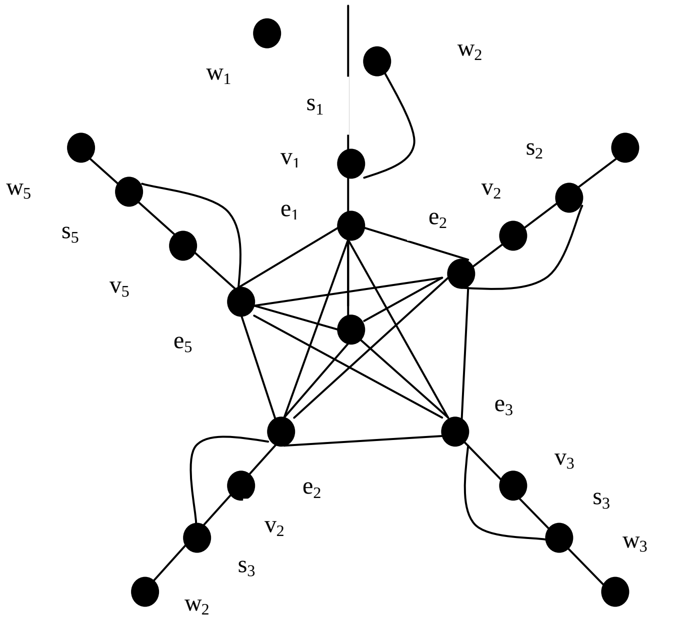

Figure.1 . a (M[$K_{1,5,5}$]) = 6.

Case(ii). Consider c_j and c_k ($1 \leq j < k \leq n$). The color classes of c_j is {e_j, s_{j-1} } and that of c_k is {e_k, s_{k-1}}.The induced sub graph of these color classes contains only the bicolored path, e_k e_j s_k when k= j+1 and the edge e_k e_j when k > j+1. In both cases, M[K $_{1,n,n}$] has no bicolored (c_j-c_k) cycle. So, the coloring C is acyclic and therefore,

a [M($K_{1,n,n}$)] = n+1 , $n \geq 3$.

3.ACYCLIC COLORING OF C[K $_{1,n,n}$]

3.1Theorem

The acyclic chromatic number of C[K $_{1,n,n}$] is

a (C[K $_{1,n,n}$]) = 2n , $n \geq 3$.

Proof:

By the definition of central graph,C[$K_{1,n,n}$] has the following structuralcharacteristics.

1. < v_i ; i=1 to n > form a clique of order n.
2. < v, w_i ; i=1 to n > form a clique of order n+1.
3. < v_i, w_j ; j=1 to n and $j \neq i$ > form a clique of order n.
4. {e_i ;i=1 to n} and {s_i ;i=1 to n} are independent sets.

Now, consider the coloring of C[$K_{1,n,n}$] as follows.

Assign c_i to w_i, i=1 to n and c_{n+1} to v. Assign c_{n+1} to s_i,i=1 to n. Assign c_2 to e_i ,i=1 to n and c_1 to v_1. If we assign c_i to v_i for any i=2 to n, then $v_1v_iw_1w_iv_1$ will form a bicolored (c_1-c_i) cycle. Hence, assign c_{n+i} to v_i, i=2 to n. we can show that the above coloring is acyclic by the following analysis.

As the path ve_iv_i(i=1 to n) is colored with three colors, it can not be a part of any bicolored cycle. Similarly, the path $v_is_iw_i$(i=2 to n) can not be a part of any bicolored cycle. So, consider $v_1s_1w_1$,a bicolored (c_1-c_{n+1}) path. As v and v_1 are non adjacent in C[$K_{1,n,n}$], it is only a bicolored path, but not cycle.

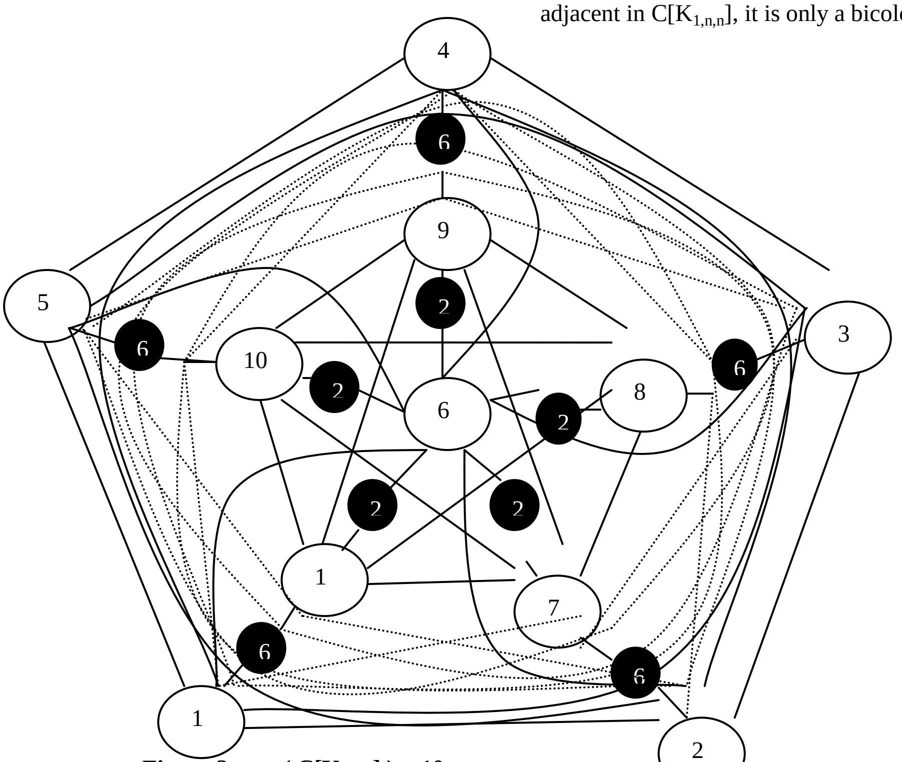

Figure.2. a (C[$K_{1,5,5}$]) = 10

Thus, C[$K_{1,n,n}$] has no bicolored cycle in the above coloring. Therefore,

a (C[$K_{1,n,n}$]) = 2n , $n \geq 3$.

4. ACYCLIC COLORING OF T[$K_{1,n,n}$]

4.1 Theorem

For any Double star graph $K_{1,n,n}$,

A (T[$K_{1,n,n}$]) = n+1, $n \geq 3$.

Proof:

In $T[K_{1,n,n}]$,

1. < v, e_i ;i=1 to n > form a clique of order n+1.
2. v and v_i (i=1 to n), v_i and w_i(i=1to n) are adjacent.
3. e_i and s_i (i=1 to n) are adjacent.

Now, color the vertices of $T[K_{1,n,n}]$ as follows

Assign c_i to e_i and w_i(i=1 to n) and $c_{n+1}$to v. Assign c_{i+1} to v_i(i=1 to n-1) and c_1 to v_n. Assign $c_{n+1}$to s_i(i=1 to n). consider the following cases in order to show that the given coloring is acyclic.

Case(i)

Consider c_{n+1} and c_i (i=1 to n). In the induced sub graph of these color classes, we get only the bicolored path, $w_i s_i e_i v$ $v_{i-1} s_{i-1}$, not cycle. Therefore, $T[K_{1,n,n}]$ has no bicolored cycle in the given coloring.

Case(ii)

Consider c_i and c_{i+1}(i=1 to n-1). The induced sub graph of these color classes contains the bicolored path,$e_{i+1} e_i v_i w_i$, but not cycle. So, $T[K_{1,n,n}]$ has no bicolored (c_i-c_{i+1}) cycle.

Case(iii)

Consider c_k and c_j where $1 \leq k < j-1 \leq n$. The induced sub graph of these color classes contains the edge $e_k e_j$ and not any bicolored cycle.

So, the given coloring is acyclic. Therefore,

$a(T[K_{1,n,n}]) = n+1$, $n \geq 3$

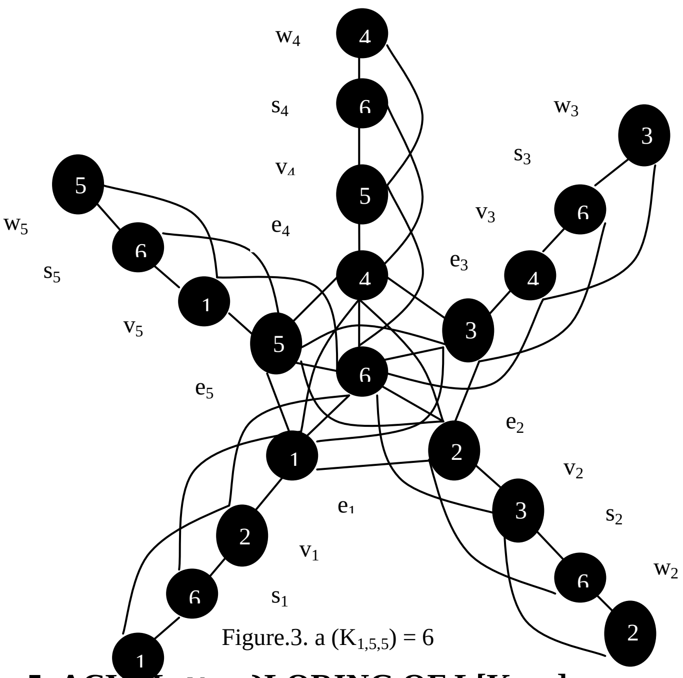

Figure.3. a $(K_{1,5,5})$ = 6

5. ACYCL OLORING OF $L[K_{1,n,n}]$

5.1 Theorem

For any Double star graph $K_{1,n,n}$,

$a(L[K_{1,n,n}]) = n$.

Proof:

In $L[K_{1,n,n}]$,

1. <e_i; i=1 to n> form a clique of order n
2. e_i s_i is the pendent edge at e_i.

Now color $L[K_{1,n,n}]$ as follows:

Assign c_i to e_i, i = 1 to n and c_{i-1} to s_i, i = 2 to n-1 and c_n to s_1. The induced sub graph of any two color classes c_i and c_j contains a bicolored path e_i e_j s_j, if | i-j | = 1 and the edge e_i e_j if |i –j | > 1. So, the coloring is acyclic and therefore,

$a(L[K_{1,n,n}]) = n$.

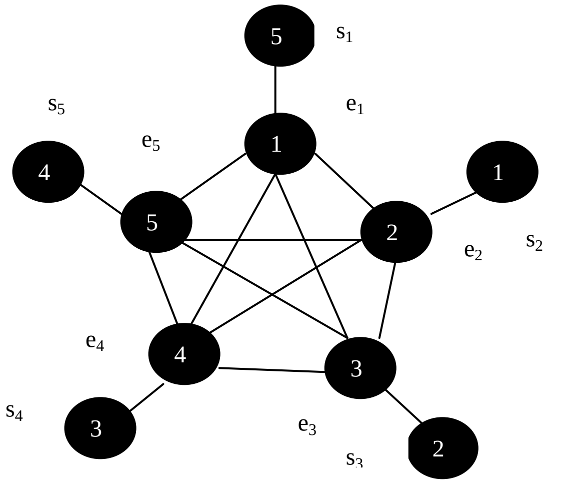

Figure 4 . a($L[K_{1,5,5}]$) = 5

Conclusion

Using the result of the study and the result of Vernold [9], we have the following results:

1. X_s {$M[K_{1,n,n}]$} = n + 1 = a($M[K_{1,n,n}]$}.
2. X_S ($C[K_{1,n,n}]$} = 2n + 1 = a($C[K_{1,n,n}]$} + 1.
3. X_S ($T[K_{1,n,n}]$} = n + 1 = a($T[K_{1,n,n}]$}.
4. X_S ($L[K_{1,n,n}]$} = n = a($L[K_{1,n,n}]$}.

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