Semi-compatibility in Non-Archimedean Fuzzy Metric spaces

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ABSTRACT

The purpose of this paper is to prove a common fixed point theorem for six self maps using the concept of semi-compatibility and weak compatibility of pair of self maps in Non- Archimedean fuzzy metric space.

AMS SUBJECT CLASSIFICATION: 47H10, 54H25

KEYWORDS

Semicompatible and weakly compatible mappings,

Non-Archimedean Fuzzy Metric Spaces.

1. INTRODUCTION

In [7], Istratescu initiated the study of fixed point theory in non-Archimedean Menger spaces and proved some fixed point theorems in this space using the same method as Cain and Kasriel in [5] for probabilistic metric space. The concept of fuzzy sets was coined by Zadeh [8] in his seminal paper in 1965. Since then, to use this concept in topology and analysis many authors have expansively developed the theory of fuzzy sets and application. Zadeh[9] anticipated that medical diagnosis would be the most likely application domain of Fuzzy set theory. The theory of fuzzy set is of fundamental importance in Medical diagnosis, effect of drugs, diagnosis process. Recently, a number of fixed point theorems and their applications in fuzzy set theory have been proved by several authors. George and Veeramani[1] and Kramosil and Michalek[6] have introduced the concept of fuzzy topological spaces induced by fuzzy metric which has very important applications in quantum particle physics particularly in connections with both string and $\varepsilon^{(\infty)}$ theory which were given and studied by El. Naschie [10- 12, 14].Many authors[2-3,13] have proved fixed point theorem in fuzzy(probabilistic) metric spaces. Recently, Dorel Mihet[4] proved a common fixed point theorem in Non- Archimedean fuzzy metric space. The purpose of this paper is to prove a common fixed point theorem for six self maps using the concept of semi-compatibility and weak compatibility of pair of self maps in Non- Archimedean fuzzy metric space.

2. PREMINILARIES

Definition 1.1 : A binary operation * : $[0, 1] \times [0, 1] \rightarrow [0, 1]$ is a continuous tnorm
if it satisfies the following conditions:
(1) * is associative and commutative,
(2) * is continuous,
(3) a * 1 = a for all a $\in$ [0, 1],

(4) a * b $\leq$ c * d whenever a $\leq$ c and b $\leq$ d,
  for each a, b, c, d $\in$ [0, 1].
Example 1.1 : Two typical examples of continuous t-norm are a * b = ab and a * b = min(a, b).
Definition 1.2[4]: The 3-tuple (X, M, *) is called a non-Archimedean fuzzy metric space (shortly, N.A. FM-space) if X is an arbitrary set, * is a continuous t norm and M is a fuzzy set in $X^2 \times [0, \infty)$ satisfying the following conditions:
For all x,y,z $\in$ X and s,t > 0,
(NFM-1) M(x, y, 0) = 0,
(NFM-2) M(x, y, t) = 1, for all t > 0 if and only if x = y,
(NFM-3) M(x, y, t) = M(y, x, t),
(NFM-4) M(x, y, t) * M(y, z, s) $\leq$ M(x, z, max{t, s})
Or equivalently M(x, y, t) * M(y, z, t) $\leq$ M(x, z, t)
(NFM-5) M(x, y, .) : $[0, \infty) \rightarrow [0, 1]$ is left continuous.
Definition 1.3 : For t $\in (0, \infty)$, we define the closed ball B[x, r, t] with centre x$\in$ X and radius r $\in (0,1)$ as
B[x, r, t] = {y $\in$ X ,M(x, y, t) > 1 − r}.
Definition 1.4 : A N.A. FM-space (X, M, *) is said to be of type (C)g if there exists a g$\in \Omega$ such that
g(M(x,y , t)) $\leq$ g(M(x,z , t)) + g(M(z,y , t))
for all x,y,z $\in$ X and t $\geq$ 0, where Ω = { g : g : [0,1] $\rightarrow$ [0,∞) is continuous, strictly decreasing , g(1) = 0 and g(0) < ∞ }.
Definition 1.5: A N. A. FM-space (X, M, *) is said to be of type (D)g if there exists a g $\in \Omega$ such that
g(* (s,t)) $\leq$ g(s) + g(t) for all s,t $\in$ [0,1].
Definition 1.6: A pair of maps A and B is called weakly compatible pair if they commute at coincidence points i.e.,
 Ax = Bx if and only if ABx = BAx .
Definition 1.7: Let A and S be mappings from N.A. FM-space (X, M, *) in to itself. The mappings A and S are said to be compatible if $\lim_{n\to\infty}$g(M(ASx$_n$, SAx$_n$, t)) = 0,

for all t > 0, whenever {x$_n$} is a sequence in X such that

$\lim_{n\to\infty}$Ax$_n$ = $\lim_{n\to\infty}$Sx$_n$ = z for some z $\in$ X.

Definition 1.8 : Let A, S : X $\rightarrow$ X be mappings. The pair

(A, S) is said to be weak-compatible if they commute at their

coincidence points, i.e., if Az = Sz for some w $\in$ X then such

that ASw = SAw.

Definition 1.9 : Let A and S be mappings from an N.A. FM-space (X, M, ∗) into itself. Then the mappings are said to be semi-compatible if

$\lim_{n\to\infty}g(M(ASx_n, Sx, t)) = 0$, for all t > 0,whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty}Ax_n = \lim_{n\to\infty}Sx_n = x$ for some $x \in X$.

Proposition 1.1: Let A and S be self-maps on an N.A. FM-space (X, M, ∗). If S is continuous, then (A,S) is semi-compatible if and only if (A, S) is compatible.

Proof : Consider a sequence $\{x_n\}$ in X such that $\{Ax_n\}$ and $\{Sx_n\}$ converges to u $\in$ X. Suppose that (A, S) is compatible, then

$g(M(ASx_n, Su, t)) \leq g(M(ASx_n, SAx_n , t)) + g(M(SAx_n, Su\ t))$

letting $n\to\infty$, Since (A, S) is compatible, we have

$\lim_{n\to\infty}g(M(ASx_n, Su, t)) = 0$. Hence $ASx_n \to Su$,i.e., (A,S) is semi-compatible.

Conversely, Suppose that (A,S) is semi-compatible, then

$g(M(ASx_n, SAx_n, t)) \leq g(M(ASx_n, Su , t))+g(M(SAx_n, Su\ t))$.

letting $n\to\infty$, Since (A, S) is semi-compatible and S is continuous, we have

$SAx_n \to Su$ and this implies

$\lim_{n\to\infty}g(M(ASx_n, Su, t)) = 0$.

Hence the pair (A, S) is compatible.

The following is an example of a pair of self-maps (A, S) which is compatible but not semi-compatible. Further, it is also seen here that the semi-compatibility of the pair (A, S) need not imply the semi-compatibility of (S,A).

Example 1.2. Let X = [0, 1] and let (X, M, ∗) be the N.A. FM-space with

$g(M(x, y, t)) = g\left[\exp\dfrac{|x-y|}{t}\right]^{-1}$ for all x, y $\in$ X, t > 0.

Define self-map S as follows:

$$Sx = \begin{cases} x \ if \quad 0 \leq x \prec \dfrac{1}{2} \\[2mm] 1 \ if \quad x \geq \dfrac{1}{2} \end{cases}$$

Let I be the identity map on X and $x_n = \dfrac{1}{2} - \dfrac{1}{n}$. Then,

$\{Ix_n\} = \{x_n\} \to \dfrac{1}{2}$

And $\{Sx_n\} = \{x_n\} \to \dfrac{1}{2}$. Thus $\{ISx_n\} = \{Sx_n\} \to \dfrac{1}{2} \neq S\left(\dfrac{1}{2}\right)$. Hence (I, S) is not semicompatible.

Again as (I, S) is commuting, it is compatible. Futher, for any sequence $\{x_n\}$ in X such that $\{x_n\} \to$ x, we have $\{SIx_n\} = \{Sx_n\} \to$ x = Ix. Hence (S, I) is always semicompatible.

Remark 1.1. The above example gives an important aspect of semicompatibility as the pair of self-maps (I,S) is commuting, hence it is weakly commuting, compatible and weak compatible yet it is not semicompatible. Further, it is to be noted that the pair (S, I) is semicompatible but (I,S) is not semicompatible here.

The following is an example of a pair of self-maps (A, S) which is semicompatible but not compatible.

Example 1.2 : Let (X, M, ∗) be the N.A. FM-space, where X = [0, 2], with t-norm defined by a ∗ b = min{a, b}, for all a, b $\in$ [0, 1] and

$g(M(x, y,t)) = g\left(\dfrac{t}{t + d(x, y)}\right)$ for all t > 0 and

$g(M(x, y,0)) < \infty$, for all x, y $\in$ X. Define self-maps A and S on X follows :

$$Ax = \begin{cases} 2 \ if \quad 0 \leq x \leq 1 \\[2mm] \dfrac{x}{2} \ if \quad 1 \prec x \leq \dfrac{1}{2} \end{cases}, Sx = \begin{cases} 2 \ if \quad x = 1 \\[2mm] \dfrac{x+3}{5} \ otherwise \end{cases}$$

And $x_n = 2 - \dfrac{1}{2n}$. Then we have S(1) = A(1) = 2 and S(2) = A(2) = 1. Also SA(1) = AS(1) = 1 and SA(2) = AS(2) = 2. Thus (A, S) is weak compatible. Again,

$Ax_n = 1 - \dfrac{1}{4n}$, $Sx_n = 1 - \dfrac{1}{10n}$.

Thus, $Ax_n \to 1, \ Sx_n \to 1$.

Hence u = 1.

Further, $SAx_n = \dfrac{4}{5} - \dfrac{1}{20n}$, $ASx_n = 2$.

Now,

$\lim_{n\to\infty} g(M(ASx_n, Su, t)) = g(M(2, 2, t)) = 0$,

$\lim_{n\to\infty} g(M(ASx_n, SAx_n, t)) = \lim_{n\to\infty} g(M(2, \frac{4}{5} - \frac{1}{20n}, t))$

$= g\left(\dfrac{t}{t + \dfrac{6}{5}} \right) \neq 0$, for all t > 0. Hence (A, S) is semicompatible but it is not compatible.

Remark 1.2. (1) If a N.A.FM-space (X,M, *) is of type (D)g then (X,M, *) is of type (C)g .
(2) If (X, M, *) is a N.A. FM-space and $* \geq *$ m , where * m (s, t) =max{s + t-1,0}, then (X,M, *) is of type (D)g for $g \in \Omega$ defined by g(t) = 1 - t .
Throughout this paper, let (X,M, *) be a complete N.A.FM-space of type (D)g with a continuous strictly increasing t-norm D .
Let $\phi : [0,+\infty) \to [0,+\infty)$ be a function satisfying the following condition (Φ) :

(Φ) ϕ is upper-semicontinuous from the right and ϕ(t) < t for all t > 0.

Lemma 1.1. [16] If a function $\phi : [0,+\infty) \to [0, +\infty)$ satisfies the condition (Φ), then we have

(1) For all t ≥ 0, $\lim_{n\to\infty} \phi^n (t) = 0$, where $\phi^n (t)$ is the n-th iteration of ϕ(t).

(2) If {t_n} is a non-decreasing sequence of real numbers and $t_{n+1} \leq \phi(t_n)$, n = 1,2,..., then $\lim_{n\to\infty} t_n = 0$. In particular, if t ≤ ϕ(t) for all t ≥ 0, then t = 0.

Lemma 1.2[15] Let {y_n} be a sequence in X such that $\lim_{n\to\infty}$ F(y_n,y_{n+1},t)=1 for all t > 0. If the sequence {y_n} is not a Cauchy sequence in X, then there exist ∈$_0$> 0, t_0 >0, two sequences {m_i}, {n_i} of positive integers such that

(1) $m_i > n_i + 1$, and $n_i \to \infty$ as i → ∞,

(2) F(y_{m_i},y_{n_i} ,t_0) < 1 − ∈$_0$ and F(y_{m_i-1},y_{n_i} ,t_0) ≥ 1−∈$_0$, i = 1,2,...

3. RESULTS
Theorem 1 : Let A, B, S ,T, P and Q :X→X be mappings such that
(1) P(X) ⊂ ST(X) and Q(X) ⊂ AB(X),

(2)   AB = BA, ST = TS, PB = BP, QS = SQ, QT = TQ,

(3) g(M(Px, Qy, t)) ≤ ϕ(max{g(M(ABx,Ty, t)), (M(ABx,Px,t)),g(M(STy,Qy,t)), 1/2(g(M(ABx,Qy, t))+ g(M(STy, Px, t)))}) for all t > 0,

where a function $\phi : [0, \infty) \to [0,\infty)$ satisfies the condition(Φ).

(4) the pair (P,AB) is semicompatible and (Q,ST) is weakly compatible

(5) Either AB or P is continuous

Then A, B, S, T, P and Q have a unique common fixed point in X.

Proof. Let $x_0 \in X$, then by (1), there exists a point $x_1 \in X$ such that $Px_0 = STx_1$ since P(X) ⊂ ST(X). Since Q(X) ⊂ AB(X), for this point x_1, we can choose a point $x_2 \in X$ such that $Qx_1 = ABx_2$ and so on. Inductively, we can define a sequence {y_n} in X such that

(6) $y_{2n} = Px_{2n} = STx_{2n+1}$ and $y_{2n+1} = ABx_{2n+2} = Qx_{2n+1}$ for n = 0, 1,2..., Now we prove the sequence {y_n}, defined by (6), such that

$\lim_{n\to\infty} g(M(y_n,y_{n+1},t))=0$ for all t >0 is a Cauchy sequence in X.

Since g ∈ Ω , it follows that $\lim_{n\to\infty} M(y_n, y_{n+1}, t) = 1$ for all t > 0 if and only if $\lim_{n\to\infty} g(M(y_n, y_{n+1}, t))=0$ for all t > 0. By Lemma 1.2. if {y_n} is not a Cauchy sequence in X , then there exist ∈$_0$ > 0 , t_0 > 0 and two sequences {m_i}, {n_i} of positive integers such that

(A) $m_i > n_i + 1$, and $n_i \to \infty$ as i → ∞,

(B) F(y_{m_i},y_{n_i} ,t_0) > 1 − ∈$_0$ and F(y_{m_i-1},y_{n_i} ,t_0) ≤ 1−∈$_0$, i = 1,2,...
Thus we have

$g(1-∈_0) < g(M(y_{m_i},y_{n_i} ,t_0))$

(7) $\leq g(M(y_{m_i}, y_{m_i+1} ,t_0)) + g(M(y_{m_i+1},y_{n_i} ,t_0))$

$\leq g(M(y_{m_i}, y_{m_i+1} ,t_0)) + g(1-\mathcal{E}_0)$

Letting i → ∞ in (7), we have

(8) $\lim_{n\to\infty} g(M(y_{m_i},y_{n_i} ,t_0))= g(1-∈_0)$.

On the other hand, we have

$g(1- ∈_0) < g(M(y_{m_i},y_{n_i} ,t_0))$

(9) $\leq g(M(y_{n_i}, y_{n_i+1} ,t_0))+ g(M(y_{n_i+1}, y_{m_i} ,t_0))$

Now, consider g(M(y_{n_i+1}, y_{m_i} , t_0)) in (9).Without loss of generality assume that both n_i and m_i are even.

Then , by (3), we have

g(M(y_{n_i+1}, y_{m_i} , t_0)) = g(M(Px$_{m_i}$, Qx$_{n_i+1}$,t_0))

$\leq \phi$ (max{g(M(ABx$_{m_i}$, STx$_{n_i+1}$, t_0)) ,

g(M(ABx$_{m_i}$, Px$_{m_i}$,t_0)),g(M(STx$_{n_i+1}$, Qx$_{n_i+1}$,t_0)),

1/2(g(M(ABx$_{m_i}$,Qx$_{n_i+1}$,t_0))+g(M(STx$_{n_i+1}$, Px$_{m_i}$,t_0)))})

(10)   = ϕ(max{g(M(y_{m_i-1} y_{n_i} n$_i$,,t_0)),

g(M(y_{m_i-1}, y_{m_i} ,,t_0)), g(M(y_{n_i}, y_{n_i+1} ,t_0)),

1/2(g(M(y_{m_i-1},y_{n_i+1} ,t_0)) + g(M(y_{n_i}, y_{m_i} ,t_0)))})

By (8), (9) and (10), letting i $\rightarrow \infty$ in (10), we have

$$g(1-\epsilon_0) \le \phi \ (\max \ \{g(1-\epsilon_0) \ , \ 0, \ 0, \ g \ (1-\epsilon_0)\})$$

$$= \phi \ (g(1-\epsilon_0)) < g(1-\epsilon_0)$$

which is a contradiction. Therefore, $\{y_n\}$ is a Cauchy-sequence in X .

Now, we prove $\lim_{n\rightarrow\infty} g(M(y_n, y_{n+1}, t_0)) = 0$ for all $t > 0$. In fact , by (3) and (4), we have

$$g(M(y_{2n}, y_{2n+1}, t)) = g(M(Px_{2n}, Qx_{2n+1}, t))$$

$$\le \phi(\max\{g(M(ABx_{2n}, STx_{2n+1}, t)),$$

$$g(M(ABx_{2n}, Px_{2n}, t)), g(M(STx_{2n+1}, Qx_{2n+1}, t)) \ ,$$

$$1/2(g(M(ABx_{2n}, Qx_{2n+1}, t))+g(M(STx_{2n+1}, Px_{2n}, t)))\})$$

$$= \phi(\max\{g(M(y_{2n-1}, y_{2n}, t)), g(M(y_{2n-1}, y_{2n}, t)),$$

$$g(M(y_{2n}, y_{2n+1}, t)), 1/2 \ (g(M(y_{2n-1}, y_{2n+1}, t)) + g(1)))\})$$

$$\le \phi(\max\{g(M(y_{2n-1}, y_{2n}, t)), g(M(y_{2n}, y_{2n+1}, t)),$$

$$1/2(g(M(y_{2n-1}, y_{2n}, t)) \ + \ g(M(y_{2n}, y_{2n+1}, t)))\}) \ .$$

If $g(M(y_{2n-1}, y_{2n}, t)) \le g(M(y_{2n}, y_{2n+1}, t))$ for all $t > 0$, then, by (3),

$g(M(y_{2n}, y_{2n+1}, t)) \le \phi \ (g(M(y_{2n}, y_{2n+1}, t)))$ which means that , by Lemma 1.1.,

$g(M(y_{2n}, y_{2n+1}, t)) = 0$ for all $t > 0$. Similarly, we have $g(M(y_{2n+1}, y_{2n+2}, t)) = 0$ for all $t > 0$. Thus we have

$\lim_{n\rightarrow\infty} g(M(y_n y_{n+1}, t)) = 0$ for all $t > 0$. On the other hand, if

$g(M(y_{2n-1}, y_{2n}, t)) \ge g(M(y_{2n}, y_{2n+1}, t))$, then by (3), we have

$g(M(y_{2n}, y_{2n+1}, t)) \le \phi \ (g(M(y_{2n-1}, y_{2n}, t))$ for all $t > 0$.

Similarly, $g(M(y_{2n+1}, y_{2n+2}, t)) \le \phi(g(M(y_{2n}, y_{2n+1}, t)))$, for all $t > 0$ .Thus we have

$g(M(y_n, y_{n+1}, t)) \le \phi(g(M(y_{n-1}, y_n, t))$ for all $t > 0$ and

n = 1,2,3,... Therefore by Lemma 1.1,

$\lim_{n\rightarrow\infty} g(M(y_n, y_{n+1}, t)) = 0$ for all $t > 0$, which implies that $\{y_n\}$ is a Cauchy sequence in X by Lemma 1.2. Since $(X,M,*)$ is complete, the sequence $\{y_n\}$ converges to a point $z \in X$ and so the subsequences $\{Px_{2n}\}$, $\{Qx_{2n+1}\}$, $\{ABx_{2n}\}$ and $\{STx_{2n+1}\}$ of $\{y_n\}$ also converges to the same limit z, i.e., $\lim_{n\rightarrow\infty} Px_{2n} = \lim_{n\rightarrow\infty} STx_{2n+1} = \lim_{n\rightarrow\infty} Qx_{2n+1} = \lim_{n\rightarrow\infty} ABx_{2n+2} = z.$

Case I (P is continuous). In this case, we have

$PPx_{2n} \rightarrow Pz$, $P(AB)x_{2n} \rightarrow Pz$.

The semi-compatibility of the pair (P, AB) gives

$P(AB)Px_{2n} = ABz$. As the limit of a sequence in N.M. Fuzzy metric Space is unique we have, $ABz = Pz$

Step I. By putting $x = Px_{2n}$, $y = x_{2n+1}$ in (3), we obtain that

$$g(M(PPx_{2n}, Qx_{2n+1}, t)) \le \phi(\max\{g(M((AB)Px_{2n}, STx_{2n+1}, t)),$$

$$g(M((AB)Px_{2n} PPx_{2n}, t)), g(M(STx_{2n+1}, Qx_{2n+1}, t)),$$

$$1/2(g(M((AB)Px_{2n}, Qx_{2n+1}, t))+ g(M(STx_{2n+1}, PPx_{2n}, t)))\})$$

Letting n $\rightarrow \infty$, we have

$$g(M(Pz, z, t)) \le \phi(\max\{g(M(Pz, z, t)), g(M(Pz, Pz, t)),$$

$$g(M(z, z, t)), 1/2(g(M(Pz, z, t))+ g(M(z, Pz, t)))\})$$

$$\Rightarrow g(M(Pz, z, t)) \le \phi(\max\{g(M(Pz, z, t)), 0, 0,$$

$$g(M(Pz, z, t)))\})$$

$$\Rightarrow g(M(Pz, z, t)) \le \phi\{g(M(Pz, z, t)\}$$

$$\Rightarrow g(M(Pz, z, t)) = 0 \ .i.e., \ M(Pz, z, t) = 1$$

$$\Rightarrow z = Pz.$$

Step II. As $P(X) \subset ST(X)$, there exists a point $w \in X$ such that $z = Pz = STw$. By putting $x = Px_{2n}$, $y = w$ in (3), we obtain that

$$g(M(PPx_{2n}, Qw, t)) \le \phi(\max\{g(M((AB)Px_{2n}, STw, t)),$$

$$g(M((AB)Px_{2n}, PPx_{2n}, t)), g(M(STw, Qw, t)),$$

$$1/2(g(M((AB)Px_{2n}, Qw, t))+ g(M(STw, PPx_{2n}, t)))\})$$

Letting n $\rightarrow \infty$, we have

$$g(M(z, Qw, t)) \le \phi(\max\{g(M(z, z, t)), g(M(z, z, t)),$$

$$g(M(z, Qw, t)), 1/2(g(M(z, Qw, t))+ g(M(z, z, t)))\})$$

$$g(M(z, Qw, t)) \le \phi(\max\{0, 0, g(M(z, Qw, t)),$$

$$1/2(g(M(z, Qw, t)))\})$$

$$\Rightarrow g(M(z, Qw, t)) \le \phi(g(M(z, Qw, t))$$

$$\Rightarrow g(M(z, Qw, t)) = 0 \ .i.e., \ M(z, Qw, t) = 1$$

$$\Rightarrow z = Qw.$$

Therefore Qw = STw = z. Since (Q,ST) is weakly compatible, we get that

$(ST)Qw = Q(ST)w$, that is, $Qz = STz$.

Step III. By putting $x = x_{2n}$, $y = z$ in (3), we obtain that

$$g(M(Px_{2n}, Qz, t)) \le \phi(\max\{g(M(ABx_{2n}, STz, t)),$$

$$g(M(ABx_{2n}, Px_{2n}, t)), g(M(STz, Qz, t)),$$

$$1/2(g(M(ABx_{2n}, Qz, t))+ g(M(STz, Px_{2n}, t)))\})$$

$$g(M(z, Qz, t)) \le \phi(\max\{g(M(z, Qz, t)), g(M(z, z, t)),$$
$$g(M(Qz, Qz, t)), 1/2(g(M(z, Qz, t))+ g(M(Qz, z, t)))\})$$

$$g(M(z, Qz, t)) \le \phi(\max\{g(M(z, Qz, t)), 0, 0, g(M(z, Qz, t))\})$$

$$\Rightarrow g(M(z, Qz, t)) \le \phi(g(M(z, Qz, t))$$

$$\Rightarrow g(M(z, Qz, t)) = 0 \ .i.e., \ M(z, Qz, t) = 1$$

$$\Rightarrow z = Qz.$$

Step IV. Putting $x = x_{2n}$, $y = Tz$ in (3) we get,

$$g(M(Px_{2n}, QTz, t)) \le \phi(\max\{g(M(ABx_{2n}, STTz, t)),$$

g(M(ABx$_{2n}$, Px$_{2n}$,t)),g(M(STTz ,QTz ,t)),

1/2(g(M(ABx$_{2n}$, QTz, t))+ g(M(STTz, Px$_{2n}$, t)))})

As QT = TQ and ST = TS we have QTz = TQz = Tz and ST(Tz) = T(ST)z = Tz.

Letting n $\longrightarrow \infty$ we get,

g(M(z, Tz, t)) ≤ ϕ(max{g(M(z, Tz , t)), g(M(z, z, t)),

g(M(Tz, Tz ,t)), 1/2(g(M(z, Tz, t))+ g(M(Tz, z, t)))})

g(M(z, Tz, t)) ≤ ϕ(max{g(M(z, Tz , t)), 0, 0,g(M(z, Tz, t))})

$\Longrightarrow$ g(M(z, Tz, t)) ≤ ϕ(g(M(z, Tz ,t))

$\Longrightarrow$ g(M(z, Tz, t)) = 0 .i.e., M(z, Tz, t) = 1

$\Longrightarrow$ z = Tz.

Now STz = Tz = z implies Sz = z. Hence Sz = Tz = Qz = z.

Step V. By putting x = Bz , y = x$_{2n+1}$ in (3), we obtain that

g(M(PBz, Qx$_{2n+1}$, t)) ≤ ϕ(max{g(M((AB)Bz,STx$_{2n+1}$, t)),

g(M((AB)Bz ,PBz,t)),g(M(STx$_{2n+1}$,Qx$_{2n+1}$,t)),

1/2(g(M((AB)Bz ,Qx$_{2n+1}$, t))+ g(M(STx$_{2n+1}$, PBz, t)))})

As BP = PB, AB = BA so we have P(Bz) = B(Pz) = Bz and

AB(Bz) = B(ABz) = Bz.

Letting n $\longrightarrow \infty$, we have

g(M(Bz, z, t)) ≤ ϕ(max{g(M(Bz, z , t)), g(M(Bz ,Bz,t)),

g(M(z ,z ,t)), 1/2(g(M(Bz, z , t))+ g(M(z , Bz, t)))})

$\Longrightarrow$ g(M(Bz, z, t)) ≤ ϕ(max{g(M(Bz, z , t)), 0, 0,

g(M(Bz ,z , t)))})

$\Longrightarrow$ g(M(Bz, z, t)) ≤ ϕ{g(M(Bz, z , t)}

$\Longrightarrow$ g(M(Bz, z, t)) = 0 .i.e., M(Bz, z, t) = 1

$\Longrightarrow$ z = Bz and ABz = z implies Az = z. Therefore, Az = Bz = Pz =z.

Hence we have Az = Bz = Pz = Sz = Tz = Qz = z that is, z is a common fixed point of A, B, S, T, P and Q.

Case II (AB is continuous). In this case, we have

(AB)^2 x$_{2n}$ $\longrightarrow$ ABz and (AB)Px$_{2n}$ $\longrightarrow$ ABz.

As (P, AB) is semicompatible, we have P(AB)x$_{2n}$ $\longrightarrow$ ABz.

Step VI. By putting x = ABx$_{2n}$, y = x$_{2n+1}$ in (3), we obtain that

g(M(PABx$_{2n}$, Qx$_{2n+1}$, t)) ≤ ϕ(max{g(M(ABABx$_{2n}$,STx$_{2n+1}$, t)), g(M(ABABx$_{2n}$, PABx$_{2n}$, t)),g(M(STx$_{2n+1}$,Qx$_{2n+1}$,t)),

1/2(g(M(ABABx$_{2n}$,Qx$_{2n+1}$, t))+

g(M(STx$_{2n+1}$, PABx$_{2n}$, t)))})

Letting n $\longrightarrow \infty$, we have

g(M(ABz, z, t)) ≤ ϕ(max{g(M(ABz, z , t)),

g(M(ABz ,ABz,t)),g(M(z ,z ,t)),

1/2(g(M(ABz ,z , t))+ g(M(z , ABz, t)))})

$\Longrightarrow$ g(M(ABz, z, t)) ≤ ϕ(max{g(M(ABz, z , t)), 0, 0, (g(M(z , ABz, t)))})

$\Longrightarrow$ g(M(ABz, z, t)) ≤ ϕ{g(M(ABz, z , t)}

$\Longrightarrow$ g(M(ABz, z, t)) = 0 .i.e., M(ABz, z, t) = 1

$\Longrightarrow$ z = ABz and rest of the proof follows from **Step III** onwards of the previous case.

Uniqueness. Let u be another common fixed point of A, B, S, T, P and Q.

Then u = Au = Su = Bu = Tu = Pu = Qu.

Putting x = z and y = u in (3), we have

g(M(Pz, Qu, t)) ≤ ϕ(max{g(M(ABz, STu, t)), g(M(ABz, Pz,t)),g(M(STu, Qu,t)),
1/2(g(M(ABz, Qu, t))+ g(M(STu, Pz, t)))}),

$\Longrightarrow$ g(M(z, u, t)) ≤ ϕ(max{g(M(z, u, t)), g(M(z, z, t)),g(M(u, u, t)), 1/2(g(M(z, u, t))+ g(M(u, z, t)))}),

$\Longrightarrow$ g(M(z, u, t)) ≤ ϕ(max{g(M(z, u, t)), 0, 0, g(M(z, u, t))}),

$\Longrightarrow$ g(M(z, u, t)) ≤ ϕ{g(M(z, u , t)}

$\Longrightarrow$ g(M(z, u, t)) = 0 .i.e., M(z, u, t) = 1

$\Longrightarrow$ z = u.

Therefore, z is the unique common fixed point of the self-maps A, B, S, T, P and Q.

Corollary 1.1. Let A, B, S , T :X→X be mappings such that

(1) A(X) ⊂ T(X) and B(X) ⊂ S(X),

(2) g(M(Ax, By, t)) ≤ ϕ(max{g(M(Sx,Ty, t)), g(M(Sx,Ax,t)),g(M(Ty,By,t)),
1/2(g(M(Sx,By, t))+ g(M(Ty, Ax, t)))}) for all t > 0,

where a function ϕ : [0, ∞) → [0,∞) satisfies the condition (Φ).

(3) the pair (A,S) is semicompatible and (B,T) is weakly compatible.

(4) one of A or S is continuous

Then A, B,S and T have a uniqe common fixed point in X.

Proof. As semicompatibility implies weak compatibility, the proof follows from theorem 1.1.

Now, taking S = I and T = I in theorem in 1.1, we get the following corollary.

Corollary 1.2. Let A, B :X→X be mappings such that

(1) g(M(Ax, By, t)) ≤ ϕ(max{g(M(x, y, t)),

g(M(x, Ax, t)),g(M(y, By, t)),

1/2(g(M(x, By, t))+ g(M(y, Ax, t)))}) for all t > 0, where a function $\phi : [0, \infty) \rightarrow [0,\infty)$ satisfies the condition (Φ) .

Then A , B have a unique common fixed point in X.

4. CONCLUSION

In this work we have dealt with fixed points of six mappings in which one pair of three mappings is semicompatible and other pair of three mappings is weakcompatible. The future scope is to obtained quartet fixed points for mappings in this space.

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