

# Robust PID Control of Mobile Satellite Dish Network within Nigeria

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## ABSTRACT

Automatically, locking a load to a desired position as quickly and precisely as possible using Proportional-Integral-Derivative controller can be a major problem even in direct control depending on the amount of delay involved. This is because the formulated corrective action is based on the past output and not the current output being corrected. Hence, the effectiveness of formulated control action is determined largely by the amount of delay between the feedback link and the time of delivery of the control action to the load's actuator. The large and variable time delays incurred by remotely controlling network of satellite dishes mounted on moving vehicles in Nigeria with land area of 923,677 km<sup>2</sup> further makes the control problem daunting. Therefore, the principal objective of this research is to develop a robust PID controller with satisfactory real-time optimal control performance of satellite dish network mounted on mobile vehicles spread all over Nigeria.

## General Term

PID Control System

## Keywords

PID controller, variable time delay, satellite dish network, mobile vehicle, Nigeria

## 1. INTRODUCTION

The use of Proportional-Integral-Derivative, PID, controller to automatically lock a load to a desired position as quickly and precisely as possible can be a major problem even in direct control depending on the amount of delay involved. The reason is that the formulated corrective action is based on the past output and not the current output being corrected. Hence, the effectiveness of a formulated control action is determined largely by the amount of delay between the feedback link and the time of delivery of that control action to the load's actuator. Therefore, the large and variable time delays incurred by remotely controlling network of satellite dishes mounted on moving vehicles that can be anywhere in Nigeria further makes the control problem daunting. This paper presents our work in the development of a robust PID controller with satisfactory real-time optimal control performance of satellite dish network mounted on mobile vehicles cruising or stationed within Nigeria.

The two major sources of time delay are the plant's relative position to the controller, and the speed of the mobile vehicle. The plant, i.e. the satellite dish, can be located anywhere within Nigeria, while the maximum vehicle speed supported is 240 km/hr. Hence, our objective is the effective control and delay management of this *national area network* of dishes mounted on vehicles either in parking position or in motion. The time delay incurred can be colossal considering the fact that Nigeria has a vast land area of 923,677 sq km and

extensive geographical coordinates between longitude 2°43.207'E and 14°54.685'E and latitude 4°17.825'N and 13°52.837'N. The Central Control Office, CCO, is at Ilorin (4.675°E, 8.485°N), and the satellite to lock to is at 27.5°W.

The two parameters needed to formulate the control law are the round trip delay, and the plant's transfer function.

Round trip time delay is the sum total of delays from the plant to the satellite, the satellite to the CCO, CCO to the satellite, and the satellite back to the plant; or vice versa. Therefore, a model for predicting the end-to-end delays was developed; and the plant's transfer function was empirically determined. The plant is the Outdoor Unit consisting of a dish, BUC/LNB, and a jack actuator. The round trip delay model is based on calculating the round trip distance divided by the speed of light at  $30 \times 10^9$  m/s. The transfer function is determined from the plant total mass, spring constant, and damping coefficient. Ordinarily these parameters should be read off the plant, however, these information are not available on our plant. Therefore, the parameters were empirically determined by experimentation. The principal performance index for the formulated control action is based on the settling time,  $t_s$ , of the composite system dynamic's time response. After putting together the composite system transfer function, it was then subjected to a step input forcing function which yielded an output with a settling time value for the uncompensated system assume to be the worse case settling time. This worse case settling time form the basis for the determination of PID controller's parameters.

The three PID parameters required are the proportional gain value,  $K_p$ ; integral gain value,  $K_i$ ; and the derivative gain value,  $K_d$  that guarantee system stability in spite the control action. Firstly the acceptable value of  $K_p$  was obtained using Root locus method. Then for the determined value of  $K_p$ , the region of acceptable settling time for a stable system was graphically determined in the  $K_i - K_d$  plane. Given this acceptable region of stability, values for  $K_p$ ,  $K_i$  and  $K_d$  were determined for an optimum settling time,  $t_s$  using linear programming as a novel method. The possible minimum and maximum time delays within Nigeria, which is the region of interest, were determined.

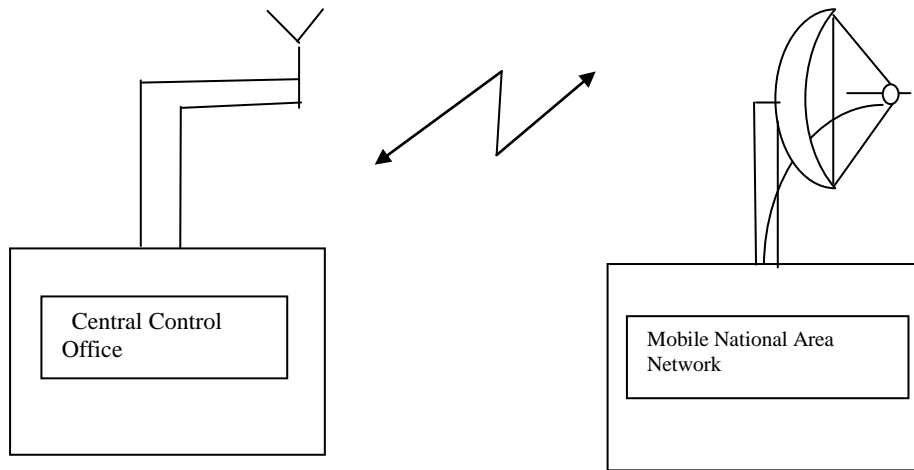
The obtained values of  $K_p$ ,  $K_i$ , and  $K_d$  were used to test the designed system for robustness by varying the time delay from minimum to maximum values. The testing was done using MATLAB as a simulation tool. The performance indices are the settling time, time to peak overshoot, percentage overshoot and rise time. The range of the time delay obtained was between 0.2461 seconds at Gulf of Guinea (2.73°E, 4.3°N) and 0.2496 seconds at Lake Chad (14.94°E, 13.91°N). The system transfer function obtained empirically shows that the dish antenna unit has a damping ratio of 0.3

and natural frequency of 1.5 radians/second, while the actuator has a critical damping ratio. The result of the simulation gave a  $K_p$  gain value of 20, and settling time of 158 seconds for the uncompensated system. The optimum parameter values obtained from the experiment are 15.9 seconds for  $t_s$ , 20 for  $K_p$ , 4 for  $K_i$ , and 0 for  $K_d$  for the compensated system. These obtained values for  $K_p$ ,  $K_i$ , and  $K_d$  were then used to test the designed system for robustness by varying the time delay from 0.2461 seconds to 0.2496 seconds. The corresponding performance indices obtained were 19.1 seconds settling time, 4.38 seconds time to peak overshoot, 69% percentage overshoot and 2.82 seconds rise time. These performance indices remain constant over the entire time delay range, thus confirming the robustness of the developed system. The system settling time of 19.1 seconds and 158 seconds obtained for the compensated and uncompensated systems respectively show an improvement in the steady state performance of the system with PID controller due to reduction in settling time value. The main contributions of this work include the determination of the minimum and maximum round trip time delays for

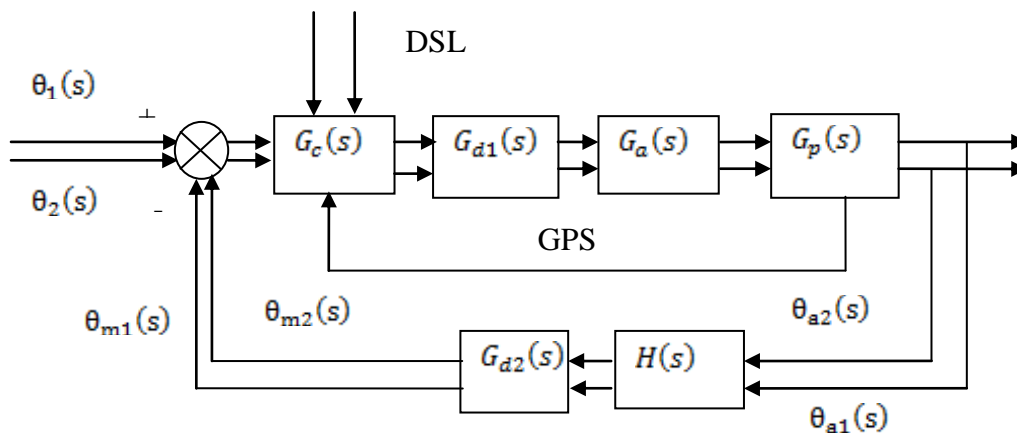
vehicle carrying satellite dish and moving at a maximum speed of 240 km/hr within Nigeria, and the empirical determination of the plant transfer function's parameters. Also, as part of our contribution is the empirical determination of the region of acceptable settling time in the stability region of  $K_i - K_d$  plane; and the optimisation of the settling time. Finally, the work serves as a springboard for various applications including a platform for implementation of tele-medicine in Nigeria.

## 2. PROBLEM STATEMENT

Given the location of central control office, satellite's longitude, and a mobile national area dish network, formulate control action to lock quickly and precisely all the dishes in the network to the given satellite. This problem is represented by system model of Fig.1. The system model can be reduced to control system problem of Fig.2.



**Fig.1 The problem definition in block diagram**



**Fig.2: Reduction of the system model to control system problem**

where:  $G_c(s) \Rightarrow$  Controller transfer function

$G_{d1}, G_{d2} \Rightarrow$  Time delays

$G_a \Rightarrow$  Actuator transfer function

$G_p \Rightarrow$  Plant transfer function

$H(s) \Rightarrow$  Feedback transfer function

### 3. SYSTEM MODELLING

#### (i) Time Delay Modelling

The round trip delay is estimated from the total distance of signal travel divided by the speed of light. The distance,  $d_{sr}$ , between an earth station and a geostationary satellite is given by eqn (1):

$$d_{sr} = \sqrt{D^2 + R^2 - 2DR \cos(\alpha_{sn}) \cos(\Delta_1)} + \sqrt{D^2 + R^2 - 2DR \cos(\alpha_{rn}) \cos(\Delta_2)} \quad (1)$$

$$\Delta_1 = \Delta_{sn} - \Delta_s$$

$$\Delta_2 = \Delta_{rn} - \Delta_s$$

Where:

$R$  = radius of the earth in km

$D$  = sum of the radius of the earth and satellite altitude in km

$\Delta_s$  = angle of longitude of the satellite location in space in degrees

$\alpha_{sn}$  = latitude of the sending node location on the earth surface in degrees.

$\alpha_{rn}$  = latitude of the receiving node location on the earth surface in degrees.

$\Delta_{sn}$  = angle of longitude of the sending node location on the earth surface in degrees.

$\Delta_{rn}$  = angle of longitude of the receiving node location on the earth surface in degrees.

Therefore, the round trip time-delay,  $T_{rt}$ , is given by eqn (2):

$$T_{rt} = \frac{2 \cdot d_{sr}}{v} \quad (2)$$

Where:

$v = 3 \times 10^9$  m/s speed of light

The round trip delay in Nigeria has a minimum delay of 0.2461 seconds at Gulf of Guinea (2.73°E, 4.3°N); and maximum delay at Lake Chad (14.9°E, 13.91°N).

#### (ii) Outdoor Dish Unit Modelling [1]

The dynamic of the dish structure system is given in eqn (3) as:

$$I_A \ddot{\theta}_A + C_A \dot{\theta}_A + K_A \theta_A = K_A \theta_g \quad (3)$$

This equation can be reduced into standard second order form:

$$\ddot{\theta}_A + 2 \zeta \omega_n \dot{\theta}_A + \omega_n^2 \theta_A = \omega_n^2 \theta_g \quad (4)$$

where:

$$\omega_n^2 = \frac{K_A}{I_A}$$

$$\zeta = \frac{C_A}{2 \sqrt{K_A I_A}}$$

$\theta_A$  = dish angular displacement in radian;

$\theta_g$  = angular displacement of the gear output shaft

in radian;

$I_A$  = dish moment of inertial about a given axis in  $\text{kgm}^2$ ;

$C_A$  = damping coefficient in  $\text{Nm/s/radian}$ ;

$K_A$  = torsional spring stiffness in  $\text{Nm/radian}$ ;

$\omega_n$  = undamped natural frequency in  $\text{radian/second}$ ;

$\zeta$  = damping ratio

And this results in closed loop transfer function expressed in eqn (5):

$$\frac{\theta_A(s)}{\theta_g(s)} = \frac{\omega_n^2}{s^2 + 2 \zeta \omega_n s + \omega_n^2} \quad (5)$$

The relationship of the parameters in eqn (4) to the parameters of the physical dish structure is given in eqn (6):

$$I_A = \frac{M_A r^2}{4}, \quad K_A = \frac{G J}{L}, \quad J = \frac{\pi D^4}{32} \quad (6)$$

where:

$M_A$  = dish mass in kg

$$r = 1.25 \times r_A$$

$r_A$  = dish radius in metre

$G$  = modulus of rigidity in Pa;

$J$  = polar second moment of area in  $\text{m}^4$  for a material of circular sectional area

$D$  = diameter of the circular section in  $\text{m}^2$ ;

$L$  = length of the material in m;

Ordinarily, the values of these parameters are suppose to be available on the plant, however in our case they are not available. Hence they are were empirically determined from the 18 inch Eurostar jack actuator used. And for a selected value of damping ratio of  $\zeta = 0.3$ , the values of the parameters experimentally obtained are:

$$M_A = 250kg, \quad r_A = 1.2m, \quad G = 77 \times 10^9 Pa,$$

$$D = 0.013m, \quad L = 0.68m$$

$$I_A = 140.60kgm^2$$

$$J = 2.8040 \times 10^{-9} m^4$$

$$K_A = 317.5Nm/rad \cdot$$

$$\omega_n = 1.503radian/sec \cdot$$

$$C_A = 126.78Nms/rad.$$

$$\frac{C_A}{I_A} = 0.9016$$

Substituting these parameter values in equations (3) and (5) yields equations (7) and (8).

$$\ddot{\theta}_A + 0.9016 \dot{\theta}_A + 2.2578 \theta_A = 2.2578 \theta_g \quad (7)$$

The resulting transfer function is:

$$\frac{\theta_A(s)}{\theta_g(s)} = \frac{2.2578}{s^2 + 0.9016s + 2.2578} \quad (8)$$

Fig.3 shows the block diagram of the components of the outdoor unit with their corresponding transfer function for a determined actuator jack gear ratio of 30.

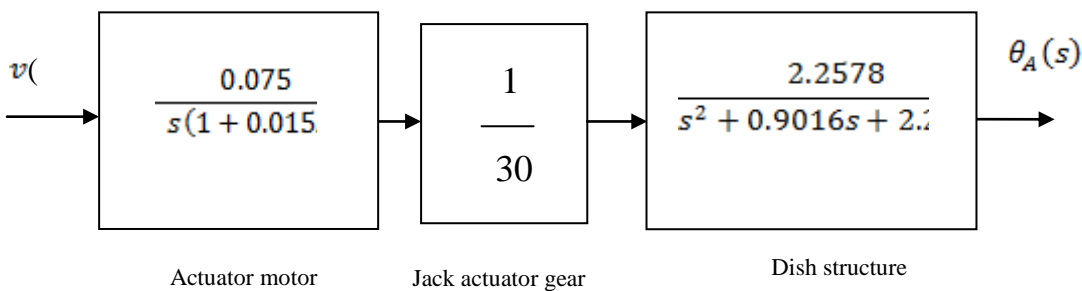


Fig.3: Block diagram of transfer functions of the components of the outdoor dish unit

The simulation result of the step input response of the outdoor unit shows that the unit is stable. Assuming unity feedback the open loop and closed loop transfer function of the composite system are given in eqn (9) and eqn (10) respectively.

The open loop transfer function of the outdoor unit is :

$$\frac{\theta_A(s)}{v(s)} = \frac{3.76}{s^4 + 67.56s^3 + 62.36s^2 + 150.52s} \quad (9)$$

And its corresponding closed loop transfer function is:

$$\frac{\theta_A(s)}{\theta_r(s)} = \frac{3.76}{s^4 + 67.56s^3 + 62.36s^2 + 150.52s + 3.76} \quad (10)$$

Where;  $\theta_r$  = reference dish position in radian

(iii) PID Controller Modelling [2,3,4,5,6]

The standard PID controller transfer function is:

$$G_c(s) = \frac{K_d s^2 + K_p s + K_i}{s} \quad (11)$$

The composite system closed loop transfer function of Fig.1. is:

$$\frac{\theta_A(s)}{\theta_r(s)} = \frac{G_c(s)G_p(s)G_{d1}(s)}{1 + G_c(s)G_p(s)G_{d1}(s)G_{d2}(s)} \quad (12)$$

This gives characteristic equation of :

$$1 + G_c(s)G(s) = 0 \quad (13)$$

where :  $G(s) = G_p(s)G_{d1}G_{d2}$

The problem is, therefore, to find  $G_c(j\omega)$  that will make the characteristic equation of eqn(13) Hurwitz stable. The design of PID controller for any arbitrary order system can be achieved by finding the general solution to eqn (14) in the (Ki, Kd) plane for a fixed value of Kp. [ ].

$$(\omega R_e(\omega))K_p + (I_m(\omega))K_i = K_d(I_m(\omega)\omega^2) - \omega \quad (14)$$

$$(\omega I_m(\omega))K_p - (R_e(\omega))K_i = -K_d(R_e(\omega)\omega^2)$$

where:

$$G(j\omega) = G_p(j\omega)G_d(j\omega);$$

$$R_e(\omega) = \text{Real part of } G(j\omega);$$

$$I_m(\omega) = \text{Imaginary part of } G(j\omega);$$

The stability boundaries in the (Kp, Ki) plane for fixed value of Kd is obtained by solving eqn(15) for the values of  $\omega = [0, \omega_c]$ .

$$K_p(w) = \frac{R_e(w)}{|G(jw)|^2} \quad (15)$$

$$K_i(w) = w^2 K_d(w) - \frac{wI_m(w)}{|G(w)|^2}$$

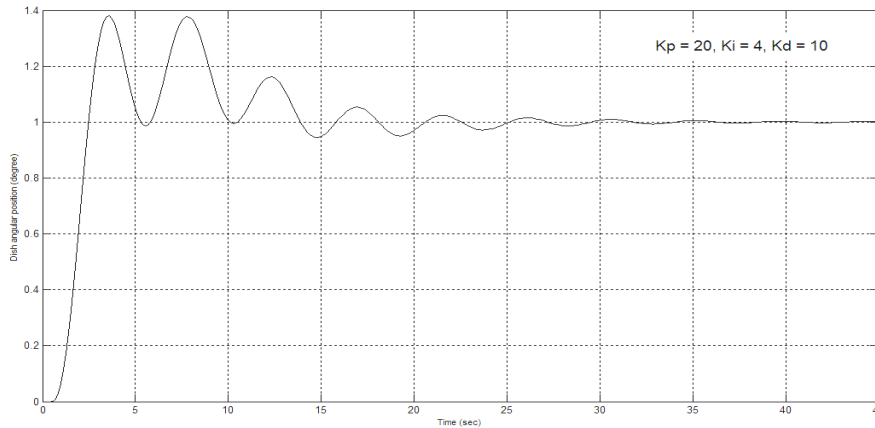
And the stability boundaries in the (Kp, Kd) plane for fixed value of Ki is obtained by solving eqn(16) for the values of  $\omega = [0, \omega_c]$ .

$$K_p(w) = \frac{R_e(w)}{|G(jw)|^2}$$

$$K_d(w) = \frac{K_i(w)}{w^2} + \frac{I_m(w)}{w|G(jw)|^2} \quad (16)$$

#### 4. EXPERIMENT AND SIMULATION

Simulation of the various models was carried out in MATLAB. The system transfer function obtained empirically shows that the dish antenna unit has a damping ratio of 0.3 and natural frequency of 1.5 radians/second, while the actuator has a critical damping ratio. The result of the simulation gave a Kp gain value of 20, and settling time of 158 seconds for the uncompensated system. The optimum parameter values obtained from the experiment are 15.9 seconds for for ts, 20 for Kp, 4 for Ki, and 0 for Kd for the compensated system. These obtained values for Kp, Ki, and Kd were then used to test the designed system for robustness by varying the time delay from 0.2461 seconds to 0.2496 seconds. The corresponding performance indices obtained were 19.1 seconds settling time, 4.38 seconds time to peak overshoot, 69% percentage overshoot and 2.82 seconds rise time. These performance indices remain constant over the entire time delay range, thus confirming the robustness of the developed system. The system settling time of 19.1 seconds and 158 seconds obtained for the compensated and uncompensated systems respectively show an improvement in the steady state performance of the system with PID controller due to reduction in settling time value. Fig.4 shows the step response of the composite system dynamic.



**Fig.4. The step response of composite system for values of  $K_p = 20$ ,  $K_i = 4$  and  $K_d = 10$ .**

#### 5. CONCLUSION

A supervisory control using PID controller for satellite dish network mounted on mobile vehicles either in fixed positions or cruising at maximum speed of 240/km/hr within Nigeria was designed and simulated. The performance indices remain constant over the entire time delay range, thus confirming the robustness of the developed system. Also, the system settling time of 19.1 seconds and 158 seconds obtained for the compensated and uncompensated systems respectively show an improvement in the steady state performance of the system with PID controller due to reduction in settling time value.

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