On $\mathcal{F}^{-\alpha\delta}$ Continuous Multifunctions

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ABSTRACT
In this paper we introduce the notion of Upper $\mathcal{F}^{-\alpha\delta}$-Continuous and Lower $\mathcal{F}^{-\alpha\delta}$-Continuous Multifunctions. The basic properties and characterizations of such functions are established.

Keywords
$\alpha\delta$-open sets, $\alpha\delta$-closed sets, faintly $\alpha\delta$-continuous multifunctions, $\alpha\delta_\theta$-closed.

1. INTRODUCTION
It is well known that various types of functions play a significant role in the theory of classical point set topology. A great number of papers dealing with such functions have appeared, and a good number of them have been extended to the setting of multifunctions [8]. This implies that both functions and multifunctions are important tools for studying properties of spaces and for constructing new spaces from previously existing ones. R. Devi, V. Kokilavani P. Basker [3] has introduced and studied the notion of $\alpha\delta$-closed sets in topological spaces. In this paper, we introduce and study upper and lower faintly $\alpha\delta$-continuous (briefly. $\mathcal{F}^{-\alpha\delta}$-Continuous) multifunctions in topological spaces. The main purpose of this paper is to define faintly $\alpha\delta$-continuous multifunctions and to obtain several characterizations and basic properties of such multifunctions.

2. PRELIMINARIES
Throughout this present paper, spaces X and Y always mean topological spaces. Let X be a topological space and A, a subset of X. The closure of A and the interior of A are denoted by $cl(A)$ and $int(A)$, respectively. A subset A is said to be regular open (resp. regular closed) if $A = int(cl(A))$ (resp. $A = cl(int(A))$), The δ-interior [11] of a subset A of X is the union of all regular open sets of X contained in A and is denoted by $Int_\delta(A)$.

The subset A is called δ-open [11] if $A = Int_\delta(A)$, *i.e.*, a set is δ-open if it is the union of regular open sets. The complement of a δ-open set is called δ-closed. Alternatively, a set $A \subset (X, \tau)$ is called δ-closed [11] if $A = cl_\delta(A)$, where $cl_\delta(A) = \{x : x \in U \in \tau \Rightarrow int(cl(A)) \cap A \neq \varphi\}$.

The family of all δ-open (resp. δ-closed) sets in X is denoted by $\delta O(X)$ (resp. $\delta C(X)$). A subset A of X is called α-open [9] if $A \subset int(cl(int(A)))$ and the complement of a α-open are called α-closed. The intersection of all α-closed sets containing A is called the α-closure of A and is denoted by $\alpha cl(A)$, Dually, α-interior of A is defined to be the union of all α-open sets contained in A and is denoted by $\alpha int\ (A)$.

A point $x \in X$ is called a θ-cluster point of A if $cl(V) \cap A \neq \varphi$ for every open subset V of X containing x. The set of all θ-cluster points of A is called the θ-closure of A and is denoted by $cl_\theta(A)$. If $A = cl_\theta(A)$, then A is said to be θ-closed [10]. The complement of a θ-closed set is said to be θ-open. Clearly, A is θ-open if and only if for each $x \in A$, there exists an open set U such that $x \in U \subset cl(U) \subset A$.
We recall the following definition used in sequel.

DEFINITION 2.1. A subset A of a space X is said to be

(a) An α-generalized closed [1] (αg-closed) set if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is α-open in (X, τ)
(b) An $\alpha\delta$-closed [3] set if $cl_\delta(A) \subseteq U$ whenever $A \subseteq U$ and U is αg-open in (X, τ).

The complement of a $\alpha\delta$-*closed* set is said to be $\alpha\delta$-*open*. The intersection of all $\alpha\delta$-*closed* sets of X containing A is called $\alpha\delta$-closure [4] of A and is denoted by $\alpha\delta_{Cl}(A)$. The union of all $\alpha\delta$-*open* sets of X contained in A is called $\alpha\delta$-interior [4] of A and is denoted by $\alpha\delta_{Int}\,(A)$.

The family of all $\alpha\delta$-open subsets of (X, τ) will be denoted by $\alpha\delta O(X)$. By a multifunction $: X \rightarrow Y$, we mean a point to-set correspondence from X into Y , also we always assume that $F\,(x) \neq \varphi$ for all $x \in X$. For a multifunction $F : X \rightarrow Y$, the upper and lower inverse of any subset A of Y are denoted by $F^+(A)$ and $F^-(A)$ respectively[2], where $F^+(A) = \{x \in X : F\,(x) \subset A\}$ and $F^-(A) = \{x \in X : F\,(x) \cap A \neq \varphi\}$. In particular, $F^-(A) = x \in X : y \in F\,(x)$ for each point $\in Y$. A multifunction $F : X \rightarrow Y$ is said to be surjective if $F\,(X) = Y$. A multifunction $F : (X, \tau) \rightarrow (Y, \sigma)$ is said to be lower $\alpha\delta$-continuous (resp. upper $\alpha\delta$-continuous) multifunction if $F^-(A) \in \alpha\delta O(X)$ (resp. $F^+(A) \in \alpha\delta O(X)$) for every $V \in \sigma$.

3. FAINTLY $\alpha\delta$-CONTINUOUS MULTIFUNCTIONS
DEFINITION 3.1. A multifunction $F : X \rightarrow Y$ is said to be:

(a) Upper faintly $\alpha\delta$-continuous (briefly. Upper $\mathcal{F}^{-\alpha\delta}$-Continuous) at $x \in X$ if for each θ-open subset V of Y containing $F\,(x)$, there exists $U \in \alpha\delta O(X)$ containing x such that $F(U) \subset V$;
(b) Lower faintly $\alpha\delta$-continuous (briefly. Lower $\mathcal{F}^{-\alpha\delta}$-Continuous) at $x \in X$ if for each θ-open subset V of Y such that $F\,(x) \cap V \neq \varphi$, there exists $U \in \alpha\delta O(X)$ containing x such that $F\,(u) \cap V \neq \varphi$ for every $u \in U$;

(c) Upper (resp. Lower) faintly $\alpha\delta$-continuous if it is Upper (resp. Lower) faintly $\alpha\delta$-continuous at each point of X.

REMARK 3.2. Since every θ-open set is open, it is clear that every upper (lower) $\alpha\delta$-continuous multifunction is upper (lower) faintly $\alpha\delta$-continuous. However, the converse is not true as the following simple example shows.

THEOREM 3.3. For a multifunction $: X \to Y$, the following are equivalent.

(a) F is Upper $\mathcal{F}^{-\alpha\delta}$-Continuous;
(b) For each $x \in X$ and for each θ-open set V such that $x \in F^+(V)$, there exists a $\alpha\delta$-open set U containing x such that $U \subset F^+$(V)
(c) For each $x \in X$ and for each θ-closed set V such that $x \in F^+(Y-V)$, there exists a $\alpha\delta$-closed set H such that $x \in X-H$ and $F^-(V) \subset H$;
(d) $F^+(V)$ is $\alpha\delta$-open for any θ-open subset V of Y;
(e) $F^-(V)$ is $\alpha\delta$-closed for any θ-closed subset V of Y;
(f) $F^-(Y-V)$ is $\alpha\delta$-closed for any θ-open subset V of Y;
(g) $F^+(Y-V)$ is $\alpha\delta$-open for any θ-closed subset V of Y.

PROOF. $(a) \Leftrightarrow (b)$: Clear.

$(b) \Leftrightarrow (c)$: Let $x \in X$ and V be a θ-closed subset of Y such that $x \in F^+(Y-V)$. By (b), there exists a $\alpha\delta$-open set U containing x such that $U \subset F^+(Y-V)$. Thus $F^-(V) \subset X-U$. Take $H = X-U$. Then $x \in X-H$ and H is $\alpha\delta$-closed. The converse is similar.

$(a) \Leftrightarrow (d)$: Let $x \in F^+(V)$ and V be a θ-open subset of . By (a), there exists a $\alpha\delta$-open set U_x containing x such that $U_x \subset F^+(V)$. Thus, $F^+(V) = \cup_{x \in F^+(V)} U_x$. Since any union of $\alpha\delta$-open sets is $\alpha\delta$-open, $F^+(V)$ is $\alpha\delta$-open. The converse is clear.

$(d) \Leftrightarrow (g)$ and $(e) \Leftrightarrow (f)$: Clear.
$(d) \Leftrightarrow (f)$: Follows from the fact that $F^-(V) = X - F^+(Y-V)$.

THEOREM 3.4. For a multifunction $F : X \to Y$, the following are equivalent:

(a) F is Lower $\mathcal{F}^{-\alpha\delta}$-Continuous;
(b) For each $x \in X$ and for each θ-open set V such that $x \in F^-(V)$, there exists a $\alpha\delta$-open set U containing x such that $U \subset F^-(V)$;
(c) For each $x \in X$ and for each θ-closed set V such that $x \in F^-(Y-V)$, there exists a $\alpha\delta$-closed set H such that $x \in X-H$ and $F^+(V) \subset H$;
(d) $F^-(V)$ is $\alpha\delta$-open for any θ-open subset V of Y;
(e) $F^+(V)$ is $\alpha\delta$-closed for any θ-closed subset V of Y;
(f) $F^+(Y-V)$ is $\alpha\delta$-closed for any θ-open subset V of Y;
(g) $F^-(Y-V)$ is $\alpha\delta$-open for any θ-closed subset V of Y.

PROOF. Similar to that of Theorem 3.3.

THEOREM 3.5. Suppose that (X, τ) and (X_i, τ_i) are topological spaces where $i \in I$. Let $F: X \to \prod_{i\in I} X_i$ be a multifunction from X to the product space $\prod_{i\in I} X_i$ and let $P_i: \prod_{i\in I} X_i \to X_i$ be a projection multifunction for each $i \in I$ which is defined by $P_i((x_i)) = \{x_i\}$. If F is upper (lower) faintly $\alpha\delta$-continuous, then $P_i \circ F$ is upper (lower) $\mathcal{F}^{-\alpha\delta}$-Continuous for each $i \in I$.

PROOF. Let V_i be a θ-open set in (X_i, τ_i). Then $(P_i \circ F)^+(V_i) = F^+\left(P_i^{\,+}(V_i)\right) = F^+(V_i \times \prod_{j\neq i} X_j)$ (resp. $(P_i \circ F)^-(V_i) = F^-\left(P_i^{\,-}(V_i)\right) = F^-(V_i \times \prod_{j\neq i} X_j)$. Since F is upper (lower) faintly $\alpha\delta$-continuous and since $V_i \times \prod_{j\neq i} X_j$ is a θ-open set, it follows from Theorems 3.3 and 3.4 that $F^+(V_i \times \prod_{j\neq i} X_j)$ (resp. $F^-(V_i \times \prod_{j\neq i} X_j)$ is a $\alpha\delta$-open set in (X, τ). Hence again by Theorems 3.3 and 3.4, $P_i \circ F$ is upper (lower) $\mathcal{F}^{-\alpha\delta}$-Continuous for each $i \in I$.

COROLLARY 3.6. Let $F : X \to Y$ be a multifunction. If the graph multifunction G_F of F is upper (lower) $\mathcal{F}^{-\alpha\delta}$-Continuous, then F is upper (lower) $\mathcal{F}^{-\alpha\delta}$-Continuous, where $G_F: X \to X \times Y$, $G_F(x) = \{x\} \times F(x)$.

COROLLARY 3.7. Suppose that $(X, \tau), (Y, \sigma), (Z, \eta)$ are topological spaces and $F_1 : X \to Y$, $F_2 : X \to Z$ are multifunctions. Let $F_1 \times F_2 : X \to Y \times Z$ be the multifunction defined by $(F_1 \times F_2)(x) = F_1(x) \times F_2(x)$ for each $x \in X$. If $F_1 \times F_2$ is upper (lower) $\mathcal{F}^{-\alpha\delta}$-Continuous, then F_1 and F_2 are upper (lower) $\mathcal{F}^{-\alpha\delta}$-Continuous. The following lemma can be easily established.

LEMMA 3.8. If $A \times B \in \alpha\delta O(X \times Y)$, then $A \in \alpha\delta O(X)$ and $B \in \alpha\delta O(Y)$.

THEOREM 3.9. Suppose that (X_i, τ_i) and (Y_i, σ_i) are topological spaces for each $i \in I$. Let $F_i : X_i \to Y_i$ be a multifunction for each $i \in I$ and let $F: \prod_{i\in I} X_i \to \prod_{i\in I} Y_i$ be the multifunction defined by $F((x_i)) = \prod_{i\in I} F_i(x_i)$. If F is upper (lower) $\mathcal{F}^{-\alpha\delta}$-Continuous, then F_i is upper (lower) $\mathcal{F}^{-\alpha\delta}$-Continuous for each $i \in I$.

PROOF. Let V_i be a θ-open subset of Y_i. Then $V_i \times \prod_{j\neq i} X_j$ is a θ-open set. Since F is upper (lower) $\mathcal{F}^{-\alpha\delta}$-Continuous, it follows from Theorems 3.4 and 3.5 that $F^+\left(V_i \times \prod_{j\neq i} Y_j\right) = F_i^{\,+}(V_i) \times \prod_{j\neq i} X_j$ (resp. $F^-\left(V_i \times \prod_{j\neq i} Y_j\right) = F_i^{\,-}(V_i) \times \prod_{j\neq i} X_j$). Consequently, it follows from Lemma 3.8 that $F_i^{\,+}(V_i)$ (resp. $F_i^{\,-}(V_i)$) is a $\alpha\delta$-open set. Thus again by Theorems 3.3 and 3.4, F_i is upper (lower) $\mathcal{F}^{-\alpha\delta}$-Continuous for each $i \in I$.

COROLLARY 3.10. Suppose that $F_1 : X_1 \to Y_1$, $F_2 : X_2 \to Y_2$ are multifunctions. If $F_1 \times F_2$ is upper (lower) $\mathcal{F}^{-\alpha\delta}$-Continuous, then F_1 and F_2 are upper (lower) $\mathcal{F}^{-\alpha\delta}$-Continuous, where $F_1 \times F_2$ is the product multifunction defined as follows: $F_1 \times F_2: X_1 \times X_2 \to Y_1 \times Y_2$, $(F_1 \times F_2)(x_1, x_2) = F_1(x_1) \times F_2(x_2)$, where $x_1 \in X_1$ and $x_2 \in X_2$. Recall that a multifunction $F : X \to Y$ is said to be punctually closed if for each $x \in X$, $F(x)$ is closed. Recall also that a space X is called θ-normal if for any disjoint closed subsets F_1 , F_2 of X, there exist two disjoint θ-open subsets V_1 , V_2 of X containing F_1 , F_2 respectively.

DEFINITION 3.11. A topological space (X, τ) is said to be $T_2^{\#\alpha\delta}$ [5] (resp. θ-T_2 [8]) if for each pair of distinct points x and y of X, there exist disjoint $\alpha\delta$-open (resp. θ-open) subsets U and V of X containing x and y, respectively.

THEOREM 3.12. Let $F: X \rightarrow Y$ be an upper $\mathcal{F}^{-\alpha\delta}$-Continuous multi-function and punctually closed from a topological space X into a θ-normal space Y such that $F(x) \cap F(y) = \varphi$ for each pair of distinct points x and y of X. Then X is $T_2^{\#\alpha\delta}$.

PROOF. Let x and y be any two distinct points of X. Then $F(x) \cap F(y) = \varphi$. Since Y is θ-normal and F is punctually closed, there exist disjoint θ-open sets U and V containing $F(x)$ and $F(y)$, respectively, but F is upper $\mathcal{F}^{-\alpha\delta}$-Continuous, so it follows from Theorem 3.4 that $F^+(U)$ and $F^+(V)$ are disjoint $\alpha\delta$-open subsets of X containing x and y, respectively. Hence X is $T_2^{\#\alpha\delta}$.

DEFINITION 3.13. A topological space (X, τ) is said to be θ-compact [5] (resp. $\alpha\delta$-compact) if every θ-open (resp. $\alpha\delta$-open) cover of X has a finite subcover. A subset A of a topological space X is said to be θ-compact relative to X if every cover of A by θ-open subsets of X has a finite subcover of A.

THEOREM 3.14. Let $F : X \rightarrow Y$ be an upper $\mathcal{F}^{-\alpha\delta}$-Continuous surjective multifunction such that $F(x)$ is θ-compact relative to Y for each $x \in X$. If X is $\alpha\delta$-compact, then Y is θ-compact.

PROOF. Let $V_\alpha : \alpha \in \Lambda$ be a θ-open cover of Y. Since $F(x)$ is θ-compact relative to Y for each $x \in X$, there exists a finite subset $\Lambda(x)$ of Λ such that $F(x) \subset \cup_{\alpha\in\Lambda(x)} V_\alpha$. Put $V(x) = \cup_{\alpha\in\Lambda(x)} V_\alpha$. Then $V(x)$ is a θ-open subset of Y containing $F(x)$. Since F is upper $\mathcal{F}^{-\alpha\delta}$-Continuous, it follows from Theorem 3.4 that $F^+(V(x))$ is a $\alpha\delta$-open subset of X containing $\{x\}$. Thus the family $\{F^+(V(x)) : x \in X\}$ is a $\alpha\delta$-open cover of X, but X is $\alpha\delta$-compact, so there exist $x_1, x_2, x_3 \ldots x_n \in X$ such that $X = \cup_{i=1}^n F^+(V(x_i))$. Hence, $Y = F\left(\cup_{i=1}^n F^+(V(x_i))\right) = \cup_{i=1}^n F\left(F^+(V(x_i))\right) \subset \cup_{i=1}^n V(x_i) = \cup_{i=1}^n \cup_{\alpha\in\Lambda(x_i)} V_\alpha$. Hence, Y is θ-compact.

For a given multifunction $: X \rightarrow Y$, the graph multifunction $G_F: X \rightarrow X \times Y$ is defined as $G_F(x) = \{x\} \times F(x)$ for every $x \in X$. In [4], it was shown that for a multifunction $F : X \rightarrow Y$, $G^+{}_F(A \times B) = A \cap F^+(B)$ and $G^-{}_F(A \times B) = A \cap F^-(B)$ where $A \subseteq X$ and $B \subseteq Y$. A multifunction $F : X \rightarrow Y$ is said to be a point closed if and only if for each $x \in X$, $F(x)$ is closed in Y.

DEFINITION 3.15. Let $F: X \rightarrow Y$ be a multifunction. The multigraph $G(F) = \{(x, y): y \in F(x), x \in X\}$ of F is said to be $\alpha\delta_\theta$-closed if for each $(x, y) \in (X \times Y) - G(F)$, there exist a $\alpha\delta$-open set U and a θ-open set V containing x and y, respectively, such that $(U \times V) \cap G(F) = \varphi$, *i.e.*, $F(U) \cap V = \varphi$.

THEOREM 3.16. If the graph multifunction $F : X \rightarrow Y$ is upper (lower) $\mathcal{F}^{-\alpha\delta}$-Continuous, then F is upper (lower) $\mathcal{F}^{-\alpha\delta}$-Continuous.

PROOF. We shall only prove the case where F is upper $\mathcal{F}^{-\alpha\delta}$-Continuous. Let $x \in X$ and V be a θ-open set in Y such that $x \in F^+(V)$. Then $G_F(x) \cap (X \times Y) = (\{x\} \times F(x)) \cap (X \times Y) = \{x\} \times (F(x) \cap V) \neq \varphi$ and $X \times V$ is θ-open in $X \times Y$ by Theorem 5 in [3]. Since the graph multifunction G_F is upper $\mathcal{F}^{-\alpha\delta}$-Continuous, there exists an open set U containing x such that $z \in U$ implies that $G_F(z) \cap (X \times V) \neq \varphi$. Therefore, we obtain $U \subseteq G^+{}_F(X \times V) = \mathcal{F}^{-\alpha\delta} - \text{Continuous} \in \alpha\delta O(X)$ from the above equalities. Consequently, F is upper $\mathcal{F}^{-\alpha\delta}$-Continuous.

THEOREM 3.17. Let $: X \rightarrow Y$, be a point closed multifunction. If F is upper faintly $\alpha\delta$-continuous and assume that Y is regular, then $G(F)$ is θ-closed with respect to X.

PROOF. Suppose $(x, y) \notin G(F)$. Then we have $y \notin F(x)$. Since Y is regular, there exist disjoint open sets V_1 ,V_2 of Y such that $y \in V_1$ and $F(x) \in V_2$. By regularity of , V_2 is also θ-open in Y . Since F is upper $\mathcal{F}^{-\alpha\delta}$-Continuous at x, there exists an $\alpha\delta$-open set U in X containing x such that $F(U) \subseteq V_2$. Therefore, we obtain $x \in U, y \in V_1$ and $(x, y) \in U \times V_1 \subseteq (X \times Y) - G(F)$. So $G(F)$ is θ-closed with respect to X.

THEOREM 3.18. Let $F: (X, \tau) \rightarrow (Y, \sigma)$ be a point closed set and upper $\mathcal{F}^{-\alpha\delta}$-Continuous multifunction. If F satisfies $x_1 \neq x_2 \Rightarrow F(x_1) \neq F(x_2)$ and Y is regular space, then X will be Hausdorff.

PROOF. Let x_1, x_2 be two distinct points belong to X, then $F(x_1) \neq F(x_2)$. Since F is point closed and Y is regular, for all $y \in F(x_1)$ with $y \notin F(x_2)$, there exists θ-open sets V_1, V_2 containing y and $F(x_2)$ respectively such that $V_1 \cap V_2 = \varphi$. Since F is upper $\mathcal{F}^{-\alpha\delta}$-Continuous and $F(x_2) \subseteq V_2$, there exists an open open set U containing x_2 such that $F(U) \subseteq V_2$. Thus $x \in U$. Therefore, U and $X - U$ are disjoint open sets separating x_1 and x_2.

THEOREM 3.19. If a multifunction $F : X \rightarrow Y$ is upper $\mathcal{F}^{-\alpha\delta}$-Continuous such that $F(x)$ is θ-compact relative to Y for each $x \in X$ and Y is θ-T_2, then the multigraph $G(F)$ of F is $\alpha\delta_\theta$-closed.

PROOF. Let $(x, y) \in (X \times Y) - G(F)$. Then $y \in Y - F(x)$. Since Y is θ-T_2, for each $z \in F(x)$, there exist disjoint θ-open subsets $U(z)$ and $V(z)$ of Y containing z and y, respectively. Thus $\{U(z) : z \in F(x)\}$ is a θ-open cover of $F(x)$, but $F(x)$ is θ-compact relative to Y , so there exist $z_1, z_2, z_3 \ldots z_n \in F(x)$ such that $F(x) \subset \cup_{i=1}^n U(z_i)$. Put $U = \cup_{i=1}^n U(z_i)$ and $V = \cap_{i=1}^n V(z_i)$. Then U and V are θ-open subsets of Y such that $F(x) \subset U, y \subset V$ and $U \cap V = \varphi$. Since F is upper $\mathcal{F}^{-\alpha\delta}$-Continuous, it follows from Theorem 3.4 that $F^+(U)$ is a $\alpha\delta$-open subset of X. Also $x \in F^+(U)$ since $F(x) \subset U$ and $F\left(F^+(U)\right) \cap V = \varphi$ since $U \cap V = \varphi$. Hence, $G(F)$ is $\alpha\delta_\theta$-closed.

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