Permutation Labeling for Some Shadow Graphs

J. Shiama

Assistant Professor
Department Of Mathematics
Cms College Of Science And Commerce,
Coimbatore -641049, Tamilnadu, India

ABSTRACT

A permutation labeling of a graph G is a bijective assignment of labels from { 1,2,3, …p} to the vertices of G such that when each edge of G has assigned a weight defined by the number of permutations of f(u) things taken f(v) at a time. Such a labeling f is called permutation labeling of G. A graph which admits permutation labeling is called permutation graphs. In this paper I proved that the shadow graphs of path P_n, star $K_{1,n}$ and path union of shadow graphs of cycle C_n are permutation graphs. Further I proved that the split graphs of path P_n and star $K_{1,n}$ are permutation graphs.

Keywords: Permutation labeling, Shadow graph, Split graph, path union.

1. INTRODUCTION

Throughout this paper we consider simple, finite, connected and undirected graph **G = (V(G), E(G))** with p vertices and q edges. G is also called a (p,q) graph. For standard terminology and notation we follow Haray[4] and Gary Chartrand and Pinz zhang[6]. For a detailed survey on graph labeling we refer Gallian [5] .The brief summary of definitions and other information which are necessary for the present investigation are given below. "Graph labeling" at its heart, is a strong communication between" number theory"[3] and "structure graphs" [2,4,6]. In this paper, permutation labeling of graphs are introduced and studied.

Graph labeling , where the vertices and edges are assigned real values or subsets of a set are subject to certain conditions , have often been motivated by their utility to various applied fields and their intrinsic mathematical interest (logical- mathematical).Graph labeling were first introduced in the mid sixties

An enormous body of literature has grown around the subject, especially in the lat thirty years or so, and is still getting embellished due to increasing number of application driven concepts. Permutations play a major role in combinatorial problems. The new labeling introduced in this paper is a logical-mathematical attempt.

Definition 1.1. A (p, q) graph G(V,E) is said to be a permutation graph [7] if there exists a bijection f: V(G) → { 1,2,3, …, p} such that the induced edge function g_f : E(G) → N defined as $g_f(uv)= \begin{cases} f(u)p_{f(v)} \; if \; f(u) > f(v) \\ f(v)p_{f(u)} \; if \; f(v) > f(u) \end{cases}$

is injective, where $^{f(u)}P_{f(v)}$ denotes the number of permutations of f(u) things taken f(v) at a time. Such a labeling f is called permutation labeling of G.

Definition 1.2. The shadow graph $D_2(G)$ of a connected graph G is constructed by taking two copies of G say G' and G" join each vertex v' in G; to the neighbors of the corresponding vertex v" in G".

Definition 1.3. For a graph G the split graph is obtained by adding to each vertex v a new vertex such that v' is adjacent to every vertex that is adjacent to v in G. The resultant graph is denoted as spl(G)

Definition 1.4. Let G_1 , G_2 , …… G_n , n ≥ 2 be n copies of a fixed graph G. The graph G obtained by adding an edge between G_i and G_{i+1} for i = 1,2,…… n-1 is called path union of G.[7]

2. MAIN RESULTS

PERMUTATIONS LABELING OF SOME SHADOW GRAPHS OF PATHS, CYCLES AND STAR GRAPHS

Theorem 2.1: $D_2(P_n)$ admits a permutation labeling.

Proof: Consider two copies of P_n. Let v_1 , v_2 ,… , v_n be the vertices of the first copy of P_n and $v_{1'}$, $v_{2'}$,… , $v_{n'}$ be the vertices of the second copy of P_n. Let G be the shadow graph $D_2(P_n)$. Then |V(G)| = 2n and |E(G)| = 4 (n-1). To define f: V(G) → {1,2, … ,2n}

Case (i) n is even, n ≥ 2

$f(v_i) = i \quad (1\leq i \leq n)$

$f(u_i) = 4+ i \quad (1\leq i \leq n)$

Case (ii) n is odd , $n \geq 3$

$f(v_i) = i \quad (1 \leq i \leq n)$

$f(u_i) = 3 + i \quad (1 \leq i \leq n)$

Hence $D_2(P_n)$ is a permutation labeling

Illustration: 2.2: The Permutation labeling of the graph $D_2(P_3)$ is shown in Figure-1

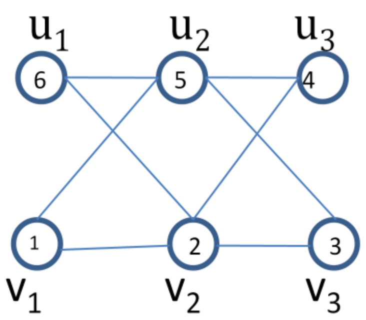


Fig-1: The Permutation labeling of the graph $D_2(P_3)$

Illustration:2.3: The Permutation labeling of the graph $D_2(P_7)$ is shown in Figure-2

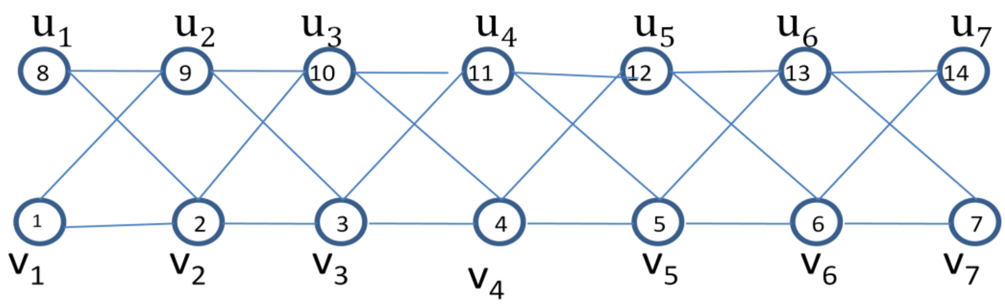


Fig-2: The Permutation labeling of the graph $D_2(P_7)$

Theorem 2.4: $D_2(K_{1,n})$ admits a permutation labeling.

Proof: Consider two copies of $K_{1,n}$. Let $v_1, v_2, \ldots, v_n$ be the pendent vertices of the first copy of $K_{1,n}$ and $v_{1'}, v_{2'}, \ldots, v_{n'}$ be the pendent vertices of the second copy of $K_{1,n}$ with v and v'respective apex vertices. Let $G = D_2(K_{1,n})$. Then $|V(G)| = 2n+2$ and $|E(G)| = 4n$. To define $f: V(G) \rightarrow \{1,2, \ldots, 2n+2\}$ as follows.

$f(v_i) = 2 + i \quad (1 \leq i \leq n)$

$f(v) = 1$
$f(v') = 2$ $f(v_i') = 6 + i \quad (1 \leq i \leq n)$

Hence $D_2(K_{1,n})$ is a permutation graph.

Illustration:2.5: Permutation labeling of the graph $D_2(K_{1,4})$ is shown in Figure-3.

$f(v) = 1$

$f(v') = 2$

$f(v_i') = 6 + i \quad (1 \leq i \leq n)$

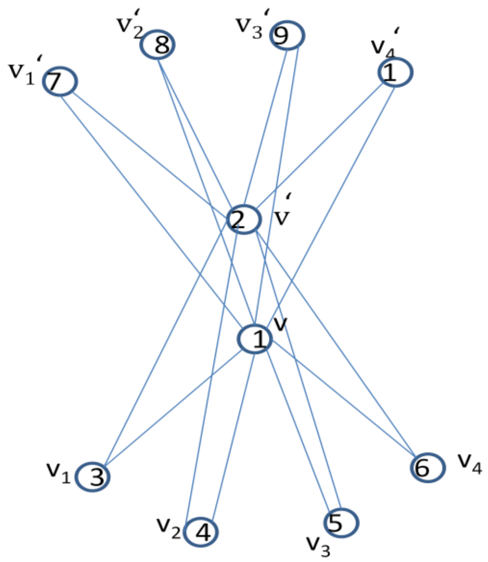


Fig-3: Permutation labeling of the graph $D_2(K_{1,4})$

Theorem 2.6: The path union of k copies of $D_2(c_n)$ is a permutation graph.

Proof: Let $D_2(C_n)$ be the shadow graph of cycle. Let G be the path union of k copies $G_1, G_2, \ldots, G_k$ of shadow graph $D_2(C_n)$. Let $G_1', G_2', \ldots G_k'$ be the first copy of cycle C_n and $G_1'', G_2'', \ldots G_k''$ be the second copy of cycle C_n in $D_2(C_n)$. Let us denote the successive vertices of $G_1'$by $u_{i1}', u_{i2}', \ldots u_{in}'$ and G_i'' by $u_{i1}'', u_{i2}'', \ldots u_{in}''$. Let $e_i = u_{i1}'' u_{(i+1)}''$ be the edge joining G_i and G_{i+1} for i = 1, 2, …., k- 1. Define f: V(G) → {1, 2, …., p } by

$f(u_{1j'}) = i, \quad 1 \le i \le n$

$f(u_{1j''}) = I + n, \quad 1 \le i \le n$

$f(u_{2j'}) = i + 2n, \quad 1 \le i \le n$

$f(u_{2j''}) = i + 3n, \quad 1 \le i \le n$

Illustration:2.7:

The permutation labeling of a graph **G** obtained by joining two copies of shadow graphs $D_2(C_4)$by a path of arbitrary length is shown in figure-4

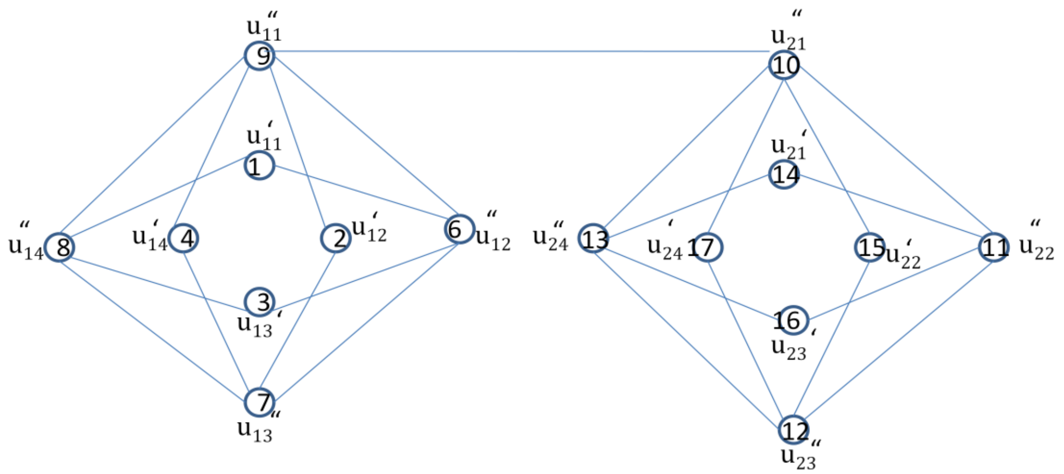


Fig-4:The permutation labeling of a graph G obtained by joining two copies of shadow graphs $D_2(C_4)$by a path of arbitrary length

Theorem 2.8: A graph **G** obtained by joining two copies of shadow graphs $D_2(C_n)$ by a path of arbitrary length is a permutation graph.

Proof: Let graphs $D_2(C_n)$be the shadow graph of cycle C_n .Let **G** be a graph obtained by joining two copies of shadow graphs $D_2(C_n)$ by a path P_k of length k – 1. Let $u_1', u_2', \ldots u_k'$ be the vertices of C_n' and $u_1'', u_2'', \ldots u_k''$ be the vertices of C_n''

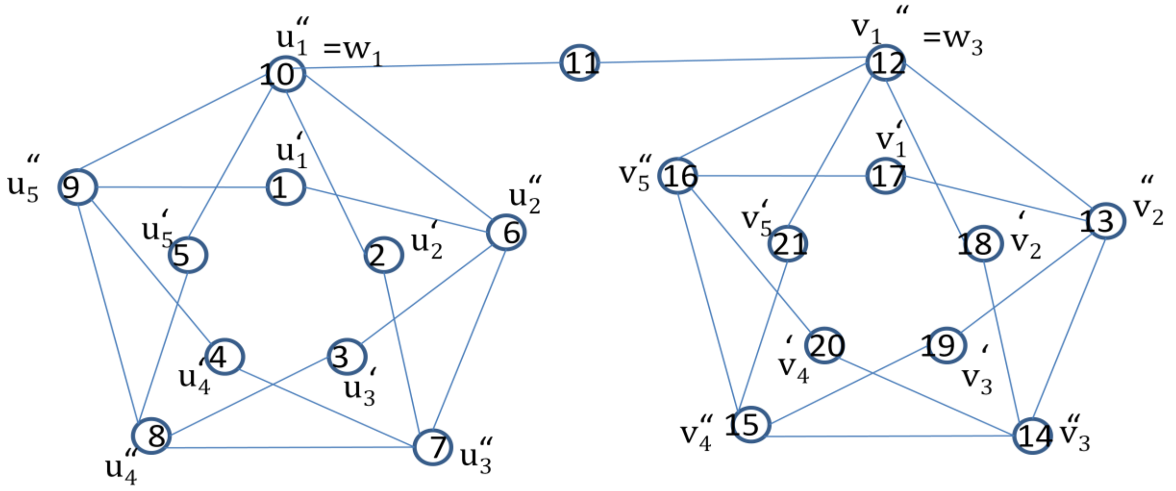


Fig-5: The permutation labeling of a graph G obtained by joining two copies of shadow graphs $D_2(C_5)$by a path of arbitrarylength

in first copy of $\mathbf{D_2}(C_n)$. Let $v_1', v_2', \ldots, v_k'$ be the vertices of C_n' and $v_1'', v_2'', \ldots, v_k''$ be the vertices of C_n'' in the second copy of $\mathbf{D_2}(C_n)$ in G. Let $w_1, w_2, \ldots w_k$ be the vertices of path P_k with $u_1'' = w_1$ and $v_1'' = w_k$. We note that $|V(G)| = 4n + k - 2$ and $|E(G)| = 8n + k - 3$.

To define f: $V(G) \rightarrow \{1, 2, \ldots, p\}$ by

$f(u_{i'}) = i, \quad 1 \leq i \leq n$

$f(u_{i''}) = i + n, \quad 1 \leq i \leq n$ and

$f(u_{i''}) = w_1$, when $i = 1$

$f(v_{i'}) = i + 3n + 1, \quad 1 \leq i \leq n$

$f(v_{i''}) = i + 2n + 1, \quad 1 \leq i \leq n$

$f(v_{i''}) = w_3$, when i = 1 and

Illustration:2.9:

The permutation labeling of a graph **G** obtained by joining two copies of shadow graphs $\mathbf{D_2}(C_5)$by a path of arbitrary length is shown in figure-5

3. PERMUTATIONS LABELING OF SOME SPLIT GRAPHS

Theorem 3.1: $Spl(P_n)$ is a permutation graph **Proof:** Let $u_1, u_2, \ldots, u_n$ be the vertices corresponding to $v_1, v_2, \ldots, v_n$ of P_n which are added to obtain $Spl(P_n)$. Let G be the graph $Spl(P_n)$. Then $|V(G)| = 2n$ and $|E(G)| = 3(n-1)$. To define f: $V(G) \rightarrow \{1, 2, \ldots, 2n\}$ as follows.

$f(v_i) = i \quad (1 \leq i \leq n)$

$f(u_i) = n + i \quad (1 \leq i \leq n)$

Hence $Spl(P_n)$ is a permutation graph

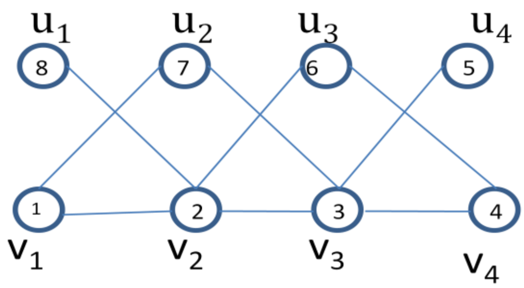


Fig-6: Permutation labeling of the graph $Spl(P_4)$

Illustration: 3.2: Permutation labeling of the graph $Spl(P_4)$ is shown in Figure-6.

Illustration: 3.3: Permutation labeling of the graph $Spl(P_7)$ is shown in Figure-7.

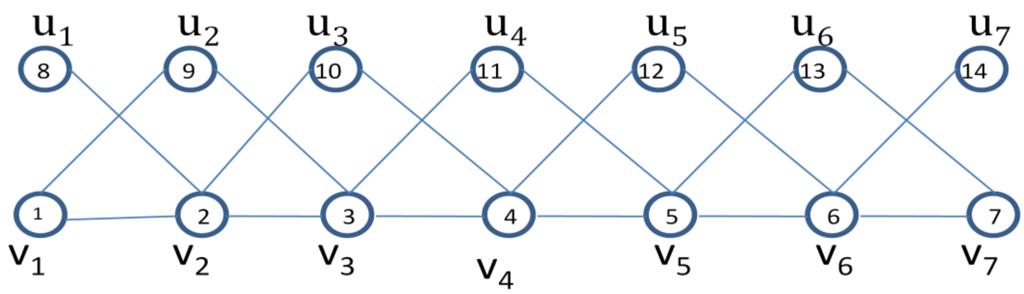


Fig-7: Permutation labeling of the graph Spl(P_7).

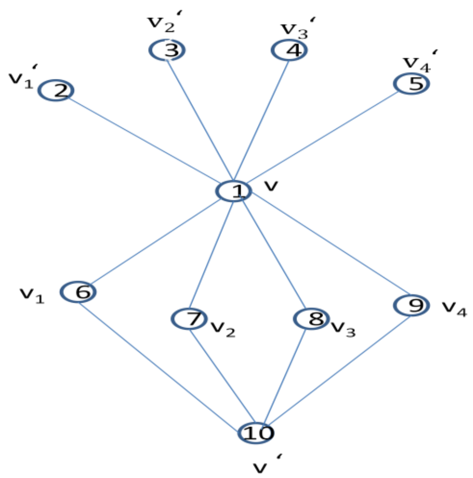
Theorem 3.4: Spl($K_{1,n}$) is a permutation graph.

Proof: Let u_1, u_2, … , u_n be the vertices corresponding to v_1, v_2, … , v_n of P_n which are added to obtain Spl(P_n). Let G be the graph Spl(P_n). Then |V(G)| = 2n+2 and |E(G)| = 3n. To define

f: V(G) → {1,2, … ,2n+2 } as follows.
$f(v_i) = i \quad (1 \le i \le n)$
$f(v_{i'}) = i \quad (1 \le i \le n)$ f(v)
= 1 f(v')
= 2n+2

and all the edges of Spl($K_{1,n}$) have different weights for the edges. Hence Spl($K_{1,n}$) is a permutation graph.

Illustration:3.5: Permutation labeling of the graph



Spl($K_{1,4}$)

Fig-8:Permutation labeling of the graph Spl($K_{1,4}$)

4. CONCLUSION

Finding shadow and split graphs seems to be a useful tool to generate new permutation graphs. Here I investigated shadow graphs of path P_n, star $K_{1,n}$ and path union of shadow graphs of cycle C_n are permutation graphs. Further I proved that the split graphs of path P_n and star $K_{1,n}$ are permutation graphs.One can investigate further more graphs which satisfies permutation labeling.

5. ACKNOWLEDGEMENTS

The author is very thankful to the referees for their valuable comments and kind suggestions.

6. REFERENCES

[1] B.D.Acharya and S.M .Hegde, Arithmetic graphs, J.Graph theory, 14(3) (1990),275 - 299

[2] L.Beineke and S.M.Hegde, Strongly multiplicative graphs , Discuss. Math. Graph theory, 21(2001), 63- 75.

[3] D.M.Burton, Elementary Number theory, Second edition, Wm.C.Brown Company Publishers, 1980.

[4] Frank Harrary, Graph theory, Narosa Publishing House-(2001).

[5] J A Gallian, A dynamic survey of graph labeling, The Electronics journal of Combinatories, 17(2010) # DS6

[6] Gary Chartrnd, Ping Zhang,Introduction to Graph theory, McGRAW- Hill International Edition

[7] S.C Shee Y.S Ho , " The cardinality of path union of n copies of a graph " , Discrete Math, 151 (1996), 221- 229

[8] Suresh Manjanath Hegde , Sudhakar Shetty, Combinatorial labeling of graphs ,Applied Mathematics E- notes,6(2006), 251-258© Revised 11th September

[9] J. Shiama" Square sum labeling for some middle and total graphs" International Journal of Computer Applications (0975-08887) Volume 37- No.4 January 2012

[10] J. Shiama "Permutation labeling for some split graphs" Proceedings of the National Conference on Mathematical Modelling and Simulation. NCMS'12 11th Feb.2012

[11] D B West, Introduction to Graph Theory, Prentice-Hall, India, 2001