

# Evaluating Robustness of Resource Allocation in Uniprocessor Real Time System

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## ABSTRACT

Real time systems (RTS) operate in an unpredictable changing environment that causing certain system performance features to degrade. Such systems need robustness to guarantee limited degradation despite variations in environmental parameters. EDF (Earliest Deadline First) scheduling has been used to evaluate the robustness of uniprocessor system with makespan as performance parameter. Robustness of real time system is directly proportional to the makespan of the resource allocation. EDF produces the optimal schedule that maximizes the robustness in the system.

## Keywords

Robustness, QoS, Resource Allocation, Real Time System.

## 1. INTRODUCTION

Real Time systems are widely used in our daily life and production industry such as the robotic control, telecommunications, chemical plant control, satellite control, flight control systems, military systems, multimedia systems, and so on. A real-time system is the one whose logical correctness is based on correctness of outputs as well as timeliness. It consists of a controlling system (computer) and a controlled system (environment). Real time systems guarantee that all the timing requirements can be met by the real time scheduling and schedulability analysis. Real time systems may operate in an environment where certain system performance features degrades due to unpredictable circumstances, such as sudden machine failures, higher than expected system load, or inaccuracies in the estimation of system performance parameters like makespan, slack etc [15, 16].

Resource allocation is very important to achieve a given level of QoS (Quality of Service). The resource allocation is defined as the act of assigning (matching) each task to a machine and ordering (scheduling) the execution of the tasks on each machine. Resource allocation is generally performed based on estimated computation time of each task on each class of machines. A resource allocation is said to be robust with respect to specified system performance features against perturbations in given system parameters if degradation in these features is within acceptable limits when certain perturbations occur [2]. For example, if a resource allocation has been declared to be robust with respect to satisfying a makespan requirement against perturbations in the estimated execution time, then the system configured under that allocation should continue to operate without a makespan violation when the actual execution time is greater than estimated execution time. Makespan (the completion time for an entire set of tasks) is mostly used as the performance feature that requires to be optimized in such systems.

Robustness has been defined in different ways by different

researchers. According to [5] robustness is the degree to which a system can function correctly in the presence of inputs different from those assumed. In a more general sense a robust system continues to operate correctly across a wide range of operating conditions. Robustness guarantees the maintenance of certain desired system characteristics despite variations in the behavior of its component parts or its environment [4]. A robust system is one that continues to perform at desired level of service in spite of perturbations in some components that constitute the system. In this paper resource allocation and scheduling are synonymous for uniprocessor system.

The rest of the paper is organized as follows. Section 2 describes system model and defines the resource allocation problem. Section 3 provides the work dealing with robustness and some robustness metrics. Section 4 presents some experiments and their results that highlight the usefulness of the robustness metric. Related work is given in section 5. Section 6 concludes the paper.

## 2. SYSTEM MODEL

It is based on periodic real-time tasks characterized by environment dependent execution time functions, as opposed to the traditional model with hard periodic real time tasks characterized by worst case execution times. In this paradigm, occasional deadline misses could be tolerated when unpredictable environmental factors drive a demand on resources beyond their limits. This section begins the system model with a traditional model; then necessary extensions are made to incorporate environmental factors.

The system model used in this work is derived from the standard real-time periodic task model by Liu (2000), where a software system consists of a set of  $n$  periodic tasks  $S = \{T_1, T_2, \dots, T_n\}$ . Each  $T_i$  is released periodically with a period of  $p_i$  and has a deadline equal to its period. The execution time of each task  $T_i \in S$  is a constant  $e_i$  that represents the worst case execution time of  $T_i$  executed on a set of  $m$  identical processors  $H = \{P_1, P_2, \dots, P_m\}$ .

### Workload Model

The traditional model does not capture dynamic and unpredictable environment since the execution times are modeled as constants. It is inadequate for some real-time systems operating in these environments. For instance, a general control system can have filter, analysis, action planning, and actuation tasks, and one or more of the tasks may contain algorithms and execution times that are affected by unpredictable environmental factors. Systems with these properties include air defense, surveillance, and intelligent vehicles. All periodic tasks arrive at the system having different period and are ready to execute any time within each period. We further assume that all periodic tasks have

deadlines greater than its period and execution time and their deadlines have to be met even in the presence of uncertainties in execution time.

We considered each task  $T_i \in S$  is represented by a tuple  $(p_i, t_i, d_i)$  where  $T_i$  is released periodically with a period of  $p_i$  and has a deadline  $d_i$  and has an estimated worst case execution time  $t_i$ . The perturbation environmental parameters that affect a system are modeled as  $pp = \{pp_1, pp_2, \dots, pp_n\}$ . Each task  $T_i$  has actual execution time  $e_i$  that is the function of environmental variable  $pp$  i.e.  $e_i = f(pp)$ . Generally, the basic parameter used for performance metric is utilization, given as  $U(pp) = \sum_{i=1}^n (e_i / p_i)$ .

**Robust Resource Allocation Problem:** A set of  $n$  periodic tasks to be allocated on uniprocessor, such that all tasks should meet their deadlines and maximizes the robustness.

### 3. ROBUSTNESS TECHNIQUES

#### 3.1 Deterministic Robustness Metric

This section contains a general procedure, called FePIA, for deriving a general robustness metric for any desired computing environment [1, 2]. The name for the above procedure stands for identifying the performance features, the perturbation parameters, the impact of perturbation parameters on performance features, and the analysis to determine the robustness. Specific examples illustrating the application of the FePIA procedure to sample systems are given in the next section. Each step of the FePIA procedure [1, 2] is now described.

1) *Describe quantitatively the requirement that makes the system robust.* Based on this robustness requirement, determine the QoS performance features like throughput, response time, makespan etc. that should be limited in variation to ensure that the robustness requirement is met. Consider an example *makespan* (the total time it takes to complete the execution of a set of applications) for a given resource allocation, the acceptable variation is up to  $\delta$  % of the makespan that was calculated using estimated execution times of applications on the machines they are assigned, and the uncertainties in system parameters are inaccuracies in the estimates of these execution times.  $\delta$  is user defined parameter for allowing variation, for example the acceptable variation is up to 35% of the makespan. Mathematically, let  $P$  be the set of system performance features that should be limited in variation. For each element  $P_i \in P$ , quantitatively describe the tolerable variation in  $P_i$ . Let  $\langle b_i^{\min}, b_i^{\max} \rangle$  be a tuple that gives the bounds of the tolerable variation in the system feature  $P_i$ . For the makespan example,  $P_i$  is the time the  $i^{\text{th}}$  machine finishes its assigned applications, and its corresponding  $\langle b_i^{\min}, b_i^{\max} \rangle$  could be  $\langle 0, 135 * (\text{estimated makespan value}) \rangle$ .

2) *Identify all of the system and environment parameters whose values may impact the QoS(Quality of Service) performance features selected in Step1.* These are called the perturbation parameters for example machine failure, higher than expected system load, execution time exceed than the calculated execution time, inaccuracies in the estimation of system parameters etc., and the performance features are required to be robust with respect to these perturbation parameters. For the makespan example above, the resource allocation where it is desired that the makespan be robust (stay within 135 percent of its estimated value) with respect to uncertainties in these estimated execution times. Mathematically, let  $Q$  be the set

of perturbation parameters. It is assumed that the elements of  $Q$  are vectors. Let  $Q_j$  be the  $j^{\text{th}}$  element of  $Q$ . For the makespan example,  $Q_j$  could be the vector composed of the actual application execution times, i.e., the  $i^{\text{th}}$  element of  $Q_j$  is the actual execution time of the  $i^{\text{th}}$  application on the machine it was assigned. In general, representation of the perturbation parameters as separate elements of  $Q$  would be based on their nature or kind (e.g., message length variables in  $Q_1$  and computation time variables in  $Q_2$ ).

3) *Identify the impact of the perturbation parameters in Step 2 on the system performance features in Step1.* For the makespan example, the sum of the actual execution times for all of the applications assigned a given machine is the time when that machine completes its applications. Note that Step1 implies that the actual time each machine finish its applications must be within the acceptable variation. Mathematically, for every  $P_i \in P$ , determine the relationship  $P_i = f_{ij}(Q_j)$ , if any, that relates  $P_i$  to  $Q_j$ . In this expression,  $f_{ij}$  is a function that maps  $P_i$  to  $Q_j$ . For the makespan example,  $P_i$  is the finishing time for machine  $m_i$ , and  $f_{ij}$  would be the sum of execution times for applications assigned to machine  $M_i$ . The rest of this discussion will be developed assuming only one element in  $Q$ . The case where multiple perturbation parameters can affect a given  $P_i$  simultaneously will be examined in Section 3.

4) *The last step is to determine the smallest collective variation in the values of perturbation parameters identified in Step 2 that will cause any of the performance features identified in Step 1 to violate its acceptable variation.* This will be the degree of robustness of the given resource allocation. For the makespan example, this will be some quantification of the total amount of inaccuracy in the execution times estimates allowable before the actual makespan exceeds 135 percent of its estimated value [1]. Let  $Q_j^{\text{orig}}$  be the value of  $Q_j$  at which the system is originally assumed to operate. However, due to inaccuracies in the estimated parameters or changes in the environment, the value of the variable  $Q_j$  might differ from its assumed value. Let the distance  $\|Q_j^*(P_i) - Q_j^{\text{orig}}\|_2$  be called the robustness radius,  $r_\omega(P_i, Q_j)$ , of  $P_i$  against  $Q_j$ .

$$r_\omega(P_i, Q_j) = \text{MIN} \|Q_j^*(P_i) - Q_j^{\text{orig}}\|_2$$

#### 3.2 Stochastic Robustness Metric

A stochastic robustness metric for a given distributed computing environment should reasonably predict the performance of the system. This is based on a mathematical model where the relationship between uncertainty in system parameters and its impact on system performance are described stochastically. This stochastic model is then used to derive a quantitative evaluation of the robustness of a given resource allocation as the probability that the resource allocation will satisfy the expressed QoS constraints.

Assume that  $M$  copies of an application arrive at computing node at wall clock time  $t$ , and  $n_j$  is the number of applications pending execution or being executed by computing node  $j$  at that time. Let  $t_0$  denote the wall-clock start time of execution for the query being processed by computing node  $j$  at time  $t$ . The functional dependence between the uncertainty parameters and the performance characteristic at time  $t$ , denoted as  $\zeta(t)$ , is

$$\zeta(t) = \max_{j=1,2,\dots,M} \{T_{1,j} - (t - t_0) + \sum_{i=1}^{n_j} T_{i,j}\}$$

Due to its functional dependence on the uncertainty parameters  $T_{ij}$ , the performance characteristic in above equation is itself a random variable. Let the QoS constraints be quantitatively described by the values  $\beta_{\min}$  and  $\beta_{\max}$  limiting the acceptable range of possible variation in system performance, i.e.,  
 $\beta_{\min} \leq \zeta \leq \beta_{\max}$ .

The stochastic robustness metric, denoted as  $\theta$ , is the probability that the performance characteristic of the system is confined to the interval  $[\beta_{\min}, \beta_{\max}]$  i.e.

$$\theta = P[\beta_{\min} \leq \zeta \leq \beta_{\max}]$$

The stochastic robustness quantitatively measures the probability that the generated system performance will satisfy the stipulated QoS constraints. Clearly, unity is the most desirable stochastic robustness metric value, i.e., there is zero probability that the system will violate the established QoS constraints [3]. There are different methods of computing the robustness metric like Fast Fourier Transform, Bootstrap method etc. by assuming functional independence among set of local uncertainty parameters  $[T_{ij} | 1 \leq i \leq n_{ij}]$  and local performance parameter  $\zeta$ .

Existing robust allocation approaches such as Gertphol et al. (2002), Ali et al. (2003), Juedes et al. (2004) and Gu et al. (2005) have the shortcoming that they employ coarse robustness metrics, which can result in poor allocations. The robustness metric of Ali et al. (2003) was based on the  $L_2$  norm. It measures the radius of maximum environment perturbation environment without violating feasible boundaries. However, the metric only partially characterizes feasible regions with an inner tangent sphere, and no algorithm was developed to optimize it. The max-of-min component of a workload vector among all feasible points was adopted as robustness metric in Gertphol et al. (2002), Juedes et al. (2004), Gu et al. (2005). But it only partially characterizes feasible regions with an inner tangent rectangle. In addition, many existing approaches do not emphasize real-time scheduling and feasibility. For instance, CPUs were assumed to be fair-shared in Gertphol et al. (2002). A special scheduler based on tightness was assumed by Shestak et al. (2005), but no feasibility guarantee was made. Gu et al. (2007) introduced a new robustness metric is accurately characterized the complete space of a feasible region. The robustness of allocation  $M$  is defined to be the volume of the entire feasible space of environmental variable  $w$ . Since each environmental variable is discrete, the volume can be expressed mathematically as

$$R(M) = \sum_{w_1=0}^{\infty} \sum_{w_2=0}^{\infty} \dots \sum_{w_l=0}^{\infty} \Theta(f_m(w_1, w_2, \dots, w_l))$$

### 3.3 Other robustness Metrics

Distributed real time system must operate in an environment replete with uncertainty while proving a required level of quality of service (QoS). The System undergoes unpredictable changes causing certain system performance features to degrade. Such system needs robustness to guarantee limited degradation despite fluctuation in the behavior of its components or environment. The robustness gives an idea of the stability of the solution with regards to another performance metric such as schedule length, load balance of an application, queue waiting time of batch scheduler etc. Some required definitions used in robustness metrics:

i. *Makespan (M)*: The total execution time of application.

- ii. *Slack of task (S)*: A time window within which the task can be delayed without affecting the makespan
- iii. *Entropy of schedule*: It is based on the probability of an execution path that will become critical.
- iv. *Expected makespan (E(M))*: The average value of the makespan.

Based on these definitions we define the following robustness metrics [15].

**Makespan standard deviation.** The standard deviation of the makespan distribution tells how narrow this distribution is. The narrower the distribution, the smaller the standard deviation is. This metric related to robustness because when there are two schedules the one for which the standard deviation is the smaller is the one for which realizations are more likely to have makespan close to the average value. Mathematically

$$\sigma_M = \sqrt{E(M^2) - E(M)^2}$$

**Makespan Differential entropy.** The differential entropy of distribution of a distribution measures the uncertainty of that distribution. If there is less uncertainty there is more chance than two realizations give a close result hence that schedule is robust.

$$h(M) = \int_{-\infty}^{+\infty} f(x) \log f(x) dx$$

**Average slack.** The slack gives the sum of space time in the schedule. It is related to robustness of makespan as a schedule with large slack is able to absorb a lot of uncertainty. For deterministic schedule the slack is defined as

$$s = \sum_{i \in V} M - Bl(i) - Tl(i)$$

Where  $Bl(i)$  is the bottom level of task  $i$  (the length of the longest path from  $i$  to an exit node including  $i$ ) and  $Tl(i)$  is the top level of node  $i$  (the length of the longest path from entry node to node  $i$  excluding  $i$ ).

**Slack standard deviation.** Each task has its own slack. Some tasks have a very large slack and other a slack of zero. As shown in [24] only task with non-zero slack can absorb uncertainty without delaying the makespan. Hence the standard deviation of all slack needs to be as small as possible. For computing the standard deviation of all slack, we use average slack  $S$  as shown above and slack of every node  $i \in V: s_i = M - Bl(i) - Tl(i)$ .

Then we have standard deviation of the slack:

$$\sigma_s = \sqrt{\sum_{i \in V} (s_i - S)^2}$$

**Average lateness.** A schedule is said late if its makespan exceeds the average makespan. The average lateness as defined in [24] is the average of the difference between the makespan of the late realization and the average makespan. If this metric is large this means that the makespan tends to be far from the average and then that the robustness is low. It is defined as:

$$L = E(M') - E(M)$$

where  $M'$  is the random variable describing the realization that have makespan larger than  $E(M)$ .

**Probabilistic metric.** This metric has been defined in [15] and gives the probability that the makespan is within two bounds. If this probability is high, this means that the

makespan of a given realization is likely to be close to the average makespan and hence that the robustness is high. An absolute probabilistic metric [15] that measures the probability of the makespan to be within  $[E(M)-\delta, E(M)+\delta]$  where  $\delta$  a positive constant given by the user. The absolute probabilistic metric is defined as:

$$A(\delta) = P(E(M) - \delta \leq m \leq E(M) + \delta)$$

The relative probabilistic metric [15] that measure the probability of the makespan to be within  $[E(M)/\gamma, E(M) * \gamma]$  where  $\gamma$  is a real number greater than 1. The relative probabilistic metric is defined as:

$$R(\gamma) = P\left(\frac{E(M)}{\gamma} \leq m \leq \gamma E(M)\right)$$

## 4. EXPERIMENTS

### 4.1 Robust EDF Algorithm

The scheduling problem is to find the schedule S that is feasible (meets all task deadlines) and maximizes robustness. Liu and Layland [21] presented a necessary and sufficient schedulability condition for EDF(Earliest Deadline First) scheduling under the assumption that all tasks' relative deadlines are equal to their periods, the schedulability condition is that the total utilization of the task set is less than or equal to 1. It has been proven to be optimal among all scheduling algorithms on a uniprocessor, in the sense that if a real-time task set cannot be scheduled by EDF, then this task set cannot be scheduled by any algorithm. We have considered, arrival time for all tasks is same. Each task is characterized by arrival time, Estimated Time to Compute (ETC), deadline and actual execution time. EDF generates the schedule for tasks according to their closest deadline first. EDF produces the optimal schedule that maximizes the robustness metric R(M). The basic idea of the algorithm (Alg. 1) generates a feasible schedule S which represents sequence of tasks. S is used to derive the robustness of the system.

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#### Algorithm 1 Robust Resource Allocation for Uniprocessor

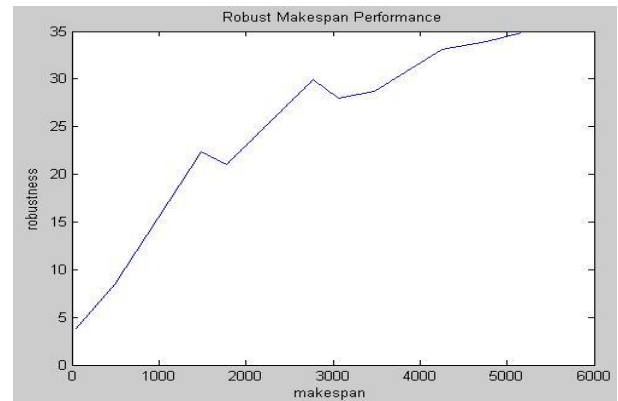
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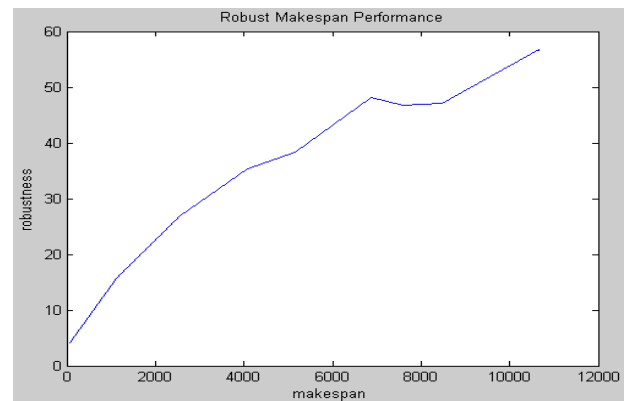
Input: MaxTask, P, T = [T1 T2 ... TMaxTask]
Output: Robustness
1: for i = 1 to MaxTask do
2: Compute Utilization  $U = \sum_{i=1}^{MaxTask} \frac{t_i}{p_i}$ 
3:     if  $U \geq 1$  then
4:         Tasks are not schedulable
5:     else
6:         Sort Tasks according to Earliest Deadline First
7:         Make Schedule S = [1:MaxTask]
8:          $M = \sum_{i=1}^{MaxTask} t_i$  // Compute Makespan M for S
9:         // Compute a tuple that gives the bounds of the tolerable
           variation in M
10:        vM = [0, 1.35 * M]
11:        Compute Finishing time F of Processor
12:        Compute Robustness
13:    end if
14: end for
    
```

ETC and deadline for each task is uniformly distributed. The task arrival pattern follows Poisson's distribution with ETC. The simulation study has been performed with the simulator designed using Matlab 7.0.1. Performance of EDF has been studied with averaging 10 instances of the allocation. The robustness value for each sample was given *point to plane*

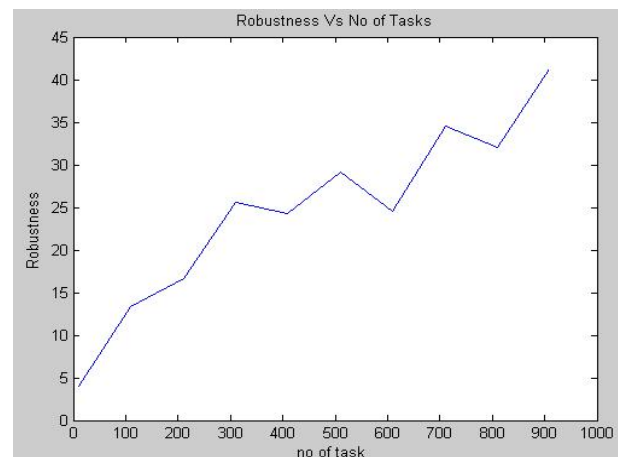
formula. Here robustness requirement is that the makespan M of the resource allocation should not increase more than 35% beyond its predicted value which is computed based on the expected execution times of tasks [1].



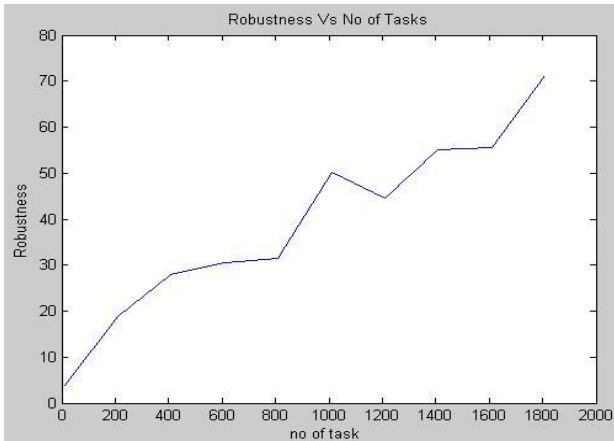
**Figure 4.1(a) Performance of EDF with makespan for 1000 tasks**



**Figure 4.1(b) Performance of EDF with makespan for 2000 tasks**

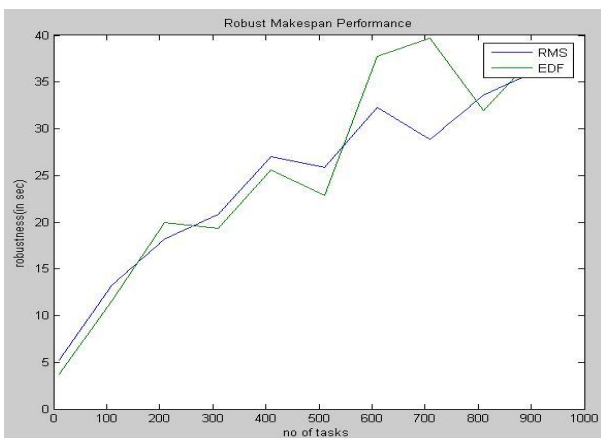


**Figure 4.2(a) Performance of EDF with 1000 tasks**

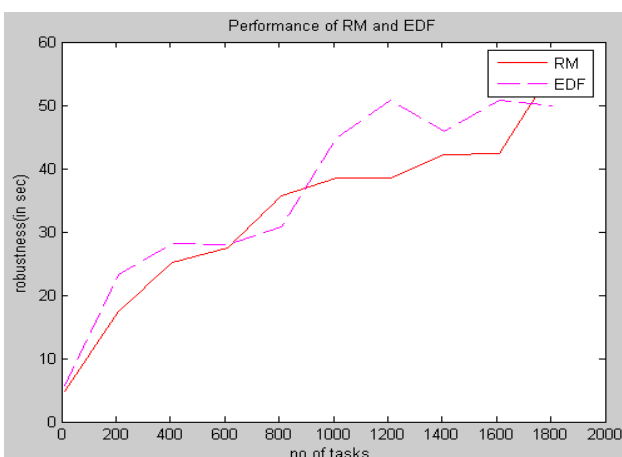


**Figure 4.2(b) Performance of EDF with 2000 tasks**

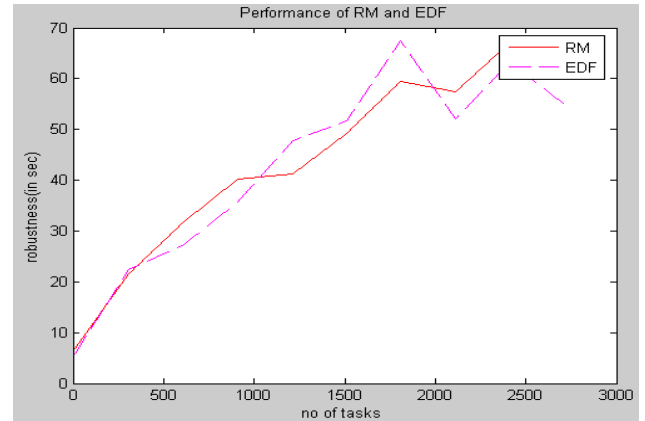
Figure 4.1(a, b) shows the performance of EDF with uniformly distributed service time of the different task set on single processor. The simulations indicates the dependency of robustness on makespan; as makespan increases the corresponding robustness also increases. The dependency of robustness on *no of the tasks* arrived with Poisson distribution is shown in Figure 4.2(a, b).



**Figure 4.3(a) Robustness for EDF and RM with 1000 tasks**



**Figure 4.3(b) Robustness for EDF and RM with 2000 tasks**



**Figure 4.3(c) Robustness for EDF and RM with 3000 tasks**

Robustness of EDF on different task sets is compared with the Rate Monotonic (RM) Scheduling on uniprocessor system. The simulation results indicates on better performance of EDF over RM when the no of task on the system greater than 600 or above. A dependency of makespan with no of task can also be observed.

## 5. RELATED WORK

A universal framework for defining robust resource allocations in heterogeneous computing systems was addressed in [2]. This work referred to a resource allocations tolerance to uncertainty as the robustness of that resource allocation. In [1, 2], a four-step procedure is established for deriving a deterministic robustness metric. The first step is defining robustness metric requires quantitatively describing what makes the system robust. This description establishes the required QoS level that must be delivered to refer to the system as robust essentially bounding the acceptable variation in system performance. In the second step, all modeled system and environmental parameters perturbation parameters that may impact the system's ability to deliver acceptable QoS are identified.

In stochastic approach [11], each perturbation parameter, or uncertainty parameter, is modeled as a random variable fully described by a probability mass function (pmf). Our second approach differs from that in [1], where a single deterministic estimated value for each of the identified perturbation parameters is used. In the third step, the impact of the identified perturbation parameters on the systems performance features is defined. This requires identifying a function that maps a given vector of perturbation parameters to a value for the performance feature of the system. Similarly in [12] stochastic environment, this involves defining the functional dependence between the input random variables and the given performance feature. However, the model this involves more complex computations to combine random variables. Finally, in the fourth step, the previously identified relation is evaluated to quantify the robustness.

As a measure of robustness, the authors in [1] use the minimum robustness radius that relies on a deterministic performance characteristic so used in a deterministic worst-case analysis.

In stochastic model [5, 6, 11, 12], more information regarding the variation in the perturbation parameters is assumed known. Representing the uncertainty parameters of the system as stochastic variables enables the robustness metric in the stochastic model to account for all possible outcomes for the

performance of the system. The stochastic robustness metric [9, 11, 12] requires more information and is far more complex to calculate than its deterministic counterpart. To handle the computational complexity, [3] considered the FFT and bootstrap approximation methods that greatly simplify the required calculations. The scheduling problem in real time systems has been explained in [8, 13, 14]. A heuristic dynamic scheduling scheme for parallel real-time jobs, modeled by directed acyclic graphs (DAG) where parallel real time jobs arrive at a heterogeneous system following a Poisson process, in a heterogeneous system is presented in [12, 15].

## 6. CONCLUSION

A robust schedule can absorb some degree of uncertainty in tasks duration while maintaining a stable solution. This paper discussed the different type of robustness metrics in real time system. The robustness of real time system becomes the vibrant research issue to provide the stability of system in presence of unpredictable operational environments. A number of experiments were conducted to examine the performance and scalability of the robust allocation algorithm by using EDF. From the experiment results we can conclude that robustness of real time system is directly proportional to the makespan. The Majority of real time system operates on multiprocessor system those used in applications like chemical plant control, satellite control, flight control systems, military systems etc. The performance of EDF can tested for the multiprocessor system on different real time system.

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