τ*-Generalized Homeomorphism in Topological Spaces

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Abstract
A.Pushpalatha et al [11] introduced the concept of τ*-g-closed set in topological spaces. S.Eswaran and A.Pushpalatha [6] introduced and studied the properties of τ*-generalized continuous maps and τ*-gc-irresolute maps in topological spaces. In this paper, we introduce and study a new class of maps called τ^*-generalized open maps and the notion of τ^*-generalized homeomorphism and τ^*-gc- homeomorphism in topological spaces.

General Terms
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Keywords
τ*-g-open map, τ*-g-homeomorphism,
τ*-gc-homeomorphisms

1. INTRODUCTION
The concept of the closed sets in topological spaces has been generalized to generalized closed sets by Levine [7]. Using the topology τ* introduced by Dunham [5], Pushpalatha et al [11] introduced τ* generalized closed sets and examined its properties. In [6], Eswaran and Pushpalatha defined the notion of τ^*-generalized continuous maps and a space called τ^*-T_g space. Using generalized closed sets, Balachandran et al [1] introduced and studied the notion of generalized continuous maps. Thivagar [14] defined and studied maps namely strongly α-open, strongly semiopen, strongly preopen, quasi α-open, quasi semiopen, quasi preopen. Mashhour et al [9], Biswas [2], Mashhour et al [10] and Cammaroto et al [3] defined and studied the maps preopen, semiopen, α-open and semipreopen respectively.

Generalized homeomorphisms via generalized closed sets and gc-homeomorphisms in terms of preserving generalized closed sets were first introduced by Maki, Sundaram and Balachandran [8]. Devi et al [4] introduced and studied sg-homeomorphism, gs- homeomorphism, sgc-homeomorphism gsc- homeomorphism

The purpose of this paper is to introduce and study the notion of τ*-g-open map, τ*-g-homeomorphism and τ*-gc-homeomorphisms in topological spaces.

Throughout this paper (X, τ*) and (Y, σ*) (or simply X and Y) represent non-empty topological spaces on which no separation axioms are assumed, unless otherwise mentioned. For a subset A of a space (X, τ*), cl (A), $cl^*(A)$ and A^c represent closure of A, closure* of A and complement of A respectively.

2. PRELIMINARIES
Since we use the following definitions and results, we recall them.

Definition 2.1. A subset A of a topological space (X, τ) is called generalized closed [7] (briefly g-closed) in X if cl(A) $\subseteq$ G whenever A $\subseteq$ G and G is open in X. A subset A is called generalized open (briefly g-open) in X if its complement A^c is g-closed.

Definition 2.2. For the subset A of a topological X,

(i) the generalized closure operator cl^* [5] is defined by the intersection of all g- closed sets containing A.

(ii) the topology τ^* [5] is defined by $\tau^* = \{G : cl^*(A^c) = A^c\}$

Definition 2.3. A subset A of a topological space X is called τ^*-generalized closed set [11] (briefly τ^*-g-closed) if $cl^*(A) \subseteq G$ whenever $A \subseteq G$ and G is τ^*-open. The complement of τ^*-generalized closed set is called the τ^*-generalized open set (briefly τ^*-g-open).

Definition 2.4.. A topological space (X, τ*) is called τ^*-T_g space [6] if every τ*-g-closed set in X is g-closed in X.

Definition 2.5. A subset A of (X, τ) is called semiopen (preopen, α-open and semipreopen) if A ⊂ cl int(A) [2] (resp. A ⊂ int cl(A) [9], A ⊂ int cl int(A) [10] and A ⊂ cl int cl(A) [3]) .

Definition 2.6. A map $f : X \to Y$ is called

i) generalized continuous [1] (g-continuous) if the inverse image of every closed set in Y is g-closed in X

ii) semigeneralized continuous [13] (sg-continuous) if $f^{-1}(V)$ is sg-closed set in X for each closed subset V of Y

iii) sg-irresolute [13] if $f^{-1}(V)$ is sg-closed set in X for each sg-closed set in Y.

Definition 2.7. A map $f : X \to Y$ is called τ*-generalized continuous [6] (briefly τ*-g-continuous) if the inverse image of every g-closed (or g-open) set in Y is τ*-g-closed (or τ*-g-open) in X.

Definition 2.8. A map $f : X \to Y$ is called a

i) generalized open map [8](g-open) if f(U) is g-open in Y for every open set U in X.

ii) strongly α-open [10] if the image of each α-open set in X is a α-open set in Y

iii) strongly semiopen [15] if the image of each semiopen set in X is semiopen in Y

iv) strongly preopen [15] if the image of each preopen set in X is a preopen set in Y

v) quasi α-open [15] if the image of each α-open set in X is open set in Y.

vi) quasi semiopen [15] if the image of each semiopen set in X is open set in Y.

vii) quasi preopen [15] if the image of each preopen set in X is open set in Y.

viii) preopen [9] if f(U) is preopen in Y for each open set U in X.

ix) semiopen [2] if f(U) is semiopen in Y for each open set U in X.

x) α-open [10] if f(U) is α-open in Y for each open set U in X.

xi) semipreopen [3] if f(U) is semipreopen in Y for each open set U in X

xii) presemiopen [15] if f(U) is semiopen in Y for each semiopen set U in X

Definition 2.9. A bijection map $f: X \to Y$ is called a

i) generalized homeomorphism [8](g-homeomorphism) if f is both g-continuous and g-open.

ii) semi-generalized homeomorphism [4] (sg-homeomorphism) if f is both sg-continuous and sg-open map

iii) generalized semi homeomorphism [4] (gs-homeomorphism) if f is both gs-continuous and gs-open

iv) sgc-homeomorphism [4] if both f and f^{-1} are sg-irresolute map

v) gsc- homeomorphism [4] if both f and f^{-1} are gs-irresolute

Remark 2.10. In [12], it has been proved in Theorem 3.2 that every closed set is τ*-g-closed

Remark 2.11. In [12], it has been proved in Theorem 3.4 that every g-closed set is τ*-g-closed

3. τ*-GENERALIZED OPEN MAP IN TOPOLOGICAL SPACES

In this chapter, we introduce the notion of τ*-generalized open mp and study some of their properties. We also investigate its relationship with some existing mappings.

Definition 3.1. A map $f: X \to Y$ is said to be τ*-generalized open map (briefly τ*-g-open map) if for each g-open set U in X, f (U) is a τ*-g-open set in Y

Theorem 3.2. Every open map is τ*-g-open map but not conversely.

Proof: Let $f: X \to Y$ be an open map. Let U be any open set in X. Since f is an open map, f(U) is open in Y. Since every open set is g-open, U is g-open in X. By Remark 2.10, f(U) is τ*-g-open in Y. Therefore f is a τ*-g-open map.

The converse of the theorem need not be true as seen from the following example.

Example 3.3. Let X = Y = {a, b, c}, τ = {X, ϕ, {b}, {c}, {b, c}}and σ = {Y, ϕ, {a}}. Let $f: X \to Y$ be an identity map. Then f is τ*-g-open map. But it is not an open map. Since for the open set V = {b}in X, f(V) = {b} is not open in Y.

Theorem 3.4.. Every g-open map is τ*-g-open map but not conversely.

Proof: Let $f: X \to Y$ be a g-open map. Let U be any open set in X. Since open set implies g-open set, U is g-open in X. Also, since f is a g-open map, f(U) is g-open set in Y. By remark 2.11, f(U) is τ*-g-open set in Y. Therefore f is a τ*-g-open map.

The converse of the theorem need not be true as seen from the following example.

Example 3.5. Let X = Y = {a, b, c}, τ = {X, ϕ, {b}, {a, b}, {b, c}}and σ = {Y, ϕ, {c}}. Let $f: X \to Y$ be an identity map. Then f is τ*-g-open map. But it is not a g-open map. Since for the open set V = {a, b}in X, f(V) = {a, b} is not g-open in Y.

Theorem 3.6. For any bijection $f: X \to Y$,the following statements are equivalent:

(a) The inverse map $f^{-1}: Y \to X$ is τ*-g-continuous.

(b) f is a τ*-g-open map

(c) f is a τ*-g-closed map.

Proof: (a) $\Longrightarrow$ (b). Let G be any g-open set in X. Since f^{-1} is τ*-g-continuous, the inverse image of G under f^{-1} is τ*-g-open in Y. That is $\left(f^{-1}\right)^{-1}(G)$ = f(G) is τ*-g-open in Y and so f is a τ*-g-open map. Hence (a) $\Longrightarrow$ (b).

(b) $\Longrightarrow$ (c) Let F be any g-closed set in X. Then F^c is g-open in X. Since f is a τ*-g-open map, f(F^c) is τ*-g-open in Y. But f(F^c) = Y – f(F). Therefore Y – f(F) is τ*-g-open in Y and so f(F) is τ*-g-closed in Y. Hence, f is a τ*-g-closed map. Thus, (b) $\Longrightarrow$ (c).

(c) $\Longrightarrow$ (a) Let F be any g-closed set in X. Since f is a τ*-g-closed map, f(F) is τ*-g-closed in Y. But f(F) = $\left(f^{-1}\right)^{-1}(F)$.Therefore the inverse map f^{-1} is τ*-g-continuous. Thus (c) $\Longrightarrow$ (a). Hence (a), (b) and (c) are equivalent.

Remark 3.7. The following example shows that τ*-g-open map is independent from the following maps.

Example 3.8. Let X = Y = {a, b, c} and let $f: X \to Y$ be an identity map.

(i) Let τ={X, ϕ, {a, b}} and σ={Y,ϕ,{b},{c},{b, c}, {a, b}}. Then f is strongly α-open. But it is not τ*-g-open, since for the g-open set V = {a}in X, f(V) = {a} is not τ*-g-open in Y.

(ii) Let τ = {X, ϕ, {a}} and σ = {Y, ϕ, {b}, {a, c}. Then f is a τ*-g-open map. But it is not strongly α-open, since for the α-open set V={a} in X, f(V) ={a} is not α-open in Y.

(iii) Let τ = {X, ϕ, {a}} and σ = {Y, ϕ, {a}, {a, b}, {a, c}. Then f is strongly semi-open. But it is not τ*-g-open, since for the g-open set V = {b}in X, f(V) = {b}is not τ*-g-open in Y.

(iv) Let τ = {X, ϕ, {a}, {b}, {a, b}, {a, c}} and σ = {Y, ϕ, {b}, {a, c}. Then f is a τ*-g-open map. But it is not strongly semi-open, since for the semi-open set V = {a, b} in X, f(V) = {a, b} is not semi-open in Y.

(v) Let τ = {X, ϕ, {a}, {a, b}} and σ ={Y, ϕ, {a}, {c}, {a, c}, {a, b}}. Then f is strongly pre-open. But it is not τ*-g-open, since for the g-open set V = {b}in X, f(V) = {b} is not τ*-g-open in Y.

(vi) Let τ = {X, ϕ, {a}} and σ = {Y, ϕ, {b}, {a, c}. Then f is a τ*-g-open map. But it is not strongly pre-open, since for the pre-open set V = {a} in X, f(V)= {a} is not pre-open in Y.

(vii) Let τ = {X, ϕ, {a}, {a, b}} and σ ={Y, ϕ, {a}, {b},{a, b}, {a, c}}. Then f is quasi α-open. But it is not τ*-g-open, since for the g-open set V = {c}in X, f(V) = {c} is not τ*-g-open in Y.

(viii) Let τ = {X, ϕ, {b}} and σ = {Y, ϕ, {c}, {a, b}}. Then f is a τ*-g-open map. But it is not quasi α-open, since for the α-open set V = {b} in X, f(V)= {b} is not open in Y.

(ix) Let τ = {X, ϕ, {b}, {a, c}} and σ ={Y, ϕ, {a}, {b},{a, b}, {a, c}}. Then f is quasi semi-open. But it is not τ*-g-open, since for the g-open set V = {c} in X, f(V)= {c} is not τ*-g-open in Y.

(x) Let τ = {X, ϕ, {c}, {a, c}, {b, c}} and σ = {Y, ϕ, {c}}. Then f is a τ*-g-open map. But it is not quasi semi-open, since for the semi-open set V = {b, c} in X, f(V)= {b, c} is not open in Y.

(xi) Let τ = {X, ϕ, {c}} and σ = {Y, ϕ, {b}, {a, b}}. Then f is quasi pre-open. But it is not τ*-g-open, since for the g-open set V = {a, c} in X, f(V) = {a, c} is not τ*-g-open in Y.

(xii) Let τ = {X, ϕ, {c}, {a, c}} and σ = {Y, ϕ, {c}}. Then f is a τ*-g-open map. But it is not quasi pre-open, since for the pre-open set V = {a, c} in X, f(V)= {a, c} is not open in Y.

(xiii) Let τ = {X, ϕ, {a}, {a, b}} and σ = {Y, ϕ, {a}, {a, b}, {a, c}}. Then f is preopen. But it is not τ*-g-open, since for the g-open set V = {b} in X, f(V) = {b} is not τ*-g-open in Y.

(xiv) Let τ = {X, ϕ, {a}, {b}, {a, b}, {a, c}} and σ = {Y, ϕ, {c}}. Then f is a τ*-g-open map. But it is not preopen, since for the open set V ={a, b} in X, f(V) = {a, b} is not preopen in Y.

(xv) Let τ ={X, ϕ,{b}} and σ={Y, ϕ,{b},{c},{b, c},{a, b}}. Then f is semiopen. But it is not τ*-g-open, since for the g-open set V= {a} in X, f(V) = {a} is not τ*-g-open in Y.

(xvi) Let τ = {X, ϕ, {a}, {c}, {a, c}} and σ = {Y, ϕ, {a, c}}. Then f is a τ*-g-open map. But it is not semiopen, since for the open set V = {a} in X, f(V) = {a} is not semiopen in Y.

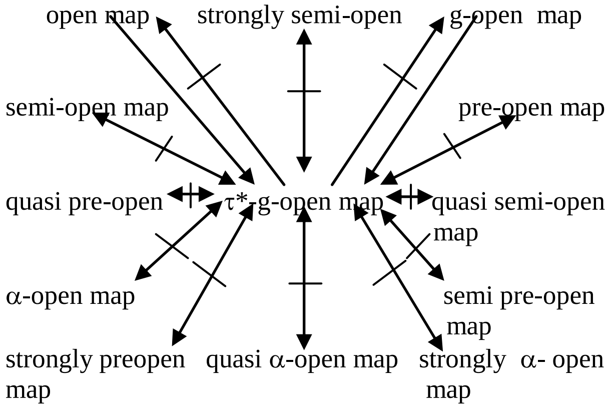
(xvii) Let τ = {X, ϕ, {a, c}} and σ = {Y, ϕ, {a},{a, b}, {a, c}}. Then f is α-open. But it is not τ*-g-open, since for the g-open set V = {c}in X, f(V) ={c} is not τ*-g-open in Y.

(xviii) Let τ = {X, ϕ, {a},{a, b}, {a, c}} and σ = {Y, ϕ, {a}, {b, c}}. Then f is a τ*-g-open map. But it is not α-open, since for the open set V = {a, b} in X, f(V) = {a, b} is not α-open in Y.

(xix) Let τ = {X, ϕ, {a}, {b, c}} and σ = {Y, ϕ, {a}, {b}, {a, b}, {b, c}}. Then f is semi pre-open. But it is not τ*-g-open, since for the g-open set V = {c} in X, f(V) = {c} is not τ*-g-open in Y.

(xx) Let τ = {X, ϕ, {b}, {a, b}, {b, c}} and σ = {Y, ϕ, {a}}. Then f is a τ*-g-open map. But it is not semi pre-open, since for the open set V = {b} in X, f(V) = {b} is not sp-open in Y.

Remark 3.9 From the above discussion, we have the following implications.



A ⟶ B means A implies B, A ↛ B means A does not imply B and A ↮ B means A and B are independent

4. τ*-GENERALIZED HOMEOMORPHISM AND τ*-GC-HOMEOMORPHISM IN TOPOLOGICAL SPACES

In this chapter, we introduce the notion of τ*-generalized homeomorphisms and τ*-gc-homeomorphisms which are generalizations of homeomorphisms and study some of their properties. We also investigate their relationship with some existing homeomorphisms.

Definition 4.1. A bijection $f:(X,\tau^*)\rightarrow(Y,\sigma^*)$ is called τ*-generalized homeomorphism (briefly τ*-g-homeomorphism) if f is both τ*-g-continuous map and τ*-g-open map.

Theorem 4.2. Every homeomorphism is τ*-g-homeomorphism but not conversely.

Proof: Let $f:X\rightarrow Y$ be homeomorphism. Then f is both continuous map and open map. In [6], it has been proved in Theorem 3.4 that every continuous map is τ*-g-continuous. Also from Theorem 3.2, f is a τ*-g-open map. That is f is both τ*-g-continuous map and τ*-g-open map. Hence f is τ*-g-homeomorphism.

The converse of the theorem need not be true as seen from the following example.

Example 4.3. Let X = Y = {a, b, c}, τ = {X, ϕ, {a}, {a, b}}and σ = {Y, ϕ, {a}, {b}, {a, b}}. Let $f:X\rightarrow Y$ be an identity map. Then f is τ*-g-homeomorphism. But it is not homeomorphism. Since for the closed set V = {a, c} in Y, $f^{-1}(V)$ = {a, c} is not closed in X. So, f is not continuous. Therefore f is not homeomorphism.

Theorem 4.4. Every g-homeomorphism is τ*-g-homeomorphism but not conversely.

Proof: Let $f:X\rightarrow Y$ be g-homeomorphism. Then f is both g-continuous map and g-open map. In [6], it has been proved in Theorem 3.6 that every g-continuous map is τ*-g-continuous. So, f is τ*-g-continuous. Also, by Theorem 3.4, f is a τ*-g-open map. Therefore f is both τ*-g-continuous and τ*-g-open. Hence f is τ*-g-homeomorphism.

The converse of the theorem need not be true as seen from the following example.

Example 4.5. Let X = Y = {a, b, c}, τ = {X, ϕ, {b}, {a, c}}and σ = {Y, ϕ, {b}}. Let $f : X \to Y$ be an identity map. Then clearly f is τ*-g-homeomorphism. But it is not g-homeomorphism. Since for the open set V = {a, c} in X, $f^{-1}(V)$ = {a, c} is not g-open in Y.

Theorem 4.6. If $f : X \to Y$ is a bijective and τ*-g-continuous map, then the following statements are equivalent:

(a) f is a τ*-g-open map.
(b) f is a τ*-g- homeomorphism
(c) f is a τ*-g-closed map.

Proof: (a) $\Rightarrow$ (b).By assumption, f is bijective, τ*-g-continuous and τ*-g-open. Then by definition, f is τ*-g- homeomorphism. Hence (a) $\Rightarrow$ (b).

(b) $\Rightarrow$ (c). Since f is τ*-g-homeomorphism, it is bijective, τ*-g-open and τ*-g-continuous. Then by Theorem 3.6, f is a τ*-g-closed map. Hence (b) $\Rightarrow$ (c).

(c) $\Rightarrow$ (a). By assumption, f is τ*-g-closed and bijective. Therefore by Theorem 3.6, f is τ*-g-open map. Hence (c) $\Rightarrow$ (a). Thus (a), (b) and (c) are equivalent.

Theorem 4.7. Let X and Z be any two topological spaces and let Y be a τ*-T_g-space. If $f : X \to Y$ and $g : Y \to Z$ be τ*-g-homeomorphisms, then the composition $g \circ f : X \to Z$ is also τ*-g-homeomorphism

Proof: Let U be a g-closed set in Z. Since $g : Y \to Z$ is τ*-g-continuous, g^{-1}(U) is τ*-g-closed in Y. Since Y is a τ*-T_g-space, g^{-1}(U) is g-closed in Y. Also, $f : X \to Y$ is τ*-g-continuous. Therefore $f^{-1}[g^{-1}(U)]$ is a τ*-g-closed in X. But $f^{-1}[g^{-1}(U)] = (g \circ f)^{-1}(U)$. Hence $g \circ f$ is τ*-g-continuous.

Again, let U be a g-open set in X. Since $f : X \to Y$ is a τ*-g-open map, f(U) is τ*-g-open in Y. And since Y is a τ*-T_g-space, f(U) is g-open in Y. Also $g : Y \to Z$ is a τ*-g-open map. Therefore g[f(U)] is a τ*-g-open set in Z. But g[f(U)] = $(g \circ f)(U)$. Hence $(g \circ f)$ is a τ*-g-open map. Thus, $g \circ f : X \to Z$ is both τ*-g-continuous and τ*-g-open map. Hence it is τ*-g-homeomorphisms.

Definition 4.8. A bijection $f : X \to Y$ is said to be τ*-gc-homeomorphisms if f is τ*-gc-irresolute and its inverse f^{-1} is also τ*-gc-irresolute. The space (X, τ*) and (Y, σ*) are said to be τ*-gc-homeomorphic if there exists a τ*-gc-homeomorphism from (X, τ*) to (Y, σ*).

Notations: The family of all τ*-gc-homeomorphisms [respectively τ*-g-homeomorphisms, homeomorphisms, g-homeomorphism, gc-homeomorphism] from a topological space X onto itself is denoted by τ*-gch(X) [respectively τ*-g(X), h(X), gh(X), gch(X)].

Theorem 4.9. Every homeomorphism is τ*-gc-homeomorphism but not conversely.

Proof: Let $f : X \to Y$ be homeomorphism. Then f is both continuous map and open map. Let U be a closed set in Y. Since f is continuous, $f^{-1}(U)$ is closed in X. By Remark 2.10, every closed set is τ*-g-closed. Thus, U in Y is a τ*-g-closed set implies $f^{-1}(U)$ in X is a τ*-g-closed set. So, f is τ*-gc-irresolute.

Again, let V be a closed set in X. Then V^c is open in X. Since f is an open map, f(V^c) is open in Y. But f(V^c) = Y – f(V). So, Y – f(V) is open in Y implies f(V) is closed in Y. By Remark 2.10, both V and f(V) are τ*-g-closed sets and $(f^{-1})^{-1}(V) = f(V)$. Thus, $(f^{-1})^{-1}(V)$ is τ*-g-closed in Y for the τ*-g-closed set V in X. This shows that $f^{-1} : Y \to X$ is τ*-gc-irresolute. Therefore f is τ*-gc-homeomorphism.

The converse of the theorem need not be true as seen from the following example.

Example 4.10. Let X = Y = {a, b, c}, τ = {X, ϕ, {a}, {a, b}}and σ = {Y, ϕ, {a}, {b}, {a, b}}. Let $f : X \to Y$ be an identity map. Then f is τ*-g-homeomorphism. But it is not homeomorphism. Since for the closed set V = {a, c} in Y, $f^{-1}(V)$ = {a, c} is not closed in X. So, f is not continuous. Therefore f is not homeomorphism.

Theorem 4.11. Let X be a topological space. Then (i) the set τ*-gch(X) is a group under the composition of maps, (ii) gh(X) is a subgroup of the group τ*-gch(X) and (iii) τ*-gch(X) ⊂ τ*-gh(X).

Proof: (i) Let f, g ∈ τ*-gch(X). Then $g \circ f$ ∈ τ*-gch(X) and so τ*-gch(X) is closed under the composition of maps. Composition of maps is always associative. The identity map $i : X \to X$ is a τ*-gc-homeomorphisms and so i ∈ τ*-gch(X). Also, $f \circ i = i \circ f = f$ for every f ∈ τ*-gch(X). If f ∈ τ*-gch(X), then f^{-1} ∈ τ*-gch(X) and $f \circ f^{-1} = f^{-1} \circ f$ = i. Hence τ*-gch(X) is a group under the composition of maps.

(ii) Let $f : X \to X$ be a g-homeomorphism. By Theorem 3.5 of [12], both f and f^{-1} are τ*-gc-irresolute and so f is τ*-gc-homeomorphism. Therefore every g-homeomorphism is a τ*-gc-homeomorphism and so gh(X) is a subset of τ*-gch(X). Also gh(X) is a group under the composition of maps. Therefore gh(X) is a subgroup of the group τ*-gch(X).

(iii) In [12], it has been proved in Theorem 3.3 that every τ*-gc- irresolute map is τ*-g-continuous. Thus, τ*-gch(X) ⊂ τ*-gh(X).

Remark 4.12. Semi-homeomorphism is independent of τ*-g-homeomorphisms as seen from the following example.

Example 4.13. Let X = Y = {a, b, c} and let $f : X \to Y$ be an identity map. Let τ = {X, ϕ, {a},{a, b}} and σ = {Y, ϕ, {a}, {a, c}}. Then f is both irresolute and pre semi-open. So f is semi-homeomorphism. But it is not τ*-g- homeomorphism, since for the g-open set V ={b} in X, f(V) = {b} is not τ*-g-open in Y.
Let τ = {X, ϕ, {b}} and σ = {Y, ϕ, {a}, {b, c}}. Then f is τ*-g-homeomorphisms. But f is not pre semi-open, since for the semi-open set V ={a, c} in X, f(V) = {a, c} is not a semi-open set in Y.

Remark 4.14. sg-homeomorphism is independent of τ*-g-homeomorphisms as seen from the following example.

Example 4.15. Let X = Y = {a, b, c} and let $f : X \rightarrow Y$ be an identity map. Let τ = {X, ϕ, {a},{a, b}, {a, c}} and σ = {Y, ϕ, {a}}. Clearly f is sg-homeomorphism But it is not τ*-g-continuous, since for the g-closed set V ={a, b} in Y, $f^{-1}(V)$ = {a, b} is not τ*-g-closed in X. Therefore f is not τ*-g- homeomorphism.
Let τ = {X, ϕ, {a}} and σ = {Y, ϕ, {c}, {a, b}}. Then f is τ*-g-homeomorphisms. But f is not sg-continuous, since for the closed set V = {a, b} in Y, $f^{-1}(V)$ = {a, b} is not sg-closed in X.

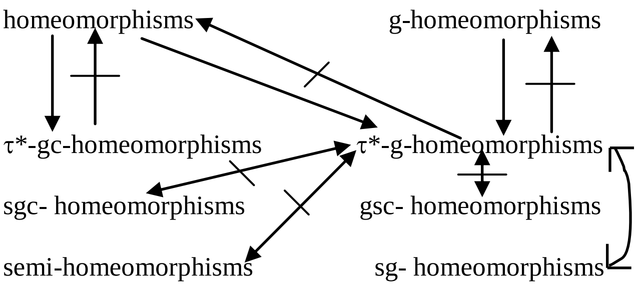
Remark 4.16. sgc-homeomorphism is independent of τ*-g-homeomorphisms as seen from the following example.

Example 4.17. Let X = Y = {a, b, c} and let $f : X \rightarrow Y$ be an identity map. Let τ = {X, ϕ, {a}} and σ ={Y, ϕ, {a}, {a, b}}. Clearly f is sgc-homeomorphism But f is not τ*-g-open, since for the g-open set V ={c} in X, f(V) = {c} is not τ*-g-open in Y. Therefore f is not τ*-g- homeomorphism.
Let τ = {X, ϕ, {c}} and σ ={Y, ϕ, {a}, {b, c}}. Then f is τ*-g-homeomorphisms. But f is not sg-irresolute, since for the sg-closed set V ={b, c} in Y, $f^{-1}(V)$ ={b, c} is not sg-closed in X.

Remark 4.18. gsc-homeomorphism is independent of τ*-g-homeomorphisms as seen from the following example.

Example 4.19. Let X = Y = {a, b, c} and let $f : X \rightarrow Y$ be an identity map. Let τ = {X, ϕ, {a}, {c}, {a, c}, {b, c}} and σ = {Y, ϕ, {c}, {a, c}, {b, c}}. Clearly f is gsc-homeomorphism But f is not a τ*-g-open map, since for the g-open set V ={a} in X, f(V) = {a} is not τ*-g-open in Y. Therefore f is not τ*-g- homeomorphism.
Let τ = {X, ϕ, {a}} and σ = {Y, ϕ, {c}, {a, b}}. Then f is τ*-g-homeomorphisms. But $f^{-1} : Y \rightarrow X$ is not gs-irresolute, since for the gs-closed set V = {a} in Y, $f^{-1}(V)$ = {a} is not gs-closed in X.

Remark 4.20 From the above discussion, we have the following implications.



A ⟶ B means A implies B, A ⟶/ B means A does not imply B and A ⟷/ B means A and B are independent

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