# On Generalized Semi Homeomorphism in Intuitionistic Fuzzy Topological Spaces

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## ABSTRACT

In this paper I have introduced intuitionistic fuzzy generalized semi closed mappings, intuitionistic fuzzy generalized semi homeomorphism and intuitionistic fuzzy M-generalized semi homeomorphism in intuitionistic fuzzy topological spaces. I have given suitable examples in intuitionistic fuzzy topological spaces for these concepts. Also I have provided some characterizations of intuitionistic fuzzy generalized semi homeomorphisms.

### **KEYWORDS**

Intuitionistic fuzzy topology, intuitionistic fuzzy generalized semi closed set, intuitionistic fuzzy generalized semi closed mapping, intuitionistic fuzzy generalized semi homeomorphism, and intuitionistic fuzzy M-generalized semi homeomorphism.

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### **1. INTRODUCTION**

The concept of intuitionistic fuzzy sets was introduced by Atanassov [1] as a generalization of fuzzy sets in 1983. Coker [3] introduced intuitionistic fuzzy topological spaces using the notion of intuitionistic fuzzy sets. In this paper I have introduced intuitionistic fuzzy generalized semi closed mappings, intuitionistic fuzzy generalized semi homeomorphism and intuitionistic fuzzy M-generalized semi homeomorphism. These concepts are analyzed with the existing intuitionistic fuzzy continuous mappings and intuitionistic fuzzy closed mappings. Also I have provided some characterizations of intuitionistic fuzzy generalized semi homeomorphism.

## 2. PRELIMINARIES

**Definition 2.1:** [1] An intuitionistic fuzzy set (IFS in short) A in X is an object having the form  $A=\{\langle x, \mu_A(x), \nu_A(x)\rangle/x \in X\}$  where the functions  $\mu_A(x): X \to [0, 1]$  and  $\nu_A(x): X \to [0, 1]$  denote the degree of membership (namely  $\mu_A(x)$ ) and the degree of non -membership (namely  $\nu_A(x)$ ) of each element  $x \in X$  to the set A, respectively, and  $0 \le \mu_A(x) + \nu_A(x) \le 1$  for each  $x \in X$ . Denote by IFS(X), the set of all intuitionistic fuzzy sets in X.

**Definition 2.2:**[3] An intuitionistic fuzzy topology (IFT in short) on X is a family  $\tau$  of IFSs in X satisfying the following axioms.

 $(i) \quad 0_{\scriptscriptstyle \sim}, \, 1_{\scriptscriptstyle \sim} \in \tau$ 

(ii)  $G_1 \cap G_2 \in \tau$  for any  $G_1, G_2 \in \tau$ (iii)  $\cup G_i \in \tau$  for any family  $\{G_i / i \in J\} \subseteq \tau$ .

Here the pair  $(X, \tau)$  is called an intuitionistic fuzzy topological space (IFTS in short)

**Definition 2.3:**[3] Let ( X,  $\tau$ ) be an IFTS and A =  $\langle x, \mu_A, \nu_A \rangle$  be an IFS in X. Then the intuitionistic fuzzy interior and intuitionistic fuzzy closure are defined by

 $int(A) = \bigcup \{ G / G \text{ is an IFOS in } X \text{ and } G \subseteq A \},\$ 

 $\begin{array}{l} cl(A) \ = \ \cap \ \{ \ K \ / \ K \ is \ an \ IFCS \ in \ X \ and \ A \subseteq K \ \}. \\ For \ any \ IFS \ A \ in \ (X, \ \tau), \ we \ have \ cl(A^c) = (int(A))^c \ and \ int(A^c) \\ = (cl(A))^c. \end{array}$ 

**Definition 2.4:** An IFS A =  $\langle x, \mu_A, \nu_A \rangle$  in an IFTS (X,  $\tau$ ) is said to be an

- (i) intuitionistic fuzzy semi closed set (IFSCS in short) if  $int(cl(A)) \subseteq A$  [5],
- (ii) intuitionistic fuzzy  $\alpha$ -closed set (IF $\alpha$ CS in short) if  $cl(int(cl(A)) \subseteq A$  [5],
- (iii) intuitionistic fuzzy  $\gamma$  closed set (IF $\gamma$ CS in short) if int(cl(A))  $\cap$  cl(int(A))  $\subseteq$  A [6].

**Definition 2.5:**[5] Let A be an IFS in an IFTS  $(X, \tau)$ . Then sint(A) =  $\cup \{ G / G \text{ is an IFSOS in } X \text{ and } G \subseteq A \}$ , scl(A) =  $\cap \{ K / K \text{ is an IFSCS in } X \text{ and } A \subseteq K \}$ .

**Definition 2.6:**[12] An IFS A in an IFTS (X,  $\tau$ ) is said to be an intuitionistic fuzzy generalized semi closed set (IFGSCS in short) if scl(A)  $\subseteq$  U whenever A  $\subseteq$  U and U is an IFOS in (X,  $\tau$ ). Every IFCS, IFSCS and IF $\alpha$ CS are IFGSCS.

**Definition 2.7:**[11] Let f be a mapping from an IFTS  $(X, \tau)$  into an IFTS  $(Y, \sigma)$ . Then f is said to be an

- (i) intuitionistic fuzzy closed mapping (IF closed mapping in short) if f(A) is an IFCS in Y for every IFCS A in X.
- (ii) intuitionistic fuzzy semi closed mapping (IFS closed mapping in short) if f(A) is an IFSCS in Y for every IFCS A in X.
- (iii) intuitionistic fuzzy  $\alpha$ -closed mapping (IF $\alpha$  closed mapping in short) if f(A) is an IF $\alpha$ CS in Y for every IFCS A in X.
- (iv) intuitionistic fuzzy  $\gamma$ -closed mapping (IF $\gamma$  closed mapping in short) if f(A) is an IF $\gamma$ CS in Y for every IFCS A in X.

**Definition 2.8:**[12] Let f be a mapping from an IFTS  $(X, \tau)$  into an IFTS  $(Y, \sigma)$ . Then f is said to be

(i) intuitionistic fuzzy continuous (IF continuous in short) if  $f^{-1}(B)$  is an IFOS in X for every IFOS B in Y.

- (ii) intuitionistic fuzzy semi continuous (IFS continuous in short) if  $f^{-1}(B)$  is an IFSOS in X for every IFOS B in Y.
- (iii) intuitionistic fuzzy generalized semi continuous (IFGS continuous in short) if  $f^{-1}(B)$  is an IFGSCS in X for every IFCS B in Y.

**Definition 2.9:**[9] Let f be a bijection mapping from an IFTS  $(X, \tau)$  into an IFTS  $(Y, \sigma)$ . Then f is said to be

- (i) intuitionistic fuzzy homeomorphism (IF homeomor -phism in short) if f and  $f^{-1}$  are IF continuous mappings.
- (ii) intuitionistic fuzzy semi homeomorphism (IFS homeomorphism in short) if f and f<sup>1</sup> are IFS continuous mappings

**Definition 2.10:**[12] An IFTS  $(X, \tau)$  is said to be an intuitionistic fuzzy  ${}_{c}T_{1/2}$  (in short  $IF_{c}T_{1/2}$ ) space if every IFGSCS in X is an IFCS in X.

## 3. INTUITIONISTIC FUZZY GENERALI -ZED SEMI CLOSED MAPPINGS

In this section I have introduced intuitionistic fuzzy generalized semi closed mappings, intuitionistic fuzzy generalized semi open mappings and studied some of its properties.

**Definition 3.1:** A mapping f:  $(X, \tau) \rightarrow (Y,\sigma)$  is called an intuitionistic fuzzy generalized semi closed (IFGS closed in short) mapping if for every IFCS A of  $(X, \tau)$ , f(A) is an IFGSCS in  $(Y, \sigma)$ .

**Example 3.2:** Let  $X = \{a, b\}, Y = \{u, v\}$  and  $T_1 = \langle x, (0.6, 0.6), (0.2, 0.3) \rangle$ ,  $T_2 = \langle y, (0.2, 0.2), (0.8, 0.7) \rangle$ . Then  $\tau = \{0_{-}, T_{1}, 1_{-}\}$  and  $\sigma = \{0_{-}, T_{2}, 1_{-}\}$  are IFTs on X and Y respectively. Define a mapping f:  $(X, \tau) \rightarrow (Y, \sigma)$  by f(a) = u and f(b) = v. Then f is an IFGS closed mapping.

**Example 3.4:** Let  $X = \{a, b\}$ ,  $Y = \{u, v\}$  and  $T_1 = \langle x, (0.2, 0.3), (0.6, 0.7) \rangle$ ,  $T_2 = \langle y, (0.5, 0.6), (0.5, 0.4) \rangle$ . Then  $\tau = \{0_{-}, T_{1,} 1_{-}\}$  and  $\sigma = \{0_{-}, T_{2,} 1_{-}\}$  are IFTs on X and Y respectively. Define a mapping f:  $(X, \tau) \rightarrow (Y, \sigma)$  by f(a) = u and f(b) = v. Then f is an IFGS open mapping.

**Theorem 3.5:** Every IF closed mapping is an IFGS closed mapping but not conversely.

**Proof:** Let f:  $(X, \tau) \rightarrow (Y, \sigma)$  be an IF closed mapping. Let A be an IFCS in X. Since f is an IF closed mapping, f(A) is an IFCS in Y. Since every IFCS is an IFGSCS, f(A) is an IFGSCS in Y. Hence f is an IFGS closed mapping.

**Example 3.6:** Let  $X = \{a, b\}$ ,  $Y = \{u, v\}$  and  $T_1 = \langle x, (0.4, 0.4), (0.3, 0.6) \rangle$ ,  $T_2 = \langle y, (0.5, 0.4), (0.5, 0.6) \rangle$ . Then  $\tau = \{0, ..., v\}$ 

 $\begin{array}{l} T_{1,} 1_{\sim} \} \text{ and } \sigma = \{ 0_{\sim}, T_{2,} 1_{\sim} \} \text{ are IFTs on X and Y respectively.} \\ \text{Define a mapping } f: (X, \tau) \rightarrow (Y, \sigma) \text{ by } f(a) = u \text{ and } f(b) = v. \\ \text{Then f is} & \text{an IFGS closed mapping but not an IF} \\ \text{closed mapping} & \text{since } A = \langle x, (0.3, 0.6), (0.4, 0.4) \rangle \text{ is an IFCS in X but} & f(A) = \langle y, (0.3, 0.6), (0.4, 0.4) \rangle \text{ is not an IFCS in Y.} \end{array}$ 

**Theorem 3.7:** Every IFS closed mapping is an IFGS closed mapping but not conversely.

**Proof:** Let f:  $(X, \tau) \rightarrow (Y, \sigma)$  be an IFS closed mapping. Let A be an IFCS in X. By hypothesis, f(A) is an IFSCS in Y. Since every IFSCS is an IFGSCS, f(A) is an IFGSCS in Y. Hence f is an IFGS closed mapping.

**Example 3.8:** Let  $X = \{a, b\}$ ,  $Y = \{u, v\}$  and  $T_1 = \langle x, (0.2, 0.2), (0.7, 0.8) \rangle$ ,  $T_2 = \langle y, (0.5, 0.4), (0.5, 0.6) \rangle$ . Then  $\tau = \{0_{-}, T_1, 1_{-}\}$  and  $\sigma = \{0_{-}, T_2, 1_{-}\}$  are IFTs on X and Y respectively. Define a mapping f:  $(X, \tau) \rightarrow (Y, \sigma)$  by f(a) = u and f(b) = v. Then f is an IFGS closed mapping but not an IFS closed mapping since  $A = \langle x, (0.7, 0.8), (0.2, 0.2) \rangle$  is an IFCS in X but  $f(A) = \langle y, (0.7, 0.8), (0.2, 0.2) \rangle$  is not an IFSCS in Y.

**Theorem 3.9:** Every  $IF\alpha$  closed mapping is an IFGS closed mapping but not conversely.

**Proof:** Let f:  $(X, \tau) \rightarrow (Y, \sigma)$  be an IF $\alpha$  closed mapping. Let A be an IFCS in X. By hypothesis, f(A) is an IF $\alpha$ CS in Y. Since every IF $\alpha$ CS is an IFGSCS, f(A) is an IFGSCS in Y. Hence f is an IFGS closed mapping.

**Example 3.10:** Let X = { a, b }, Y = { u, v } and T<sub>1</sub> =  $\langle x, (0.2, 0.2), (0.8, 0.8) \rangle, T_2 = \langle y, (0.5, 0.4), (0.5, 0.6) \rangle$ . Then  $\tau = \{0_{-}, T_1, 1_{-}\}$  and  $\sigma = \{0_{-}, T_2, 1_{-}\}$  are IFTs on X and Y respectively. Define a mapping f:  $(X, \tau) \rightarrow (Y, \sigma)$  by f(a) = u and f(b) = v. Then f is an IFGS closed mapping but not an IF $\alpha$  closed mapping since A =  $\langle x, (0.8, 0.8), (0.2, 0.2) \rangle$  is an IFCS in X but  $f(A) = \langle y, (0.8, 0.8), (0.2, 0.2) \rangle$  is not an IF $\alpha$ CS in Y.

**Remark 3.11:** An IF $\gamma$  closed mapping and an IFGS closed mapping are independent to each other.

**Example 3.10:** Let X = { a, b }, Y = { u, v } and T<sub>1</sub> =  $\langle x, (0.6, 0.2), (0.4, 0.3) \rangle, T_2 = \langle y, (0.4, 0.6), (0.2, 0.2) \rangle$ , Then  $\tau = \{0_{-}, T_1, 1_{-}\}$  and  $\sigma = \{0_{-}, T_2, 1_{-}\}$  are IFTs on X and Y respectively. Define a mapping f:  $(X, \tau) \rightarrow (Y, \sigma)$  by f(a) = u and f(b) = v. Then f is an IF $\gamma$  closed mapping but not an IFGS closed mapping since A =  $\langle x, (0.4, 0.3), (0.6, 0.2) \rangle$  is an IFCS in X but f(A) =  $\langle y, (0.4, 0.3), (0.6, 0.2) \rangle$  is not an IFGSCS in Y.

**Example 3.12:** Let X = { a, b }, Y = { u, v } and T<sub>1</sub> =  $\langle x, (0.2, 0.1), (0.7, 0.8) \rangle, T_2 = \langle y, (0.5, 0.1), (0.5, 0.9) \rangle$ . Then  $\tau = \{0_{-}, T_{1,} 1_{-}\}$  and  $\sigma = \{0_{-}, T_{2,} 1_{-}\}$  are IFTs on X and Y respectively. Define a mapping f:  $(X, \tau) \rightarrow (Y, \sigma)$  by f(a) = u and f(b) = v. Then f is an IFGS closed mapping but not an IF $\gamma$ 

closed mapping since A =  $\langle x, (0.7, 0.8), (0.2, 0.1) \rangle$  is an IFCS in X but  $f(A) = \langle y, (0.7, 0.8), (0.2, 0.1) \rangle$  is not an IF<sub>Y</sub>CS in Y.

**Theorem 3.13:** A mapping f:  $X \rightarrow Y$  is an IFGS closed mapping if and only if the image of each IFOS in X is an IFGSOS in Y.

**Proof:** Let A be an IFOS in X. This implies A<sup>c</sup> is IFCS in X. Since f is IFGS closed mapping, f(A<sup>c</sup>) is an IFGSCS in Y. Since  $f(A^c) = (f(A))^c$ , f(A) is an IFGSOS in Y.

The relations between various types of intuitionistic fuzzy closed mappings are given in the following diagram. In this diagram 'cl.map.' means closed mapping.



IFα cl.map.

The reverse implications are not true in general.

**Theorem 3.14:** Let f:  $(X, \tau) \rightarrow (Y, \sigma)$  be a mapping from an IFTS X into an IFTS Y. Then the following conditions are equivalent if Y is an  $IF_cT_{1/2}$  space.

- (i) f is an IFGS closed mapping
- (ii) If A is an IFOS in X then f(A) is an IFGSOS in Y
- (iii)  $f(int(A)) \subseteq cl(int(f(A)))$  for every IFS A in X.

**Proof:** (i)  $\Rightarrow$  (ii): is obviously true.

(ii)  $\Rightarrow$  (iii): Let A be any IFS in X. Then int(A) is an IFOS in X. Then f(int(A)) is an IFGSOS in Y. Since Y is an  $IF_cT_{1/2}$ space, f(int(A)) is an IFOS in Y. Therefore f(int(A)) = $int(f(int(A)) \subseteq cl(int(f(A))).$ 

(iii)  $\Rightarrow$  (i): Let A be an IFCS in X. Then its complement A<sup>c</sup> is an IFOS in X. By hypothesis  $f(int(A^c)) \subset cl(int(f(A^c)))$ . This implies  $f(A^c) \subset cl(int(f(A^c)))$ . Hence  $f(A^c)$  is an IFSOS in Y. Since every IFSOS is an IFGSOS, f(A<sup>c</sup>) is an IFGSOS in X. Therefore f(A) is an IFGSCS in X. Hence f is an IFGS closed mapping.

**Theorem 3.15:** Let  $f : (X, \tau) \to (Y, \sigma)$  be an IF closed mapping and  $g: (Y, \sigma) \rightarrow (Z, \delta)$  is IFGS closed mapping, then  $g \circ f: (X, \tau) \to (Z, \delta)$  is an IFGS closed mapping.

**Proof:** Let A be an IFCS in X. Then f(A) is an IFCS in Y, by hypothesis. Since g is an IFGS closed mapping, g(f(A)) is an IFGSCS in Z. Hence g o f is an IFGS closed mapping.

**Theorem 3.16:** Let f:  $(X, \tau) \rightarrow (Y, \sigma)$  be a mapping. Then the following conditions are equivalent if X is an  $IF_{c}T_{1/2}$  space.

- (i) f is an IFGS closed mapping
- (ii) f(A) is an IFGSCS in Y for every IFCS A in X
- (iii)  $int(cl(f(A))) \subseteq f(cl(A))$  for every IFS A in X.

**Proof:** (i)  $\Rightarrow$  (ii): is obviously true.

(ii)  $\Rightarrow$  (iii): Let A be an IFS in X. Then cl(A) is an IFCS in X. By hypothesis, f(cl(A)) is an IFGSCS in Y. Since Y is an IF<sub>c</sub>T<sub>1/2</sub> space, f(cl(A)) is an IFCS in Y. Therefore cl(f(cl(A)) =f(cl(A)). Now  $int(cl(f(A))) \subseteq cl(cl(f(cl(A)))) \subseteq f(cl(A))$ .

(iii)  $\Rightarrow$  (i): Let A be an IFCS in X. By hypothesis int(cl(f(A)))  $\subset$  f(cl(A)) = f(A). This implies f(A) is an IFSCS in Y and hence f(A) is an IFGSCS in Y. Therefore f is an IFGS closed mapping.

#### 4. INTUITIONISTIC FUZZY GENERALI -ZED HOMEOMORPHISMS

In this section I have introduced intuitionistic fuzzy generalized semi homeomorphism and studied some of its properties with suitable examples.

**Definition 4.1:** A bijection mapping f:  $(X, \tau) \rightarrow (Y, \sigma)$  is called intuitionistic fuzzy generalized an semi homeomorphism (IFGS homeomorphism in short) if f and f<sup>1</sup> are IFGS continuous mappings.

**Example 4.2:** Let  $X = \{a, b\}, Y = \{u, v\}$  and  $T_1 = \langle x, (0.3, v) \rangle$ 0.3), (0.6, 0.7)  $\rangle$ ,  $T_2 = \langle y, (0.4, 0.8), (0.4, 0.2) \rangle$ . Then  $\tau =$  $\{0_{\sim}, T_{1}, 1_{\sim}\}$  and  $\sigma = \{0_{\sim}, T_{2}, 1_{\sim}\}$  are IFTs on X and Y respectively. Define a bijection mapping f:  $(X, \tau) \rightarrow (Y, \sigma)$  by f(b) = v. Then f is an IFGS f(a) = u and continuous mapping and f<sup>1</sup> is also an IFGS continuous mapping. Therefore f is an IFGS homeomorphism.

Theorem 4.3: Every IF homeomorphism is an IFGS homeomorphism but not conversely.

**Proof:** Let f:  $(X, \tau) \rightarrow (Y, \sigma)$  be an IF homeomorphism. Then f and  $f^1$  are IF continuous mappings. This implies f and  $f^1$  are IFGS continuous mappings. That is the mapping f is IFGS homeomorphism.

**Example 4.4:** Let  $X = \{a, b\}, Y = \{u, v\}$  and  $T_1 = \langle x, (0.3, v) \rangle$ 0.3), (0.6, 0.7)  $\rangle$ , T<sub>2</sub> =  $\langle$  y, (0.5, 0.4), (0.4, 0.2)  $\rangle$ . Then  $\tau = \{0, ..., \tau \}$  $T_1, 1_{\sim}$  and  $\sigma = \{0_{\sim}, T_2, 1_{\sim}\}$  are IFTs on X and Y respectively. Define a bijection mapping f:  $(X, \tau) \rightarrow (Y, \sigma)$  by f(a) = u and f(b) = v. Then f is an IFGS homeomorphism but not an IF homeomorphism since f and f<sup>-1</sup> are not an IF continuous mappings.

**Definition 4.5:** A bijection mapping f:  $(X, \tau) \rightarrow (Y,\sigma)$  is called an intuitionistic fuzzy semi homeomorphism (IFS homeomorphism in short) if f and f<sup>1</sup> are IFS continuous mappings.

**Theorem 4.6:** Every IFS homeomorphism is an IFGS homeomorphism but not conversely.

**Proof:** Let f:  $(X, \tau) \rightarrow (Y, \sigma)$  be an IFS homeomorphism. Then f and f<sup>1</sup> are IFS continuous mappings. This implies f and f<sup>1</sup> are IFS continuous mappings. Hence f and f<sup>1</sup> are IFGS continuous mappings. That is the mapping f is an IFGS homeomorphism.

**Example 4.7:** Let X = { a, b }, Y = { u, v } and T<sub>1</sub> =  $\langle x, (0.5, 0.4), (0.5, 0.5) \rangle$ , T<sub>2</sub> =  $\langle y, (0.2, 0.2), (0.7, 0.8) \rangle$ . Then  $\tau = \{0_{-}, T_1, 1_{-}\}$  and  $\sigma = \{0_{-}, T_2, 1_{-}\}$  are IFTs on X and Y respectively. Define a mapping f:  $(X, \tau) \rightarrow (Y, \sigma)$  by f(a) = u and f(b) = v. Then f is an IFGS homeomorphism. Consider an IFCS A =  $\langle y, (0.7, 0.8), (0.2, 0.2) \rangle$  in Y. Then f<sup>1</sup>(A) =  $\langle x, (0.7, 0.8), (0.2, 0.2) \rangle$  is not an IFSCS in X. This implies f is not an IFS continuous mapping. Hence f is not an IFS homeomorphism.

**Theorem 4.8:** Let f:  $(X, \tau) \rightarrow (Y, \sigma)$  be an IFGS homeomorphism, then f is an IF homeomorphism if X and Y are  $IF_cT_{1/2}$  space.

**Proof:** Let B be an IFCS in Y. Then  $f^{-1}(B)$  is an IFGSCS in X, by hypothesis. Since X is an  $IF_cT_{1/2}$  space,  $f^{-1}(B)$  is an IFCS in X. Hence f is an IF continuous mapping. By hypothesis  $f^{-1}$ :  $(Y, \sigma) \rightarrow (X, \tau)$  is a IFGS continuous mapping. Let A be an IFCS in X. Then  $(f^{-1})^{-1}(A) = f(A)$  is an IFGSCS in Y, by hypothesis. Since Y is an  $IF_cT_{1/2}$  space, f(A) is an IFCS in Y. Hence  $f^{-1}$  is an IF continuous mapping. Hence the mapping f is an IF homeomorphism.

**Theorem 4.9:** Let  $f: (X, \tau) \rightarrow (Y, \sigma)$  be a bijective mapping. If f is an IFGS continuous mapping, then the following are equivalent.

- (i) f is an IFGS closed mapping
- (ii) f is an IFGS open mapping
- (iii) f is an IFGS homeomorphism.

**Proof:** (i)  $\rightarrow$ (ii): Let f:  $(X, \tau) \rightarrow (Y, \sigma)$  be a bijective mapping and let f is an IFGS closed mapping. This implies f<sup>-1</sup>:  $(Y, \sigma) \rightarrow (X, \tau)$  is IFGS continuous mapping. That is every IFOS in X is an IFGSOS in Y. Hence f is an IFGS open mapping.

**Proof:** (ii)  $\rightarrow$ (iii): Let f:  $(X, \tau) \rightarrow (Y, \sigma)$  be a bijective mapping and let f is an IFGS open mapping. This implies  $f^{-1}$ :  $(Y, \sigma) \rightarrow (X, \tau)$  is IFGS continuous mapping. But f is an IFGS continuous mapping. by hypothesis. Hence f and  $f^{-1}$  are IFGS continuous mappings. That is f is an IFGS homeomorphism.

(iii)  $\rightarrow$ (i): Let f is an IFGS homeomorphism. That is f and f<sup>1</sup> are IFGS continuous mappings. Since every IFCS in X is an IFGSCS in Y, f is an IFGS closed mapping.

**Remark 4.10:** The composition of two IFGS homeomorphisms need not be an IFGS homeomorphism in general.

**Example 4.11:** Let  $X = \{a, b\}$ ,  $Y = \{c, d\}$  and  $Z = \{u, v\}$ . Let  $T_1 = \langle x, (0.7, 0.6), (0.2, 0.4) \rangle$ ,  $T_2 = \langle y, (0.6, 0.1), (0.4, 0.3) \rangle$ 

and  $T_3 = \langle z, (0.4, 0.4), (0.6, 0.2) \rangle$ . Then  $\tau = \{0_{-}, T_{1,} 1_{-}\}, \sigma = \{0_{-}, T_2, 1_{-}\}$  and  $\Omega = \{0_{-}, T_3, 1_{-}\}$  are IFTs on X,Y and Z respectively. Define a bijection mapping f:  $(X, \tau) \rightarrow (Y, \sigma)$  by f(a) = c, f(b) = d and g:  $(Y, \sigma) \rightarrow (Z, \Omega)$  by f(c) = u, f(d) = v. Then f and f<sup>1</sup> are IFGS continuous mappings. Also g and g<sup>-1</sup> are IFGS continuous mappings. But the composition g  $\circ$  f:  $X \rightarrow Z$  is not an IFGS homeomorphism since g  $\circ$  f is not an IFGS continuous mapping.

**Definition 4.12:** A bijection mapping f:  $(X, \tau) \rightarrow (Y, \sigma)$  is called an intuitionistic fuzzy M-generalized semi homeomorphism (IFMGS homeomorphism in short) if f and f<sup>-1</sup> are IFGS irresolute mappings.

**Theorem 4.13:** Every IFMGS homeomorphism is an IFGS homeomorphism but not conversely.

**Proof:** Let f:  $(X, \tau) \rightarrow (Y, \sigma)$  be an IFMGS homeomorphism. Let B be IFCS in Y. This implies B is an IFGSCS in Y. By hypothesis f<sup>1</sup>(B) is an IFGSCS in X. Hence f is an IFGS continuous mapping. Similarly we can prove f<sup>1</sup> is an IFGS continuous mapping. Hence f and f<sup>1</sup> are IFGS continuous mappings. This implies the mapping f is an IFGS homeomorphism.

**Example 4.14:** Let  $X = \{a, b\}$ ,  $Y = \{u, v\}$  and  $T_1 = \langle x, (0.3, 0.4), (0.6, 0.1) \rangle$ ,  $T_2 = \langle y, (0.2, 0.1), (0.5, 0.5) \rangle$ . Then  $\tau = \{0_{-}, T_1, 1_{-}\}$  and  $\sigma = \{0_{-}, T_2, 1_{-}\}$  are IFTs on X and Y respectively. Define a bijection mapping f:  $(X, \tau) \rightarrow (Y, \sigma)$  by f(a) = u and f(b) = v. Then f is an IFGS homeomorphism. Let us consider an IFS G =  $\langle y, (0.3, 0.2), (0.7, 0.7) \rangle$  in Y. Clearly G is an IFGSCS in Y. But  $f^{-1}(G) = \langle x, (0.3, 0.2), (0.7, 0.7) \rangle$  is not an IFGSCS in X. That is f is not an IFGS irresolute mapping. Hence f is not an IFM $\alpha$ G homeomorphism.

**Theorem 4.15**: If  $f: X \to Y$  is an IFMGS homeomorphism, then  $scl(f^{-1}(B)) = f^{-1}(scl(B))$  for every IFS B in Y.

**Proof:** Since f is an IFMGS homeomorphism, f is an IFGS irresolute mapping. Consider an IFS B in Y. Clearly scl(B) is an IFSCS in Y. This implies scl(B) is an IFGSCS in Y. By hypothesis f<sup>1</sup>(scl(B)) is an IFGSCS in X. Since f<sup>1</sup>(B) ⊆ f<sup>1</sup>(scl(B)), scl(f<sup>1</sup>(B)) ⊆ scl(f<sup>1</sup>(scl(B))) = f<sup>1</sup>(scl(B))). This implies scl(f<sup>1</sup>(B)) ⊆ f<sup>1</sup>(scl(B)). Since f is an IFMGS homeomorphism, f<sup>1</sup>: Y → X is an IFGS irresolute mapping. Consider an IFS f<sup>1</sup>(B) in X. Clearly scl(f<sup>1</sup>(B)) is an IFGSCS in X. This implies (f<sup>1</sup>)<sup>-1</sup>(scl(f<sup>1</sup>(B))) = f(scl(f<sup>1</sup>(B))) is an IFGSCS in X. This implies (f<sup>1</sup>)<sup>-1</sup>(scl(f<sup>1</sup>(B))) = f(scl(f<sup>1</sup>(B))) is an IFGSCS in Y. Clearly B = (f<sup>1</sup>)<sup>-1</sup>(f<sup>1</sup>(B)) ⊆ (f<sup>1</sup>)<sup>-1</sup>(scl(f<sup>1</sup>(B))) = f(scl(f<sup>1</sup>(B))), Therefore scl(B) ⊆ scl(f(scl(f<sup>1</sup>(B)))) = f(scl(f<sup>1</sup>(B))), since f<sup>1</sup> is an IFGS irresolute mapping. Hence f<sup>1</sup>(scl(B)) ⊆ f<sup>1</sup>(f(scl(f<sup>1</sup>(B))) = scl(f<sup>1</sup>(B)). That is f<sup>1</sup>(scl(B)) ⊆ scl(f<sup>1</sup>(B)). This implies scl(f<sup>1</sup>(B)).

## **5. CONCLUSION**

In this paper I have introduced intuitionistic fuzzy generalized semi closed mappings, Intuitionistic fuzzy generalized semi homeomorphisms and studied some of its properties. Also I have provided some characterizations of intuitionistic fuzzy generalized semi homeomorphisms.

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