A New Approach of Computation of Outage Probability for Rician/ Rayleigh Fading Environment

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ABSTRACT

In this paper proposes a novel mathematical method to express the outage probability for a desired radio signal received from a mobile transmitter in the presence of multipath interfering signals with rician and Rayleigh fading. New Rician/Rayleigh outage probabilities curves against the normalized reuse distance without lognormal shadowing effect are presented and discussed. The effect of SIR, Protection ratio R_I , Rice factor K of signals, number of cochannel interferers L on the outage probability and reuse distance D have been investigated.

Key words – Mobile radio system, Rayleigh fading, Outage probabilities, Rician fading.

1. INTRODUCTION

Channel models in which the signals are affected by fading only for microcellular mobile radio systems have been studied in [1]–[3]. In [1], the desired signal is modeled to be Rician-faded, while the co channel interferers are modeled to be Rayleigh-faded. This model is based on the consideration that a direct line-of-sight (LOS) signal component exits in within-cell transmission, while the interferers are assumed to be Rayleigh-faded because a direct LOS path between co channel cells is unlikely to exist. Closed-form outage probability expressions are derived for both single and multiple Rayleigh interferers. In [2], a closed-form outage probability expression for the case of a Rayleigh desired signal with a single Rician interferer was derived. The Rician interferer model addresses the case of spectrum sharing with competing microcellular communication services as well as with other microwave users in the same band [2].

In spite of the above fading-only assumptions, many propagation measurements have characterized the microcellular environments as composing a path loss, slow lognormal shadowing, and fast Rician fading components [5]. Outage probability for channels including all these effects has been presented in [4] and [5]. In [4], an exact outage probability expression for lognormal-shadowed Rician/Rician channels has been derived with the provision that at least one of the signals is Rayleigh distributed.In [5], the desired signal is modeled as Rician faded and lognormally shadowed, while the interferers are Rayleigh-faded and lognormally shadowed.

In this paper, we consider a microcellular environment, where both the desired signal and interferers are subjected to path loss, slow lognormal shadowing, and fast Rician fading. First, we derive a new closed-form outage probability expression for the case of Rician fading-only channels and show that the outage probability expressions presented in [1] and [2] are just special cases of our result. We also present new outage probability curves for different values of the Rice factor K and different values of the protection ratio R_I.We also study the effects of the number of active interferers L, various values of Rice factor K, protection ratio, and lognormal shadowing σ_{nk} on the reuse distance.

2. OUTAGE PROBABILITY FOR RICIAN FADING ENVIRONMENT

In this section, we start from a new approach and obtain a closed-form expression for this case for the Rayleigh and Rician channel.

2.1 Derivation of Outage Probability for Rician/Rician Fading Channels [1]

Let X_k (k = 0,……, L) be a set of independent complex Gaussian random variables, with mean value m_k and variance σ_0^2 . The derivation below is reffered from [11]. Let X_0represent the desired Rician signal and $\sum_{k=1}^{L} X_k$represent the sum of L Rician interferers. The outage probability for L interferers is expressed as

$$P\ (\text{outage}|L) = \Pr\ (Y \leq 0) \qquad (1)$$

Where

$$Y = Y_0 - \sum_{k=1}^{L} Y_k \qquad (2)$$

And $Y_0=|X_0|^2$and $Y_k = R_I|X_k|^2$;k =1,………,L.Here, R_I is the signal to interference protection ratio. In order to derive P(outage|L) given in (1), it is necessary to find the probability density function (pdf) of Y . For this, it is convenient to find the characteristic function of Y. Turin [6] has evaluated the characteristic function of a set of N complex Gaussian random variables. In our case, we need to adopt Turin's result, but only for. For each of the Y_k(k=0… L), we have

$$F_k(s) = \frac{exp\left(\frac{sC_k}{1-s\beta_k}\right)}{1-s\beta_k} \qquad (3)$$

Where $C_0 = |m_0|^2$, $\beta_0=\sigma_0^2$, $C_k = R_I|m_k|^2$, $\beta_k = R_I\sigma_0^2$, k= 1,……,L. It is to be noted that C_0 and β_0 are,respectively, the average power of the specular and diffused components of the desired Rician signal, whereas C_k and β_k are, respectively, the average power multiplied by the protection ratio of the specular and diffused components of the interferer. Now, the characteristic function F(s) of Y, assuming that all the signals X_k are independent, is given by

$$F(s) = F_0(s)\ \prod_{k=1}^{L} F_k(-s) \qquad (4)$$

If we assume further that all the interferers have the same mean $m_k = m_I$ and the same variance $\sigma_k^2 = \sigma_I^2$, which also implies that the Rice factor of each of the interfering signals is the same, $K_I = |m_I|^2 / \sigma_I^2$, then

$$F(s) = F_0(s)\,[F_k\,[\,(-s)^L] \qquad (5)$$

The pdf of the sum of the interferers $Y_I = \sum_{k=1}^{L} Y_k$, where Y_k is defined in (2), can be obtained as the inverse Fourier transform of $[F_k\,[\,(s)^L]$. The inverse Fourier transform of was obtained by Bello [7], who applied the result of Campbell and Foster [8]. The result is shown below:

$$p y_I(Y_I) = \frac{1}{2\pi j}\int [F_k\,[\,(s)^L]\, e^{-sY_I}\, ds$$

$$= \begin{cases} \dfrac{1}{\beta_k}\left(\dfrac{Y_I}{LC_k}\right)^{L-1/2} \exp\left(-\dfrac{Y_I + LC_k}{\beta_k}\right) \\ \mathrm{x} I_{L-1}\left(\dfrac{2}{\beta_k}\sqrt{Y_I LC_k}\right), & Y_I > 0 \\ 0 & Y_I < 0 \end{cases} \qquad (6)$$

The pdf of the desired signal is a special case of (6), with L = 1 and K = 0:

$$p y_0(Y_0) = \begin{cases} \dfrac{1}{\beta_0}\exp\left(-\dfrac{Y_0 + C_0}{\beta_0}\right) I_0\left(\dfrac{2}{\beta_0}\sqrt{Y_0 C_0}\right), & Y_0 > 0 \\ 0, & Y_0 < 0 \end{cases} \qquad (7)$$

Where $I_m(x)$ is the modified Bessel function of the first kind and order m. The pdf of Y and p(Y), defined in (2), can be found by convolving (6) and (7). The outage probability is then

$$\text{P (outage|L)} = \int_{-\infty}^{0} p(Y)\,dY$$

$$= \int_{-\infty}^{0} dY \int_{0}^{\infty} p y_I(Z)\, p y_0(Z+Y)\,dZ, \qquad (8)$$

Now, making the change of variables, Y = γ – Z, in (8) and interchanging the order of integration, we have

$$\text{P (outage|L)} = \int_0^\infty \frac{1}{\beta_k}\left(\frac{Z}{LC_k}\right)^{\frac{(L-1)}{2}}$$

$$\begin{cases} x\ \exp\left(-\dfrac{Z + LC_k}{\beta_k}\right) I_{L-1}\left(\dfrac{2}{\beta_k}\sqrt{Y_I LC_k}\right) \\ \mathrm{x}\displaystyle\int_0^Z \frac{1}{\beta_0}\exp\left(-\frac{\gamma + C_0}{\beta_k}\right) \\ \mathrm{x}\, I_0\left(\dfrac{2}{\beta_0}\sqrt{\gamma C_0}\right) \mathrm{d}\,\gamma \mathrm{d}Z \end{cases} \qquad (9)$$

Again, making the change of variables

$$Z = \frac{\beta_k x^2}{2};\ \ \gamma = \frac{\beta_0 y^2}{2} \qquad (10)$$

In (9) and defining the parameters

$$LC_k = \frac{\beta_k a^2}{2}\,;\ C_0 = \frac{\beta_0 b^2}{2}\,;\ \mathrm{r} = \sqrt{\frac{\beta_k}{\beta_0}} \qquad (11)$$

Where $a^2/2 = K_I L$ and $b^2/2 = K_0$ = Rice factor of the desired signal, we arrive at

$$\text{P (outage|L)} = a^{-L+1}\int_0^\infty x^L \exp\left(-\frac{x^2+a^2}{2}\right) I_{L-1}(ax)$$

$$\mathrm{x}\ \ \exp\left(-\frac{y^2+b^2}{2}\right) I_0(by)\,\mathrm{dydx}. \qquad (12)$$

This double integral is a special case of the integral evaluated by Price [9, eq. (2.5)]. Using [9, eq. (3.23)] and [9, eq. (3.24)], a new closed-form expression of the outage probability for the Rician/Rician fading channel is found to be

$$\text{P (outage|L)}$$

$$= \mathrm{Q}\left[\sqrt{\frac{2LK_I R_I}{b_1 + R_I}}\ ;\ \sqrt{\frac{2K_0 b_1}{b_1 + R_I}}\right]$$

$$+ \exp\left(-\frac{LK_I R_I + K_0 b_1}{b_1 + R_I}\right)$$

$$\mathrm{x}\ \sum_{m=0}^{L-1}\left(\frac{K_0 R_I}{LK_I R_I}\right)^{m/2} I_m\left(\sqrt{\frac{4LK_I K_0 b_1 R_I}{b_1 + R_I}}\right)$$

$$\mathrm{x}\ \left\{ \left(1 + \frac{b_1}{R_I}\right)^{-L} \sum_{k=m}^{L-1}\binom{L}{k-m}\left(\frac{b_1}{R_I}\right)^k - \delta_{m0} \right\}$$

$$m \geq 0 \qquad (13)$$

where $b_1 = \sigma_0^2/\sigma_I^2$ and δ_{m0} are Kronecker delta: $\delta_{m0} = 1$ for m=0, $\delta_{m0} = 0$ for m ≠ 0, and

$$\mathrm{Q}\,(\mathrm{u,v}) = \int_v^\infty x\ exp\left[-\frac{1}{2}(x^2 + u^2)\right] I_0(xu)\,dx \qquad (14)$$

is the Marcum's Q function.

3. COMPUTATIONAL RESULTS AND DISCUSSIONS

Finally by putting L = 1and K_0 = 0 in (13), we obtain Rayleigh desired signal and single Rician interferer, as follows:

$$\text{P(outage|L)} = 1 - \frac{b_1}{R_I + b_1}\exp\left[-\left(\frac{K_I R_I}{R_I + b_1}\right)\right] \qquad (15)$$

And by putting K_I = 0 and L =1, we obtain Rician desired signal and single Rician interferer, as follows:

$$\text{P (outage|L)} = \frac{R_I}{R_I + b_1}\exp\left[-\left(\frac{K_0 b_1}{R_I + b_1}\right)\right] \qquad (16)$$

In our derivation of (13), we have assumed that all the interferers have the same mean $m_k = m_I$ and k = 1,...,L. If all the interferers have distinct means m_k, but with identical variance $\sigma_k^2 = \sigma_I^2$, the Rice factors of the L interferers are distinct with $K_k = m_k|^2/\sigma_I^2$and k = 1,…,L.

Figs. 1 and 2 present outage probability curves as a function of the signal-to-interference power ratio (SIR), defined as

$$\mathrm{SIR} = \frac{(K_0+1)\sigma_0^2}{L(K_I+1)\sigma_L^2} = \frac{(K_0+1)}{L(K_I+1)} b_1 \qquad (17)$$

It is to be noted that when comparing the curves of single and multiple interferers, for a given SIR, and given K_0, σ_0^2, and K_I and the value of $L\sigma_I^2$ is then fixed. Therefore, for the larger L value, the corresponding σ_I^2 is smaller. This means that the power of the single interferer is distributed among the

L interferers. In fig 1, Outage probability curves for R_I = 5 and for various values of K_0, K_I and L are shown.

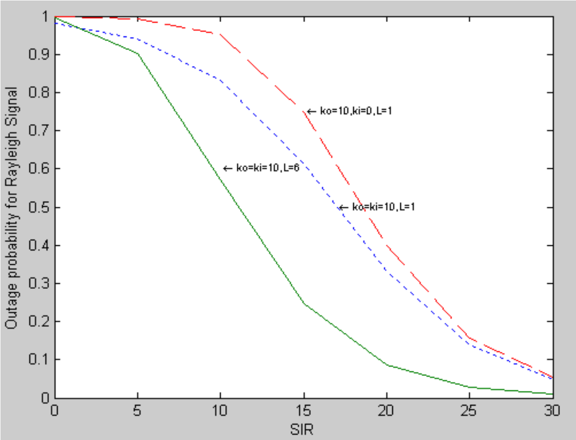

Fig. 1. Outage probability versus SIR for various L, K0, and KI, With RI = 5.

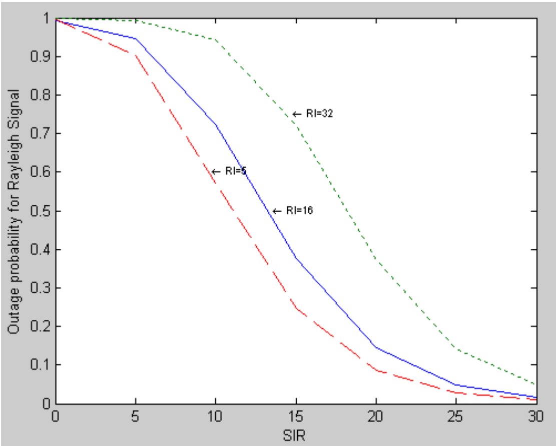

Fig.2. Outage probability versus SIR for various and RI, with K0 = 10 and KI = 5 for desired Rayleigh signal

For the range of SIR of interest comparing the curves with K_0 = 10, L=1 and different K_I values, it can be seen that as K_I decreases, P (Outage|L) increases and the largest P (Outage|L) occurs when K_I = 0 which is Rayleigh interferer case. Similarly in fig 2 and fig 3, Outage probability curves for Rician and Rayleigh interfere case is shown for different values of R_I. As expected, the outage probability increases as R_I increases.

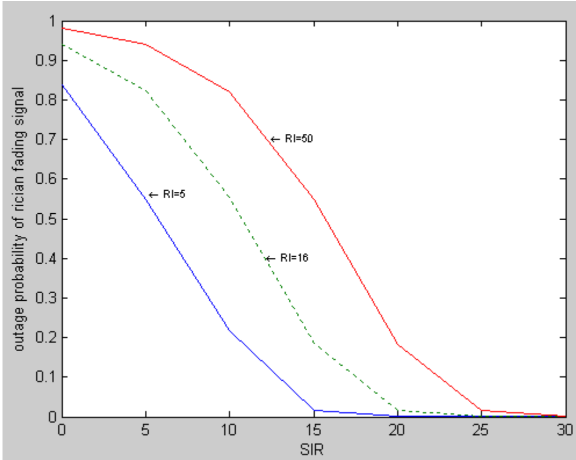

Fig.3. Outage probability versus SIR for various and RI, with K0 = 10 and KI = 5 for desired Rician signal.

Fig. 4 and fig 5 shows P(outage) curves as a function of reuse distance D at the cell boundary r_0= R, with the protection ratio R_I,K_0, and K_I,number of frequency channels N_s,and blocking probability B as parameters for the Rayleigh and Rician desired signal.As expected the result shows that in order to avoid a large P(outage) , a larger reuse distance is required.

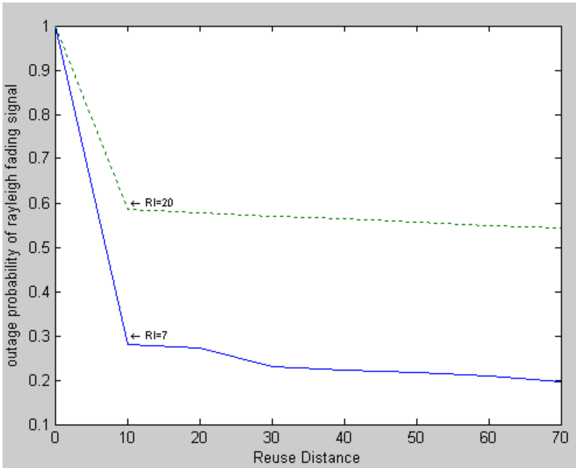

Fig.4. Outage probability versus reuse distance D for Rayleigh fading channel with different values of RI.

On the other hand, as K_I decreases while $K_0\sigma_0^2$ and reuse distance D remains constant, it can be seen from (32) that the variance σ_I^2 of the interferers becomes larger, which means that the corresponding S becomes smaller. Therefore, a larger outage probability is observed.

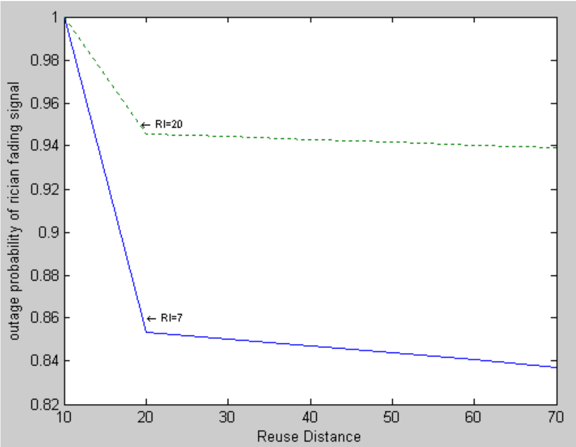

Fig.5. Outage probability versus reuse distance D for Rician fading channel with different values of RI.

4. CONCLUSIONS

In this paper, we have presented new exact form of outage probability for the Rician as well as Rayleigh fading environment without any type of shadowing as a special case. The effect of SIR, Protection ratio (R_I), Rice factor of signals, number of cochannel interferes L on the outage probability and reuse distance D have been investigated. New Rician/Rayleigh outage probabilities curves against the normalized reuse distance without lognormal shadowing effect are presented and discussed. The results are almost same as previous results by this technique.

5. REFERENCES

[1] Tjeng Thiang Tjhung, Chin Choy Chai, and Xiaodai Dong "Outage Probability for Lognormal-Shadowed RicianChannels" Ieee Transactions on Vehicular Technology, vol. 46, no. 2, may 1997,p.p 400-407.

[2] Y. D. Yao and A. U. G. Sheika, "Outage probability analysis for microcell mobile radio systems with co-channel interferers in Rician/Rayleigh fading environment," Electron. Lett., vol. 26, June 1990.pp. 864–866.

[3] J. R. Haug and D. R. Ucci, "Outage probability of microcellular radio systems in a Rayleigh/Rician fading environment," in Int. Conf. Comm., ICC '92, pp. 312.4.1–312.4.5.

[4] R. Muammar, "Co-channel interference in microcellular mobile radiosystem," in IEEE Veh. Technol. Conf. 1991, pp. 198–203.

[5] M. D. Austin and G. L. Stuber, "Exact cochannel interference analysis for log-normal shadowed Rician fading channels," Electron. Lett., vol. 30, May 1994, pp. 748–749.

[6] R. Prasad and A. Kegel, "Effects of Rician faded and log-normal shadowed signals on spectrum efficiency in microcellular radio," IEEE Trans. Veh. Technol., vol. 42, Aug. 1993, pp. 274–281.

[7] G. L. Turin, "The characteristic function of Hermitian quadratic forms in complex normal variables," Biometrika, vol. 4, June 1960, pp. 199–201.

[8] P. A. Bello, "Binary error probabilities over selectively fading channels containing specular components," IEEE Trans. Commun. Technol., vol. comm-14, Aug. 1966, pp. 400–406.

[9] G. A. Campbell and R. M. Foster, "Fourier Integrals for PracticalApplications". New York: Van Nostrand, 1957.

[10] R. Price, "Some noncentral F-distributions expressed in closed form," Biometrika, vol. 51, , June 1964, pp. 107–122.

[11] Tjeng Thiang Tjhung, Chin Choy Chai, and Xiaodai Dong "Outage Probability for Lognormal-Shadowed Rician Channels",in IEEE Veh. Technol. Conf. vol. 46,no 2, May 1997, pp.400-407.