MHD Convection Slip Fluid Flow with Radiation and Heat Deposition in a Channel in a Porous Medium

I. J. Uwanta
Department of Mathematics,
Usmanu Danfodiyo University,
Sokoto, Nigeria

M. Sani
Dept. of Maths & Computer Sci.
Umar Musa Yar'adua
University, Katsina
Nigeria

M.O Ibrahim
Department of Mathematics,
Usmanu Danfodiyo University,
Sokoto, Nigeria

ABSTRACT
This paper examines the MHD convection slip fluid flow with radiation and heat deposition in a channel in a porous medium. The dimensionless governing equations were solved using the perturbation technique to obtained the analytical expressions of velocity, temperature and concentration profiles of the fluid with expression of Skin friction, mass and heat transfer in terms of Shear stress, Nusselt number and Sherwood number respectively. The effects of various parameters associated with flow like Prandtl number Pr, Peclet number Pe, Reynold number Re, Grashof number Gr, Porous medium shape factor parameter S, Eckert number Ec, Hartmann number H, Radiation parameter N, and time t are studied with the help of graphs and tables. It follows that the velocity increase with increasing H and S, and decrease with increase in Pr, Pe, Gr, Re and Ec, while the temperature increase with increasing Pe and decrease when there is increase in Pr and Ec.

Keywords: MHD (Magnetohydrodynamics), Porous medium, Slip fluid flow and heat deposition

1.INTRODUCTION
A survey of magnetohydrodynamics (MHD) studies in the technological fields can be found in Moreau (1990). In recent years, the flow of fluids through porous media has become an important topic because of the recovery of crude oil from the pores of the reservoir rocks, in this case, Darcy's law represents the gross effect. Meanwhile, there has been a renewed interest in studying magnetohydrodynamic (MHD) flow and heat transfer in porous media because of the effect on magnetic fields on the performance of many systems Makinde (1998). Aldoss *et al* (1995) have studied mixed convection flow from a vertical plate embedded in a porous medium in the presence of a magnetic field. Chamkha (2000) has considered MHD free convection flow from a vertical plate embedded in a thermally stratified porous medium with Hall effects. In the present paper, we investigate the combined effects of a transverse magnetic field and radiative heat transfer on unsteady flow of a conducting optically thin fluid through a channel filled with saturated porous medium and non-uniform walls temperature. In the following sections, the problem is formulated, solved and the pertinent results are discussed. Panda *et al* (2003) have analyzed the unsteady free convective flow and mass transfer of a rotating elastico-viscous liquid through porous media past a vertical porous plate. Sattar (1994) has discussed the free convection and mass transfer flow through a porous medium past an infinite vertical porous plate with time dependent temperature and concentration. Rashed (2007) studied the effect of radiation on the heat transfer from a stretching surface in a porous medium. Elbashbeshy *et al* (2004) have analyzed the effect of internal heat generation and suction or injection on the heat transfer in a porous medium over a stretching surface. Sultana *et al* (2009) have analyzed the effects of internal heat generation, radiation and suction or injection on the heat transfer in a porous medium over a stretching surface. Rajeswari *et al* (2009) have studied the effect of chemical reaction, heat and mass transfer on nonlinear MHD boundary layer flow through vertical porous surface with heat source in the presence of suction. Mahmoud (2010) studied the effect of slip velocity at the wall on flow and mass transfer of an electrically conduction visco-elastic fluid past a stretching sheet embedded in a porous medium in the presence of chemical reaction and concentration dependent viscosity. Kandasamy *et al* (2007) have studied the effect of variable viscosity, heat and mass transfer on nonlinear mixed convection flow over a porous wedge with chemical reaction in the presence of heat radiation. Khanafer and Chamkha (1999) examined numerically mixed convection flow in a lid-driven enclosure filled with a fluid saturated porous medium and reported on the effects of the Darcy and Richardson numbers on the flow and heat transfer characteristics. Also, the porous media heat transfer problems have several practical engineering applications, such as the crude oil extraction, the ground water pollution, and many other practical applications, i.e., in biomechanical problems (e.g., blood, flow in the pulmonary alveolar sheet) and in the filtration transpiration cooling. Hiremath and Patil (1993) studied the effect of free convection currents on the oscillatory flow of the polar fluid through a porous medium, which is bounded by the vertical plane surface with a constant temperature. The unsteady hydromagnetic free convection flow of a Newtonian and polar fluid has been investigated by Helmy (1998). El-Hakien *et al* (1999) studied the effects of the viscous and Joule heating on MHD free convection flows with variable plate temperatures in a micropolar fluid. El-Amin (2001) considered MHD free convection and mass transfer flow in a micropolar fluid over a stationary vertical plate with a constant suction. Kim (2001) investigated the unsteady free convection flow of a micropolar fluid past a vertical plate embedded in a porous medium, and extended his work in (2004) to study the effects of heat and mass transfer in the MHD micropolar fluid flow past a vertical moving plate. At the macroscopic level, it is well accepted that the boundary condition for a viscous fluid at a solid wall is one of no-slip, i.e., the fluid velocity matches the velocity of the solid boundary. While the no-slip condition has been processed experimentally to be accurate for a number of macroscopic flows, it remains an assumption that is not based on physical principles. In many practical applications, the particle adjacent

to a solid surface no longer takes the velocity of the surface. The particle at the surface has a finite tangential velocity. It slips along the surface. The flow regime is called a slip-flow regime, and this effect can not be neglected. The study of magneto-micropolar fluid flows in the slip-flow regimes with heat transfer has important engineering applications, e.g., in power generators, refrigeration coils, transmission lines, electric transformers, and heating elements. Khandelwal *et al* (2003) studied the effects of permeability variation on the MHD unsteady flow of polar fluid through a porous medium in a slip-flow regime over an infinite porous flat plate. Sharma and Chaudhary (2003) studied the effect of variable suction on transient free convective viscous incompressible flows past a vertical plate in a slip-flow regime. Interestingly, Makinde and Mhone (2005) studied heat transfer to MHD oscillatory flow in a channel filled with porous medium.
This work will carry all along the study of MHD convection slip fluid flow with radiation and heat deposition in a channel in a porous medium which is an extension of Makinde and Mhone (2005) with addition of Eckert and Prandtl number with velocity into the energy equation.

2. PROBLEM FORMULATION

Consider the flow of a conducting optically thin fluid in a channel filled with saturated porous medium under the influence of an externally applied homogeneous magnetic field and radiative heat transfer. It is assumed that the fluid has small electrical conductivity and the electromagnetic force produced is very small. Take a Cartesian coordinate system $\left(x^*, y^*\right)$ where $0x^*$ lies along the centre of the channel, y^* is the distance measured in the normal section. Then, assuming a Boussinesq incompressible fluid model, the equations governing the motion are given as

$$\frac{\partial u^*}{\partial t^*} = -\frac{1}{\rho}\frac{\partial P^*}{\partial x^*} + \upsilon\frac{\partial^2 u^*}{\partial y^{*2}} - \frac{\upsilon}{k}u^* - \frac{\sigma_e B_0^2}{\rho}u^* + g\beta\left(T^* - T_0^*\right) \quad (1)$$

$$\frac{\partial T^*}{\partial t^*} = \frac{k}{\rho c_\rho}\frac{\partial^2 T^*}{\partial y^{*2}} - \frac{1}{\rho c_\rho}\frac{\partial q}{\partial y^*} + \frac{1}{\rho c_\rho}\left(\frac{\partial u^*}{\partial y^*}\right)^2 \quad (2)$$

Subject to the boundary conditions

$$\left.\begin{aligned} & u^* = 0, T^* = T_w^* \ \ on\, y^* = a \\ & T^* = T_0^*, u^* - ar\frac{\partial u^*}{\partial y^*} = 0 \ \ on\, y^* = 0 \end{aligned}\right\} \quad (3)$$

Where u^* is the axial velocity, t^* the time, T^* the fluid temperature, P^* the pressure, g^* the gravitational force, *q* the radiative heat flux, β the coefficient of volume expansion due to temperature, *cp* the specific heat at constant pressure, *k* the thermal conductivity, *K* the porous medium permeability coefficient, $B_0 = \left(\mu_e H_0\right)$ the electromagnetic induction, μ_e the magnetic permeability, H_0 the intensity of magnetic field, σ_e the conductivity of the fluid, ρ the fluid density and ν is the kinematic viscosity coefficient. It is assumed that both walls temperature T_0, T_w are high enough to induce radiative heat transfer. Following Cogley *et al* (1968), it is assumed that the fluid is optically thin with a relatively low density and the radiative heat flux is given by

$$\frac{\partial q}{\partial y^*} = 4\alpha^2\left(T_0^* - T^*\right) \quad (4)$$

where α is the mean radiation absorption coefficient.
Substituting (4) into (2)

$$\frac{\partial T^*}{\partial t^*} = \frac{k}{\rho c_\rho}\frac{\partial^2 T^*}{\partial y^{*2}} - \frac{1}{\rho c_\rho}4\alpha^2\left(T_0^* - T^*\right) + \frac{1}{\rho c_\rho}\left(\frac{\partial u^*}{\partial y^*}\right)^2 \quad (5)$$

3. ANALYTICAL SOLUTION

To solve the governing equations in dimensionless form, we introduce the following non-dimensional quantities:

$$\left.\begin{aligned} & \mathrm{Re} = \frac{Ua}{\upsilon}, x = \frac{x^*}{a}, y = \frac{y^*}{a}, u = \frac{u^*}{U}, \theta = \frac{T^* - T_0^*}{T_w^* - T_0^*}, \\ & H^2 = \frac{a^2\sigma_e B_0^2}{\rho\upsilon}, t = \frac{t^* U}{a}, P = \frac{aP^*}{\rho\upsilon U}, D_a = \frac{k}{a^2}, \\ & Gr = \frac{g\beta a^2\left(T_w^* - T_0^*\right)}{\upsilon U}, Pe = \frac{Ua\rho c_\rho}{k}, \\ & N^2 = \frac{4\alpha^2 a^2}{k}, \mathrm{Pr} = \frac{\upsilon\rho c_\rho}{k}, \\ & Ec = \frac{U^2}{\upsilon\rho c_\rho\left(T_w^* - T_0^*\right)} \end{aligned}\right\} \quad (6)$$

We reduce the governing equations into dimensionless form using (6) into (1), (3) and (5) as follows

$$\mathrm{Re}\frac{\partial U}{\partial t} = -\frac{\partial P}{\partial x} + \frac{\partial^2 U}{\partial y^2} - (S^2 + H^2)U + Gr\theta \quad (7)$$

$$Pe\frac{\partial \theta}{\partial t} = \frac{\partial^2\theta}{\partial y^2} + N^2\theta + Ec\,\mathrm{Pr}\left(\frac{\partial U}{\partial y}\right)^2 \quad (8)$$

Subject to the boundary conditions

$$\left.\begin{aligned} & U - r\frac{\partial U}{\partial y} = 0, \theta = 0, y = 0 \\ & U = 0, \theta = 1, y = 1 \end{aligned}\right\} \quad (9)$$

Where Re, S, H, Gr, Pe, N, Ec and Pr are the Reynolds number, porous medium shape factor, Hartmann number, Grashof

number, Peclet number, radiation parameter, Eckert number and Prandtl number respectively.
To solve the equations (7) and (8) subject to (9), we assume the pressure gradient of the form

$$-\frac{\partial P}{\partial x} = A + Be^{iwt} \tag{10}$$

where A and B are constants.
Similarly, the solution of the velocity and temperature are taken as

$$\left.\begin{aligned} U(y) &= U_0(y) + U_1(y)e^{iwt} \\ \theta(y) &= \theta_0(y) + \theta_1(y)e^{iwt} \end{aligned}\right\} \tag{11}$$

Using (10) and (11) into (7) – (9), we obtain the following

(12

(13)

The boundary conditions becomes

$$\left.\begin{aligned} &U_0 + U_1 e^{iwt} - r(U_0' + U_1' e^{iwt}) = 0, \theta_0 + \theta_1 e^{iwt} = 0, y = 0 \\ &U_0 + U_1 e^{iwt} = 0, \theta_0 + \theta_1 e^{iwt} = 1, y = 1 \end{aligned}\right\} \tag{14}$$

Separating the fluctuating and non-fluctuating terms
For the non-fluctuating terms, let $a^2 = S^2 + H^2$

$$U_0'' - a^2 U_0 + Gr\theta_0 = -A$$

$$\theta_0'' + N^2\theta_0 = -Ec\Pr\left(U_0'\right)^2 \tag{15}$$

Subject to the boundary conditions

$$U_0 - rU_0' = 0, \theta_0 = 0, y = 0 \tag{16}$$

$$U_0 = 0, \theta_0 = 1, y = 1$$

To solve (15), (16) subject to (17), we assume the solution of U_0 *and* θ_0 as

$$U_0(y) = U_{00}(y) + EcU_{01}(y) \text{ and } \theta_0(y) = \theta_{00}(y) + Ec\theta_{01}(y) \tag{18}$$

Using (18) into (15) - (17), we get

$$U_{00}'' + EcU_{01}'' - a^2U_{00} - a^2EcU_{01} + Gr\theta_{00} + EcGr\theta_{01} = -A \tag{19}$$

$$\theta_{00}'' + Ec\theta_{01}'' + N^2\theta_{00} + EcN^2\theta_{01} = -Ec\Pr\left[\left(U_{00}'\right)^2 + 2EcU_{00}'U_{01}' + \left(Ec\right)^2\left(U_{01}'\right)^2\right] \tag{20}$$

Subject to the boundary conditions

$$\left.\begin{aligned} &U_{00} + EcU_{01} - rU_{00}' - EcrU_{01}' = 0, \theta_{00} + Ec\theta_{01} = 0, y = 0 \\ &U_{00} + EcU_{01} = 0, \theta_{00} + Ec\theta_{01} = 1, y = 1 \end{aligned}\right\} \tag{21}$$

Separating the coefficients of Ec^0 and Ec^1 respectively yield

$$\theta_{00}'' + N^2\theta_{00} = 0 \tag{22}$$

$$\theta_{01}'' + N^2\theta_{01} = -\Pr\left(U_{00}'\right)^2 \tag{23}$$

$$U_{00}'' - a^2U_{00} + Gr\theta_{00} = -A \tag{24}$$

$$U_{01}'' - a^2U_{01} + Gr\theta_{01} = 0 \tag{25}$$

Subject to the boundary conditions

$$\left.\begin{aligned} &U_{00} - rU_{00}' = 0, \theta_{00} = 0, y = 0 \text{ and } U_{00} = 0, \theta_{00} = 1, y = 1 \\ &U_{01} - rU_{01}' = 0, \theta_{01} = 0, y = 0 \text{ and } U_{01} = 0, \theta_{01} = 0, y = 1 \end{aligned}\right\} \tag{26}$$

Solving (23) to (25) subject to (26), we obtain the following

$$\theta_{00}(y) = g\sin(Ny) \tag{27}$$

where: $g = \dfrac{1}{\sin N}$

(10)

$$U_{00}(y) = a_2e^{ay} + a_1e^{-ay} + a_3 + a_4\sin(Ny) \tag{28}$$

where: $a_3 = \dfrac{A}{a^2}; a_4 = \dfrac{Grg}{(N^2 + a^2)}$

$$\theta_{01}(y) = b_{16}e^{iNy} + b_{15}e^{-iNy} + b_7 - b_8e^{2ay} - b_9e^{-2ay} - b_{10}\cos\left(Ny\right)e^{ay} + b_{11}\sin\left(Ny\right)e^{ay}$$

(11)

(29)

$$+b_{12}\cos\left(Ny\right)e^{-ay} - b_{13}\cos^2\left(Ny\right) - b_{14}\sin^2\left(Ny\right)\Big\}$$

And,

$$\left.\begin{aligned} U_{01}(y) &= b_{28}e^{ay} + b_{27}e^{-ay} + b_{17}e^{iNy} + b_{18}e^{-iNy} + \\ &b_{19} + b_{20}e^{2ay} + b_{21}e^{-2ay} + b_{22}\cos\left(Ny\right)e^{ay} \\ &-b_{23}\sin\left(Ny\right)e^{ay} + b_{24}\cos\left(Ny\right)e^{-ay} - \\ &b_{25}\cos^2\left(Ny\right) - b_{26}\sin^2\left(Ny\right) \end{aligned}\right\} \tag{31}$$

(15)

(16)

Furthermore, for the fluctuating terms (17)

$$\text{Re}\,iwU_1 = B + U_1'' - a^2U_1 + Gr\theta_1$$

Or $U_1'' - b^2U_1 = -B - Gr\theta_1$

$$Peiw\theta_1 = \theta_1'' + N^2\theta_1 + 2Ec\Pr U_0'U_1'$$

Or $\theta_1'' + c^2\theta_1 = -2Ec\Pr U_0'U_1'$,

where: $b = \sqrt{a^2 - \text{Re}\,iw}$; $c = \sqrt{N^2 - P\text{e}\,iw}$

Subject to the boundary conditions

$$\left.\begin{aligned} &U_1 - rU_1' = 0, \theta_1 = 0, y = 0 \\ &U_1 = 0, \theta_1 = 0, y = 1 \end{aligned}\right\} \tag{19}$$

(34)

To solve (32), (33) subject to (34), we assume the solution of U_1 *and* θ_1 as

$$U_1(y) = U_{11}(y) + EcU_{12}(y); and, \theta_1(y) = \theta_{11}(y) + Ec\theta_{12}(y) \tag{35}$$

Using (35) into (32) - (35), we have

$$U_{11}'' + EcU_{12}'' - b^2U_{11} - Ecb^2U_{12} = -B - Gr\theta_{11} - EcGr\theta_{12} \tag{36}$$

$$\theta_{11}'' + Ec\theta_{12}'' + c^2\theta_{11} + Ecc^2\theta_{12} = -2Ec\Pr U_{00}'U_{11}'$$

(23)

Subject to the boundary conditions

$$\left.\begin{aligned} &U_{11} + EcU_{12} - rU_{11}' - EcrU_{12}' = 0, \theta_{11} + Ec\theta_{12} = 0, y = 0 \\ &U_{11} + EcU_{12} = 0, \theta_{11} + Ec\theta_{12} = 0, y = 1 \end{aligned}\right\} \tag{38}$$

(24)

(25)

(26)

Separating the coefficients of Ec^0 and Ec^1 respectively

$$\theta_{11}'' + c^2\theta_{11} = 0$$

$$\theta_{12}'' + c^2\theta_{12} = -2\Pr U_{00}'U_{11}'$$

$$U_{11}^{//} - b^2 U_{11} = -B - Gr\theta_{11} \qquad (41)$$

$$U_{12}^{//} - b^2 U_{12} = -Gr\theta_{12} \qquad (42)$$

With the following boundary conditions

$$\left.\begin{aligned} &U_{11} - rU_{11}^{/} = 0, \theta_{11} = 0, y = 0 \ \ and \ \ U_{11} = 0, \theta_{11} = 0, y = 1 \\ &U_{12} - rU_{12}^{/} = 0, \theta_{12} = 0, y = 0 \ \ and \ \ U_{12} = 0, \theta_{12} = 0, y = 1 \end{aligned}\right\} \qquad (43)$$

Solving (39) to (42) subject to (43), we get

$$U_{11}(y) = d_1 e^{by} + d_2 e^{-by} + d_3 \qquad (44)$$

where: $d_3 = \dfrac{B}{b^2}$

$$\theta_{12}(y) = e_{15} e^{icy} + e_{14} e^{-icy} - e_7 e^{y(a+b)} - e_8 e^{-y(a+b)} + e_9 e^{y(a-b)} + e_{10} e^{-y(a-b)} - e_{11}\cos(Ny)e^{by}$$

$$-e_{12}\sin(Ny)e^{by} + e_{13}\cos(Ny)e^{-by}\Big\} \qquad (45)$$

$$U_{12}(y) = f_{11}e^{by} + f_{10}e^{-by} + f_1 e^{icy} + f_2 e^{-icy} + f_3 e^{y(a+b)} + f_4 e^{-y(a+b)} - f_5 e^{y(a-b)} - f_6 e^{-y(a-b)}$$

$$-f_7\cos(Ny)e^{by} + f_8\sin(Ny)e^{by} + f_9\cos(Ny)e^{-by}\Big\} \qquad (46)$$

The solution of the velocity field is

$$U = U_0(y) + U_1(y)e^{iwt}$$

Replacing, $U_0 = U_{00} + EcU_{01}$

And, $U_1 = U_{11} + EcU_{12}$

$$\Rightarrow U = U_{00} + EcU_{01} + \left(U_{11} + EcU_{12}\right)e^{iwt}$$

Substituting (29), (31), (44) and (46)

$$U = \left[a_2 e^{ay} + a_1 e^{-ay} + a_3 + a_4\sin(Ny)\right] + Ec\Big[b_{28}e^{ay} + b_{27}e^{-ay} + b_{17}\left(\cos(Ny) + i\sin(Ny)\right)$$

$$+b_{18}\left(\cos(Ny) - i\sin(Ny)\right) + b_{19} + b_{20}e^{2ay} + b_{21}e^{-2ay} + b_{22}\cos(Ny)e^{ay} - b_{23}\sin(Ny)e^{ay}$$

$$+b_{24}\cos(Ny)e^{-ay} - b_{25}\cos^2(Ny) - b_{26}\sin^2(Ny)\Big] +$$

$$\Big\{\left(d_1 e^{by} + d_2 e^{-by} + d_3\right) + Ec\Big[f_{11}e^{by} + f_{10}e^{-by}$$

$$+f_1\left(\cos(cy) + i\sin(cy)\right) + f_2\begin{pmatrix}\cos(cy) - \\ i\sin(cy)\end{pmatrix} + f_3 e^{y(a+b)} +$$

$$f_4 e^{-y(a+b)} - f_5 e^{y(a-b)} - f_6 e^{-y(a-b)}$$

$$-f_7\cos(Ny)e^{by} + f_8\sin(Ny)e^{by} + f_9\cos(Ny)e^{-by}\Big]\Big\}e^{iwt} \qquad (47)$$

Similarly,

$$\theta = g\sin(Ny) + Ec\begin{bmatrix} b_{16}\left(\cos(Ny) + i\sin(Ny)\right) + \\ b_{15}\left(\cos(Ny) - i\sin(Ny)\right) + b_7 - b_8 e^{2ay} \\ b_9 e^{-2ay} - b_{10}\cos(Ny)e^{ay} + b_{11}\sin(Ny)e^{ay} + b_{12}\cos(Ny)e^{-ay} \\ -b_{13}\cos^2(Ny) - b_{14}\sin^2(Ny) \end{bmatrix}$$

$$+ \left\{Ec\left[\begin{matrix} e_{15}\left(\cos(cy) + i\sin(cy)\right) + \\ e_{14}\left(\cos(cy) - i\sin(cy)\right) \end{matrix} - e_7 e^{y(a+b)} - e_8 e^{-y(a+b)} + e_9 e^{y(a-b)}\right.\right.$$

$$\left.\left. +e_{10}e^{-y(a-b)} \begin{matrix} -e_{11}\cos(Ny)e^{by} - \\ e_{12}\sin(Ny)e^{by} + e_{13}\cos(Ny)e^{-by} \end{matrix}\right]\right\}e^{iwt} \qquad (48)$$

The Skin friction is given by

$$\left.\frac{\partial U}{\partial y}\right|_{y=0} = a\left(a_2 - a_1\right) + a_4 N +$$

$$Ec\Big[a\left(b_{28} - b_{27} + b_{22} - b_{24}\right) + 2a\left(b_{20} - b_{21}\right) - Nb_{23}$$

$$+iN\left(b_{17} - b_{18}\right)\Big] + \left\{\begin{matrix} b\left(d_1 - d_2\right) + \\ Ec \end{matrix}\right. \Big[b\left(f_{11} - f_{10} - f_7 - f_9\right) + (a+b)\left\{f_3 - f_4\right\}$$

$$+(a-b)\left\{f_6 - f_5\right\} + Nf_8 + ic\left(f_1 - f_2\right)\Big]\Big\}e^{iwt} \qquad (49)$$

Similarly, the Nusselt number become

$$\left.\frac{\partial\theta}{\partial y}\right|_{y=0} = g + Ec\begin{bmatrix} 2a\left(b_9 - b_8\right) - a\left(b_{10} + b_{12}\right) \\ +Nb_{11} + iN\left(b_{16} - b_{15}\right) \end{bmatrix} +$$

$$\Big\{Ec\Big[(a+b)\left\{e_8 - e_7\right\}$$

$$+\left(a-b\right)\left\{e_9-e_{10}\right\}\begin{matrix}-b\left(e_{11}+e_{13}\right)\\ -Ne_{12}+ic\left(e_{15}-e_{14}\right)\end{matrix}\Bigg]\Bigg]\Bigg\}e^{iwt} \tag{50}$$

4. RESULTS AND DISCUSSION

From the analytical solutions, we obtain numerical solutions for different values of the Prandtl number Pr, Hartmann number H, radiation parameter N, Peclet number, Reynolds number Re, Grashof number Gr, Eckert number Ec. We fixed the values of the following parameters throughout the calculations, Pr = 0.71, Pe = 0.2, S = 0.1, H = 1.6, y = 0:0.001:1, Gr = 0.2, Re = 0.5, r = 0.5, N = 0.1, A = 2, B = 1, Ec = 0.2, t = 0.1, Omega = 1.

The velocity profiles have been studied and presented in Figures 1 to 7 with different values of Prandtl number (Pr = 0.60, 0.71, 0.85, 1), Hartmann number (H = 1.0, 1.5, 2.0, 2.5), Peclet number (Pe = 0.2, 0.4, 0.6, 0.8), Grashof number (Gr = 0.2, 0.12, 0.22, 0.32), Reynolds number (Re = 0.5, 1, 1.5, 2), porous medium shape factor parameter (S = 0.1, 0.3, 0.5, 0.7) and Eckert number (Ec = 0.2, 0.5, 0.8, 1.1). Figures 1and 2 reveals that the velocity increase with increase in the porous medium shape factor parameter and the Hartmann number respectively. While Figures 3 to 7 shows that the velocity decrease with increase in the Prandtl number, Peclet number, Grashof number, Reynolds number and Eckert number respectively.

The temperature profiles are illustrated in Figures 8 to 10 for different values of Prandtl number (Pr = 0.60, 0.71, 0.85,1), Eckert number (Ec = 0.2, 0.4, 0.6, 0.8) and peclet number(Pe = 1, 1.5, 2, 2.5) shown in Figures 8-10respectively. In Figures 8 and 9, we observed that the temperature decrease with increase in the Prandtl number and Eckert number. Figure 10 reveals that the temperature increase with increase in the peclet number.

Tables 1 to 6 are the tables of Skin friction and Nusselt number for the analytical solution.

The Skin frictions are illustrated in Tables 1 to 3 for different values of Hartmann number (H = 1.0, 1.5, 2.0) and porous medium shape factor parameter (S = 0.1, 0.3, 0,5), Peclet number (Pe = 0.2, 0.4, 0.6) and Prandtl number (Pr = 0.60, 0.71, 0.85) , and radiation parameter (N = 0.1, 0.11, 0.21) and Eckert number (Ec = 0.2, 0.5, 0.8) shown in Tables 1, 2 and 3 respectively. In Table 1, we observed that the Skin friction is decreasing with increasing Hartmann number and porous medium shape factor parameter. Table 2 shows that increase in the Peclet number decreases the Skin friction while increase in the Prandtl number increases the Skin friction. Table 3 indicates that the Skin friction decrease whenever the radiation parameter increase and increase in the Eckert number results to increasing Skin friction.

The Nusselt numbers are illustrated in Tables 4 to 6.

In Table 4, we noticed that the Nusselt number is increasing with increasing Hartmann number and porous medium shape factor parameter. Table 5 shows that increase in the Peclet number slightly increase the Nusselt number while increase in the Prandtl number slightly decrease the Nusselt number. More over, Table 6 indicates that the Nusselt number decrease whenever the radiation parameter increase and increase in the Eckert number leads to increase of the Nusselt number.

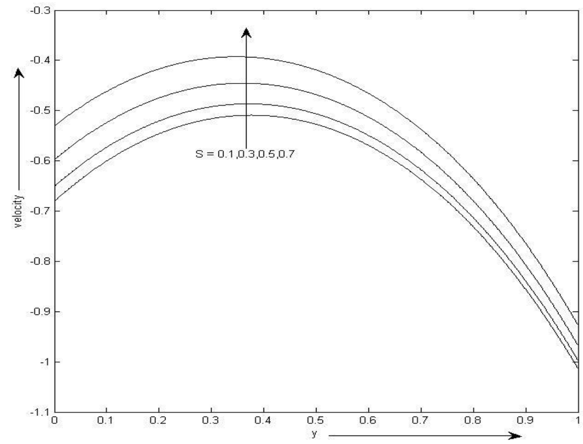

Figure 1: Variation of Velocity against y for different values of S.

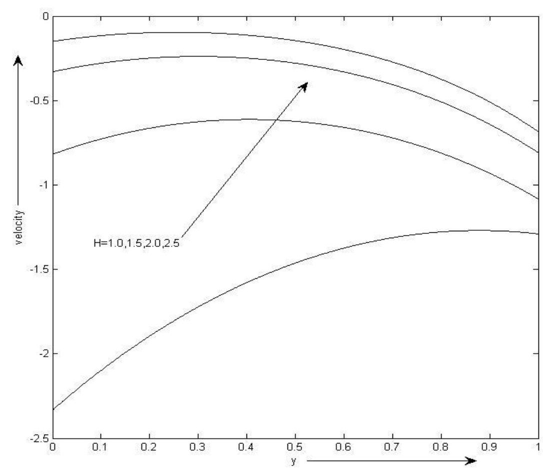

Figure 2: Variation of Velocity against y for different values of H.

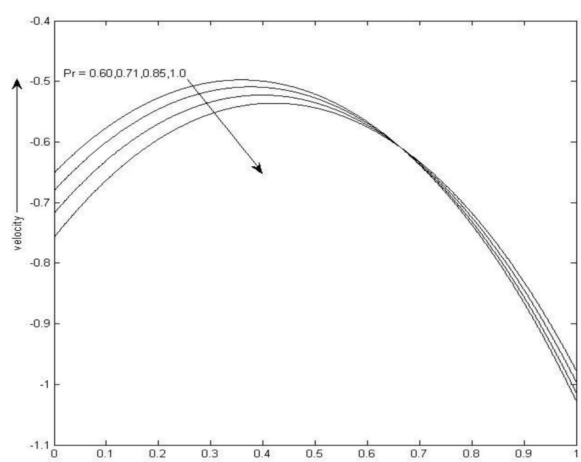

Figure 3: Variation of Velocity against y for different values of Pr.

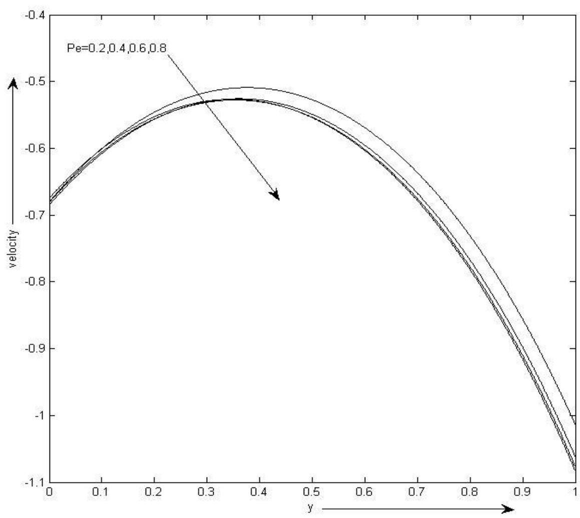

Figure 4: Variation of velocity against y for different values Pe.

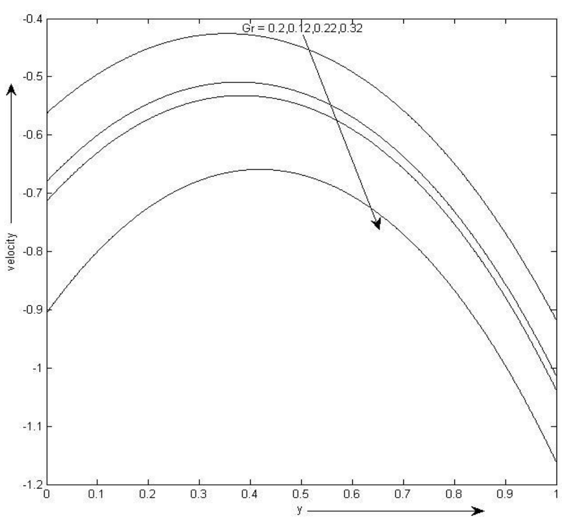

Figure 5: Variation of Velocity against y for different values of Gr.

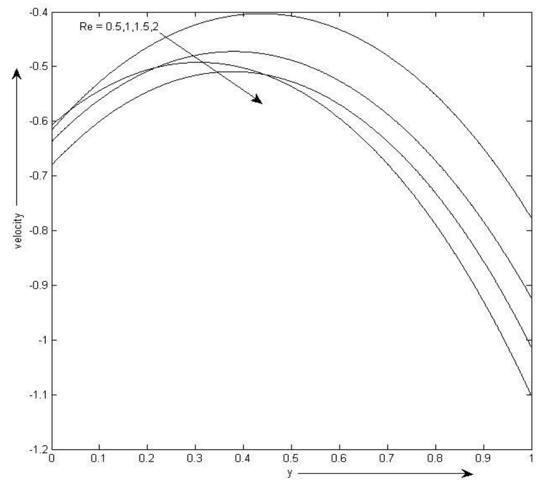

Figure 6: Variation of Velocity against y for different values of Re.

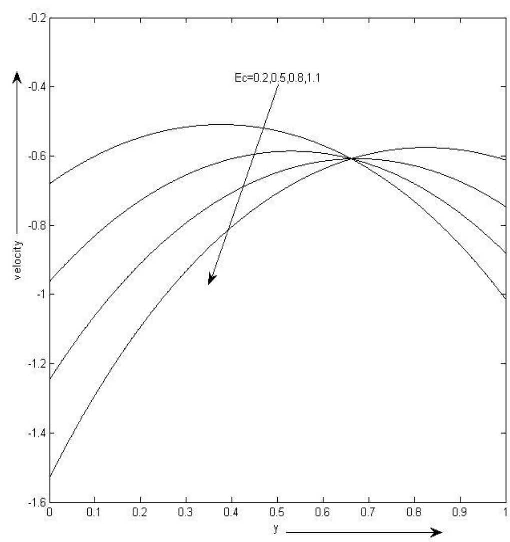

Figure 7: Variation of Velocity against y for different values of Ec.

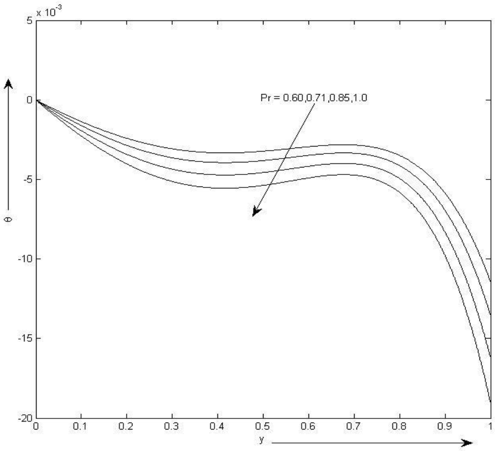

Figure 8: Variation of Temperature against y for different values of Pr.

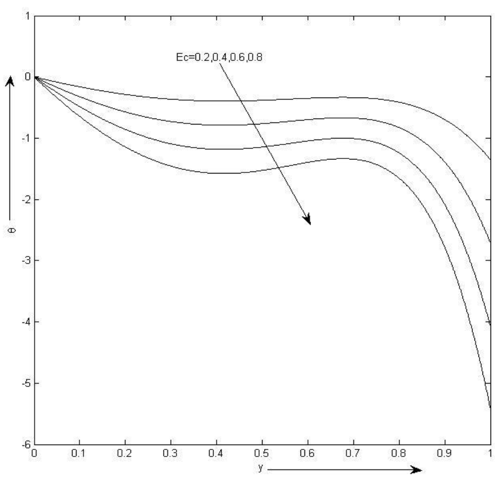

Figure 9: Variation of Temperature against y for different values of Ec.

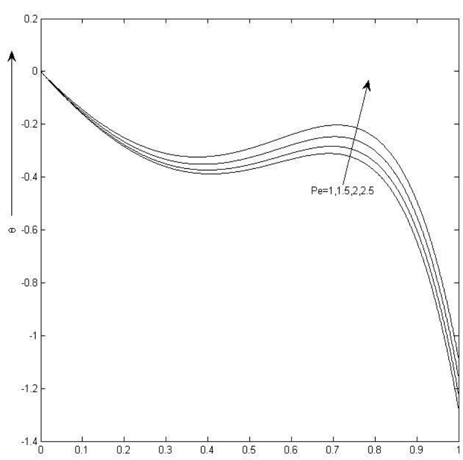

Figure 10: Variation of Temperature against y for different values of Pe.

H	1.0	1.5	2.0
SF S			
0.1	2.3211	0.9877	0.6161
0.3	2.1094	0.9598	0.5960
0.5	1.7882	0.9107	0.5670

Table 1: Skin Friction for various values of H and S.

Pe	0.2	0.4	0.6
SF Pr			
0.60	0.8423	0.8170	0.8020
0.71	0.8916	0.8617	0.8439
0.85	0.9543	0.9185	0.8972

Table 2: Skin Friction for various values of Pe and Pr.

N	0.1	0.11	0.21
SF Ec			
0.2	0.8916	0.8560	0.6997
0.5	1.3687	1.2795	0.8868
0.8	1.8459	1.7031	1.0738

Table 3: Skin Friction for various values of N and Ec.

H	1.0	1.5	2.0
Nu S			
0.1	9.6166	9.8745	9.9390
0.3	9.6528	9.8804	9.9403
0.5	9.7113	9.8907	9.9426

Table 4: Nusselt Number for various values of H and S.

Pe	0.2	0.4	0.6
Nu / Pr			
0.60	9.9136	9.9137	9.9140
0.71	9.8947	9.8948	9.8952
0.85	9.8706	9.8708	9.8713

Table 5: Nusselt Number for various values of Pe and Pr.

N	0.1	0.11	0.21
Nu / Ec			
0.2	9.8947	8.9853	4.6559
0.5	9.9593	9.0497	4.7176
0.8	10.0239	9.1141	4.7792

Table 6: Nusselt Number for various values of N and Ec.

5. CONCLUSION

This paper examines the MHD convection slip fluid flow with radiation and heat deposition in a channel in a porous medium. The dimensionless governing equations were solved using the perturbation technique. The effect of different parameters such as the Prandtl number, Grashof number, Reynolds number, Peclet number, Eckert number, radiation parameter, porous medium shape factor parameter and the Hartmann number were studied. The conclusion reveals that:

- The velocity increase with increase of the Hartmann number and porous medium shape factor parameter.
- The velocity decrease with increase in the Prandtl number, Peclet number, Grashoff number, Reynolds number and Eckert number.
- The temperature increase when the Peclet number increase
- The temperature decrease with increase of the Prandtl number and Eckert number.

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