Transient Convection Fluid Flow with Heat and Mass Flux in a Fixed Vertical Plate with Radiation

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ABSTRACT
The investigation of the transient convection fluid flow with heat and mass flux in a fixed vertical plate with radiation is presented. The dimensionless governing equations were solved using the Laplace transform method to obtained the analytical expressions of velocity, temperature and concentration profiles of the fluid. Skin friction, Nusselt number and Sherwood number are computed respectively. The effects of various parameters associated with flow like Prandtl number Pr, Schmidt number, modified Grashof number N, Radiation parameter F, chemical reaction parameter K, and time t are studied with the help of graphs and tables. It is observed that the temperature increase with increasing Pr, N and t while concentration increase with increasing K and decrease with increasing Sc and t but the velocity always increase with increasing Sc, N and t.

Keywords: Transient, Convection, Heat and mass flux.

1.INTRODUCTION
Narahari *et al* (2002) have studied the transient free convection flow between two vertical parallel plates with constant heat flux at one boundary and the other maintained at a constant temperature. Ganesan and Rani (2000) studied the unsteady free convection on vertical cylinder with variable heat and mass flux. The radiation effect on heat transfer over a stretching surface has been studied by Elbashbeshy (2000). Thermal radiation and buoyancy effects on MHD free convective heat generating flow over an accelerating permeable surface with temperature dependent viscosity has been studied by Seddeek (2001). Ghaly and Elbarbary (2002) have investigated the radiation effect on MHD free convection flow of a gas at a stretching surface with a uniform free stream. In some of these studies, only steady-state flows over semi infinite vertical plate have been considered. Chamka *et al* (2001) have studied the radiation effects free convection flow past a semi-infinite vertical plate with mass transfer. Ganesan and Loganadhan (2002) studied the radiation and mass transfer effects on the flow of incompressible viscous fluid past a moving cylinder. Abd El-Naby *et al* (2003) analyzed the radiation effects on MHD unsteady free convection flow over a vertical plate with variable surface temperature. Thermal radiation on unsteady free convection and mass transfer boundary layer over a vertical moving plate were analyzed by Raptis and Perdikis (2003). Chaudhary and Jain (2007) studied the exact solutions of incompressible couette flow with constant temperature and constant heat flux on walls in the presence of radiation. An analysis of flow formation in couette motion between two vertical parallel plates was presented by Schlichting and Gersten (2001). This problem is of fundamental importance as it provides the exact solution and reveals how the velocity profiles varies with time, approaching a linear distribution asymptotically, and how the boundary layer spreads throughout the flow field. Free convection in vertical channels has been studied widely in the last few decades under different physical effects due to its importance in many engineering applications such as cooling of electronic equipments, design of passive solar systems for energy conversion, cooling of nuclear reactors, design of heat exchangers, chemical devices and process equipment, geothermal systems, and others. However, very few papers deal with free convection in Couette motion between vertical parallel plates. This problem was further extended for magnetohydrodynamic case by Jha (2001). The unsteady free convection flow with heat flux and accelerated boundary condition was investigated by Chandran *et al* (1998). Ganesan and Palani (2004) have studied unsteady natural convection MHD flow past an inclined plate with variable surface heat and mass flux. Pathak et al (2006) concentrate on the effect of radiation on unsteady free convective flow plate with variable wall temperature.

The present work is an extension of Chaudhary and Jain (2007) by introducing concentration with modified Grashoff number in the momentum equation, mass diffusion equation with chemical reaction, and radiation parameter in the energy equation.

2. PROBLEM FORMULATION
Consider the transient free convective flow of a viscous fluid due to temperature gradient in a vertical channel with the walls at a constant distance d apart. The $x^* - axis$ is taken along one of the wall of the channel and $y^* - axis$ is normal to it. It is also considered that there is radiation only from the fluid and there is a first order chemical reaction between the diffusing species and the fluid. The fluid is a gray, emitting, and absorbing radiation, but non-scattering medium. Initially, the fluid and the channel walls are at the same temperature and there is no fluid motion. At time $t^* > 0$, the heat is supplied at a constant rate at one of the channel walls $\left(y^* = 0\right)$ while the other wall at

$y^* = d$ is maintained at a constant temperature T_d, in the stationary condition with concentration level C_d^* at all points which causes free convection currents in the channel. The mathematical model for the above free convection flow in the channel are the momentum, mass concentration and energy equations:

$$\frac{\partial u^*}{\partial t^*} = \upsilon \frac{\partial^2 u^*}{\partial y^{*2}} + g\beta\left(T^* - T_d^*\right) + g\beta^*\left(C^* - C_d^*\right) \tag{1}$$

$$\frac{\partial C^*}{\partial t^*} = D\frac{\partial^2 C^*}{\partial y^{*2}} - K^*\left(C^* - C_d\right) \tag{2}$$

$$\frac{\partial T^*}{\partial t^*} = \frac{k}{\rho C_p}\frac{\partial^2 T^*}{\partial y^{*2}} - \frac{Q_0}{\rho C_p}\left(T^* - T_d^*\right) \tag{3}$$

The initial and boundary conditions relevant to the fluid flow are:

$$\left.\begin{array}{l} For\, t^* \le 0 : u^* = 0, T^* = T_d^*, C^* = C_d^* \\ \qquad For\, 0 \le y^* \le d \\ For\, t^* > 0 : u^* = 0, \dfrac{\partial T^*}{\partial y^*} = -\dfrac{q}{k}, \dfrac{\partial C^*}{\partial y^*} = -\dfrac{q}{k} \\ \qquad at\, y^* = 0 \\ u^* = 0, T^* = T_d^*, C^* = C_d^* \\ \qquad at\, y^* = d \end{array}\right\} \tag{4}$$

Where u^* is the axial velocity, t^* is time, T^* the fluid temperature, β is coefficient of volume, C^* is the fluid concentration, C_w^* is wall concentration, D the mass diffusivity, K is chemical reaction parameter, β^* is the coefficient of concentration expansion, ρ is the density of the fluid kg.m^{-3} and q is the rate of heat transfer.

To solve the governing equations in dimensionless form, we introduce the following non-dimensional quantities:

$$\begin{array}{l} y = \dfrac{y^* U_0}{\upsilon}, U = \dfrac{u^*}{U_0}, t = \dfrac{t^* U_0^2}{\upsilon}; d = \dfrac{\upsilon}{U_0}, \\ \theta = \dfrac{T^* - T_d^*}{q\upsilon/k}, c = \dfrac{C^* - C_d^*}{q\upsilon/k}, K = \dfrac{\upsilon K^*}{U_0^2}, \\ Sc = \dfrac{\upsilon}{D}, \Pr = \dfrac{\upsilon\rho c_\rho}{k}, Gr = \dfrac{g\beta q\upsilon^2}{kU_0^4}, \\ Gc = \dfrac{g\beta^* q\upsilon^2}{kU_0^4}, F = \dfrac{\upsilon Q_0}{\rho c_\rho U_0^2} \end{array} \tag{5}$$

The governing equations on using (5) into (1) to (4) reduce to the following

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} + \theta + Nc \tag{6}$$

$$\frac{\partial c}{\partial t} = \frac{1}{Sc}\frac{\partial^2 c}{\partial y^2} - Kc \tag{7}$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{\Pr}\frac{\partial^2 \theta}{\partial y^2} - F\theta \tag{8}$$

Subject to the boundary conditions

$$\left.\begin{array}{l} t \le 0 : u = \theta = c = 0, \forall\, 0 \le y \le 1 \\ t > 0 : u = 0, \dfrac{\partial\theta}{\partial y} = -1, \dfrac{\partial c}{\partial y} = -1, at\, y = 0 \\ \qquad u = 0, \theta = 0, c = 0, at\, y = 1 \end{array}\right\} \tag{9}$$

Where Pr, Sc, K, N, and F are the Prandtl number, Schmidt number, chemical reaction parameter, modified Grashof number and radiation parameter respectively.

To obtain the solutions, we apply the Laplace transform to (6) – (9), to get

$$\frac{d^2\theta}{dy^2} - \Pr\left(P + F\right)\theta = 0 \tag{10}$$

$$\frac{d^2 C}{dy^2} - Sc\left(P + K\right)C = 0 \tag{11}$$

$$\frac{d^2 U}{dy^2} - PU = -\theta - NC \tag{12}$$

The boundary conditions are:

$$\left.\begin{aligned} & t \le 0 : U = \theta = C = 0, \forall 0 \le y \le 1 \\ & t > 0 : U = 0, \frac{d\theta}{dy} = -\frac{1}{P}, \frac{dC}{dy} = -\frac{1}{P}, at\, y = 0 \\ & U = 0, \theta = 0, C = 0, at\, y = 1 \end{aligned}\right\} \quad (13)$$

The solutions to (10) - (12) subject to (13) give

$$\theta(y,p) = \frac{1}{\sqrt{\Pr}} \sum_{n=0}^{\infty} (-1)^n \left\{ \frac{e^{-a\sqrt{Z}}}{(Z-F)\sqrt{Z}} - \frac{e^{-b\sqrt{Z}}}{(Z-F)\sqrt{Z}} \right\} \quad (14)$$

$$C(y,p) = \frac{1}{\sqrt{Sc}} \sum_{m=0}^{\infty} (-1)^m \left\{ \frac{e^{-a_1\sqrt{Z}}}{(Z-K)\sqrt{Z}} - \frac{e^{-b_1\sqrt{Z}}}{(Z-K)\sqrt{Z}} \right\} \quad (15)$$

$$U(y,p) = \frac{1}{(\Pr-1)\sqrt{\Pr}\sqrt{P+Fg}} \ldots \times \sum_{n=0}^{\infty} (-1)^n \left\{ \frac{1}{P-g} - \frac{1}{P} \right\} \Big[(e^{-b\sqrt{P+F}} - e^{-a\sqrt{P+F}}) + \sum_{h=0}^{\infty} \left(e^{-d} - e^{-c} + e^{-e} - e^{-f} \right) \Big]$$

$$+ \frac{N}{(Sc-1)\sqrt{Sc}\sqrt{P+Kl}} \sum_{m=0}^{\infty} (-1)^m \left\{ \frac{1}{P-l} - \frac{1}{P} \right\} \Big[(e^{-b_1\sqrt{P+K}} - e^{-a_1\sqrt{P+K}}) + \sum_{h=0}^{\infty} \left(e^{-d_1} - e^{-c_1} + e^{-e_1} - e^{-f_1} \right) \Big] \quad (16)$$

The Laplace inversion of (14), (15) and (16) yield the following

$$\theta(y,t) = \frac{1}{\sqrt{\Pr}} \sum_{n=0}^{\infty} (-1)^n \left[\frac{e^{Ft}}{2\sqrt{F}} \left\{ e^{-a\sqrt{F}} erfc\left(\frac{a}{2\sqrt{t}} - \sqrt{Ft} \right) + e^{a\sqrt{F}} erfc\left(\frac{a}{2\sqrt{t}} + \sqrt{Ft} \right) \right\} - \frac{e^{-\frac{a^2}{4t}}}{F\sqrt{\pi t}} - \frac{e^{Ft}}{2\sqrt{F}} e^{-b\sqrt{F}} erfc \left(\frac{b}{2\sqrt{t}} - \sqrt{Ft} \right) + e^{b\sqrt{F}} erfc\left(\frac{b}{2\sqrt{t}} + \sqrt{Ft} \right) + \frac{e^{-\frac{b^2}{4t}}}{F\sqrt{\pi t}} \right] \quad (17)$$

$$c(y,t) = \frac{1}{\sqrt{Sc}} \sum_{m=0}^{\infty} (-1)^m \left[\frac{e^{Kt}}{2\sqrt{K}} \left\{ e^{-a_1\sqrt{K}} erfc\left(\frac{a_1}{2\sqrt{t}} - \sqrt{Kt} \right) + e^{a_1\sqrt{K}} erfc\left(\frac{a_1}{2\sqrt{t}} + \sqrt{Kt} \right) \right\} - \frac{e^{-\frac{a_1^2}{4t}}}{K\sqrt{\pi t}} - \frac{e^{Kt}}{2\sqrt{K}} \left\{ e^{-b_1\sqrt{K}} \ldots \times erfc\left(\frac{b_1}{2\sqrt{t}} - \sqrt{Kt} \right) + e^{b_1\sqrt{K}} erfc\left(\frac{b_1}{2\sqrt{t}} + \sqrt{Kt} \right) \right\} + \frac{e^{-\frac{b_1^2}{4t}}}{K\sqrt{\pi t}} \right] \quad (18)$$

$$u(y,t) = \frac{1}{(\Pr-1)\sqrt{\Pr g}} \sum_{n=0}^{\infty} (-1)^n \Bigg[\Bigg[\left\{ \frac{e^{-b\sqrt{g+F}} e^{gt}}{\sqrt{g+F}} - \frac{e^{-b\sqrt{F}}}{\sqrt{F}} + \frac{e^{-a\sqrt{F}}}{\sqrt{F}} - \frac{e^{-a\sqrt{g+F}} e^{gt}}{\sqrt{g+F}} \right\} + \sum_{h=0}^{\infty} \Bigg\{ \frac{e^{gt}}{\sqrt{g+F}} \Big(e^{-\{v_1\sqrt{g}+v_2\sqrt{g+F}\}}$$

$$-e^{-\{v_3\sqrt{g}+v_4\sqrt{g+F}\}} - e^{-\{v_3\sqrt{g}+v_4\sqrt{g+F}\}} - e^{-\{v_7\sqrt{g}+v_8\sqrt{g+F}\}} + e^{-\{v_5\sqrt{g}+v_6\sqrt{g+F}\}} \Big) - \frac{e^{-Ft}}{(F+g)} \Big(e^{-\{v_1\sqrt{-F}\}} - e^{-\{v_3\sqrt{-F}\}}$$

$$+ e^{-\{v_5\sqrt{-F}\}} - e^{-\{v_7\sqrt{-F}\}} \Big) - \frac{1}{\sqrt{F}} \Big(e^{-\{v_2\sqrt{F}\}} - e^{-\{v_4\sqrt{F}\}} + e^{-\{v_6\sqrt{F}\}} - e^{-\{v_8\sqrt{F}\}} \Big) + \frac{e^{-Ft}}{F} \ldots$$

$$\times \Big(e^{-\{v_1\sqrt{-F}\}} - e^{-\{v_3\sqrt{-F}\}} + e^{-\{v_5\sqrt{-F}\}} - e^{-\{v_7\sqrt{-F}\}} \Big) \Bigg\} \Bigg] + \frac{N}{(Sc-1)\sqrt{Scl}} \ldots$$

$$\sum_{m=0}^{\infty} (-1)^m \Bigg[\Bigg[\left\{ \frac{e^{-b_1\sqrt{l+K}} e^{lt}}{\sqrt{l+K}} \frac{e^{-b_1\sqrt{K}}}{\sqrt{K}} + \frac{e^{-a_1\sqrt{K}}}{\sqrt{K}} - \frac{e^{-a_1\sqrt{l+K}} e^{lt}}{\sqrt{l+K}} \right\}$$

$$+\sum_{h=0}^{\infty}\left\{\frac{e^{lt}}{\sqrt{l+K}}\left(e^{-\left\{v_9\sqrt{l}+v_{10}\sqrt{l+K}\right\}}-e^{-\left\{v_{11}\sqrt{l}+v_{12}\sqrt{l+K}\right\}}+\right.\right.$$
$$\left.e^{-\left\{v_{13}\sqrt{l}+v_{14}\sqrt{l+K}\right\}}-e^{-\left\{v_{15}\sqrt{l}+v_{16}\sqrt{l+K}\right\}}\right)$$
$$-\frac{e^{-Kt}}{(K+l)}\left(e^{-\left\{v_9\sqrt{-K}\right\}}-e^{-\left\{v_{11}\sqrt{-K}\right\}}+e^{-\left\{v_{13}\sqrt{-K}\right\}}\right.$$
$$\left.-e^{-\left\{v_{15}\sqrt{-K}\right\}}\right)-\frac{1}{\sqrt{K}}.\left(e^{-\left\{v_{10}\sqrt{K}\right\}}-e^{-\left\{v_{12}\sqrt{K}\right\}}\right.$$
$$\left.+e^{-\left\{v_{14}\sqrt{K}\right\}}-e^{-\left\{v_{16}\sqrt{K}\right\}}\right)+\frac{e^{-Kt}}{K}\left(e^{-\left\{v_9\sqrt{-K}\right\}}\right.$$
$$\left.\left.\left.-e^{-\left\{v_{11}\sqrt{-K}\right\}}+e^{-\left\{v_{13}\sqrt{-K}\right\}}-e^{-\left\{v_{15}\sqrt{-K}\right\}}\right)\right\}\right] \quad (19)$$

Where

$a=(2n+y)\sqrt{\Pr};$

$b=(2n+2-y)\sqrt{\Pr};$

$c=(2h+y)\sqrt{P}+(2n+2)\sqrt{\Pr}\sqrt{P+F};$

$d=(2h+2-y)\sqrt{P}+(2n+2)\sqrt{\Pr}\sqrt{P+F};$

$e=(2h+y)\sqrt{P}+(2n)\sqrt{\Pr}\sqrt{P+F};$

$f=(2h+2-y)\sqrt{P}+(2n)\sqrt{\Pr}\sqrt{P+F};$

$a_1=(2m+y)\sqrt{Sc};$

$b_1=(2m+2-y)\sqrt{Sc};$

$c_1=(2h+y)\sqrt{P}+(2m+2)\sqrt{Sc}\sqrt{P+K};$

$d_1=(2h+2-y)\sqrt{P}+(2m+2)\sqrt{Sc}\sqrt{P+K};$

$e_1=(2h+y)\sqrt{P}+(2m)\sqrt{Sc}\sqrt{P+K};$

$f_1=(2h+2-y)\sqrt{P}+(2m)\sqrt{Sc}\sqrt{P+K};$

The Skin friction from the velocity is given by

$$\left.\frac{\partial u}{\partial y}\right|_{y=0}=\frac{1}{(\Pr-1)\sqrt{\Pr g}}\sum_{n=0}^{\infty}(-1)^n\left[\sqrt{\Pr}\left\{e^{-(2n+2)\sqrt{\Pr}\sqrt{g+F}+gt}\right.\right.$$
$$\left.-e^{-(2n+2)\sqrt{\Pr}\sqrt{F}}-e^{-(2n)\sqrt{\Pr}\sqrt{F}}+e^{-(2n)\sqrt{\Pr}\sqrt{g+F}+gt}\right\}$$
$$+\sum_{h=0}^{\infty}\left\{\frac{2\sqrt{P}\sqrt{-F}e^{Ft}}{(F+g)}\left(e^{-\left\{(2h)\sqrt{P}\sqrt{-F}\right\}}+e^{-\left\{(2h+2)\sqrt{P}\sqrt{-F}\right\}}\right)\right.$$
$$\left.\left.-\frac{2\sqrt{P}\sqrt{-F}e^{-Ft}}{F}\left(e^{-\left\{(2h)\sqrt{P}\sqrt{-F}\right\}}+e^{-\left\{(2h+2)\sqrt{P}\sqrt{-F}\right\}}\right)\right\}\right]$$
$$+\frac{N}{(Sc-1)\sqrt{Scl}}\sum_{m=0}^{\infty}(-1)^m\left[\sqrt{Sc}\left\{e^{-(2m+2)\sqrt{Sc}\sqrt{l+K}+lt}\right.\right.$$
$$\left.-e^{-(2m+2)\sqrt{Sc}\sqrt{K}}-e^{-(2m)\sqrt{Sc}\sqrt{K}}+e^{-(2m)\sqrt{Sc}\sqrt{l+K}+lt}\right\}$$
$$+\sum_{h=0}^{\infty}\left\{\frac{2\sqrt{P}\sqrt{-K}e^{Kt}}{(K+l)}\left(e^{-\left\{(2h)\sqrt{P}\sqrt{-K}\right\}}+e^{-\left\{(2h+2)\sqrt{P}\sqrt{-K}\right\}}\right)\right.$$
$$-\frac{2\sqrt{P}\sqrt{-K}e^{Kt}}{K}\left(e^{-\left\{(2h)\sqrt{P}\sqrt{-K}\right\}}\right.$$
$$\left.\left.\left.+e^{-\left\{(2h+2)\sqrt{P}\sqrt{-K}\right\}}\right)\right\}\right] \quad (20)$$

Similarly, the Nusselt number becomes

$$\left.\frac{\partial\theta}{\partial y}\right|_{y=0}=\frac{1}{\sqrt{\Pr}}\sum_{n=0}^{\infty}(-1)^n\left[\frac{e^{Ft}}{2\sqrt{F}}\left\{\frac{\sqrt{\Pr}}{2\sqrt{t}}erfc\ldots\right.\right.$$
$$\times\left(\frac{n\sqrt{\Pr}}{\sqrt{t}}-\sqrt{Ft}\right)e^{-(2n)\sqrt{\Pr}\sqrt{F}}-\sqrt{\Pr F}e^{-(2n)\sqrt{\Pr}\sqrt{F}}\times$$
$$erfc\left(\frac{n\sqrt{\Pr}}{\sqrt{t}}-\sqrt{Ft}\right)+\sqrt{\Pr F}e^{(2n)\sqrt{\Pr}\sqrt{F}}erfc\left(\frac{n\sqrt{\Pr}}{\sqrt{t}}+\sqrt{Ft}\right)$$
$$\left.+\frac{\sqrt{\Pr}}{2\sqrt{t}}erfc\left(\frac{n\sqrt{\Pr}}{\sqrt{t}}+\sqrt{Ft}\right)e^{(2n)\sqrt{\Pr}\sqrt{F}}\right\}$$

$$+\frac{\Pr(n)}{Ft\sqrt{\pi t}}e^{-\frac{n^2\Pr}{t}}-\frac{e^{Ft}}{2\sqrt{F}}\Big\{\sqrt{\Pr F}e^{-(2n+2)\sqrt{\Pr}\sqrt{F}}\ldots$$

$$\times erfc\left(\frac{(n+1)\sqrt{\Pr}}{\sqrt{t}}-\sqrt{Ft}\right)-\frac{\sqrt{\Pr}}{2\sqrt{t}}erfc\left(\frac{(n+1)\sqrt{\Pr}}{\sqrt{t}}-\sqrt{Ft}\right)$$

$$e^{-(2n+2)\sqrt{\Pr}\sqrt{F}}-\sqrt{\Pr F}e^{(2n+2)\sqrt{\Pr}\sqrt{F}}erfc\Big(\frac{(n+1)\sqrt{\Pr}}{\sqrt{t}}$$

$$+\sqrt{Ft}\Big)-\frac{\sqrt{\Pr}}{2\sqrt{t}}erfc\left(\frac{(n+1)\sqrt{\Pr}}{\sqrt{t}}+\sqrt{Ft}\right)\ldots e^{(2n+2)\sqrt{\Pr}\sqrt{F}}\Big\}$$

$$+\frac{\Pr(n+1)}{Ft\sqrt{\pi t}}e^{-\frac{\left[(2n+2)\sqrt{\Pr}\right]^2}{4t}}\Bigg] \qquad (21)$$

While the Sherwood number is

$$\frac{\partial c}{\partial y}=\frac{1}{\sqrt{Sc}}\sum_{m=0}^{\infty}(-1)^m\Bigg[\frac{e^{kt}}{2\sqrt{K}}\Big\{\frac{\sqrt{Sc}}{2\sqrt{t}}erfc\left(\frac{(2m+y)\sqrt{Sc}}{2\sqrt{t}}-\sqrt{Kt}\right)$$

$$e^{-(2m+y)\sqrt{Sc}\sqrt{K}}-\sqrt{ScK}\times e^{-(2m+y)\sqrt{Sc}\sqrt{K}}$$

$$erfc\left(\frac{(2m+y)\sqrt{Sc}}{2\sqrt{t}}-\sqrt{Kt}\right)+\sqrt{ScK}e^{(2m+y)\sqrt{Sc}\sqrt{K}}\ldots$$

$$\times erfc\left(\frac{(2m+y)\sqrt{Sc}}{2\sqrt{t}}+\sqrt{Kt}\right)$$

$$+\frac{\sqrt{Sc}}{2\sqrt{t}}erfc\left(\frac{(2m+y)\sqrt{Sc}}{2\sqrt{t}}+\sqrt{Kt}\right)e^{(2m+y)\sqrt{Sc}\sqrt{K}}\Big\}$$

$$+\frac{Sc(2m+y)}{2tK\sqrt{\pi t}}e^{-\frac{\left[(2m+y)\sqrt{Sc}\right]^2}{4t}}-\frac{e^{Kt}}{2\sqrt{K}}\times$$

$$\Big\{\sqrt{ScK}e^{-(2m+2-y)\sqrt{Sc}\sqrt{K}}erfc\left(\frac{(2m+2-y)\sqrt{Sc}}{2\sqrt{t}}-\sqrt{Kt}\right)$$

$$-\frac{\sqrt{Sc}}{2\sqrt{t}}erfc\left(\frac{(2m+2-y)\sqrt{Sc}}{2\sqrt{t}}-\sqrt{Kt}\right)\times$$

$$e^{-(2m+2-y)\sqrt{Sc}\sqrt{K}}-\sqrt{ScK}e^{(2m+2-y)\sqrt{Sc}\sqrt{K}}\ldots$$

$$\times erfc\left(\frac{(2m+2-y)\sqrt{Sc}}{2\sqrt{t}}+\sqrt{K}\right)-\frac{\sqrt{Sc}}{2\sqrt{t}}erfc\times$$

$$\left(\frac{(2m+2-y)\sqrt{Sc}}{2\sqrt{t}}+\sqrt{Kt}\right)e^{(2m+2-y)\sqrt{Sc}\sqrt{K}}\Big\}$$

$$+\frac{Sc(2m+2-y)}{2tK\sqrt{\pi t}}e^{-\frac{\left[(2m+2-y)\sqrt{Sc}\right]^2}{4t}}\Bigg] \qquad (22)$$

3. RESULTS AND DISCUSSION

From the analytical expressions for the velocity, temperature and concentration, the computations was carried out for different values of the Prandtl number Pr, Schmidt number Sc, radiation parameter F, modified Grashof number N and chemical reaction parameter K. We fixed the following parameters values throughout the calculations, Pr = 0.71, Sc = 0.2, N = 0.1, F = 0.14, K = 0.14, t = 5, y = 0:0.001:1

Figures1 to 8 are the graphs for the analytical solutions.

The temperature profiles are illustrated in Figures 1 to 3 for different values of Prandtl number (Pr = 0.60, 0.71, 0.85,1), radiation parameter (F = 0.014, 0.024, 0.034, 0.044) and time (t = 15, 20, 25, 30) shown in Figures 1, 2 and 3 respectively. In Figure 1, we observed that the temperature increases with increase in the Prandtl number. Figure 2 shows that the temperature increase with increase of the radiation parameter and similarly for the time, Figure 3 reveals that the temperature increase with respect to increase in time.

The concentration profiles are presented in Figures 4 to 6 for different values of Schmidt number (Sc = 0.2, 0.4, 0.6, 0.8), chemical reaction parameter (K = 0.06, 0.08, 0.10,0.12) and time (t = 5, 10, 15, 20). In Figure 4, we noticed that the concentration decreases with increase in the Schmidt number, and Figure 5 shows that the concentration increases as the chemical reaction parameter increase while increase in time decrease the concentration as shown in Figure 6.

The velocity profiles are illustrated in Figures 7 and 8 for different values of Schmidt number (Sc = 0.01, 0.02, 0.03, 0.04) and modified Grashof number (N = 0.200, 0.201, 0.202, 0.203) shown in Figures 7, 8 and 9 respectively. In Figure 7, we observed that the velocity increases with increase in the Schmidt number and Figure 8 shows that the velocity increase with increase of the modified Grashof number.

Tables 1 to 5 are the tables for Skin friction, Nusselt number and Sherwood number for the analytical solution.

The Skin frictions are illustrated in Tables 1 to 3 for different values of modified Grashof number (N = 0.1, 0.11, 0.21) and time (t = 5, 10, 15), Prandtl number (Pr = 0.60, 0.71, 0.85) and radiation parameter (F = 0.02, 0.04, 0.06), and Schmidt number (Sc = 0.2, 0.4, 0.6) and chemical reaction (K = 1.0, 1.1, 1.2) shown in Tables 1, 2 and 3 respectively. In Table 1, we observed that the Skin friction is decreasing with increasing modified Grashof number while increase in time leads to decrease in the Skin friction. Table 2 shows that as the Prandtl number increase, the Skin friction also increase. Similarly, the Skin friction increase with increase in the radiation parameter. Further more, Table 3 indicates that the Skin friction decreases whenever the Schmidt number increases while increase in the chemical reaction parameter results to decrease in the Skin friction.

Table 4 shows that increasing Prandtl number results to decrease of the Nusselt number and similarly, increasing radiation parameter leads to decrease of the Nusselt number. In Table 5, we observed that increase in Schmidt number and chemical reactions parameter causes decrease of the Sherwood number.

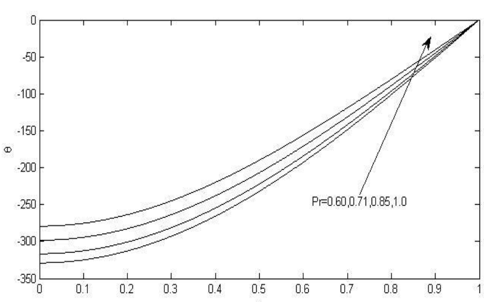

Figure 1: Variation of Temperature against y for different values of Pr.

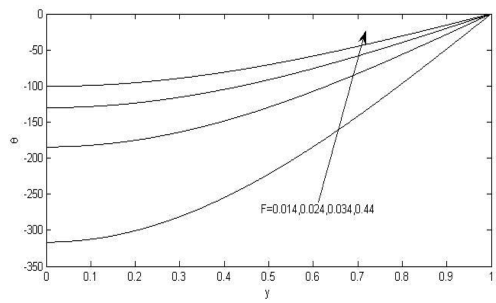

Figure 2: Variation of Temperature against y for different values of F.

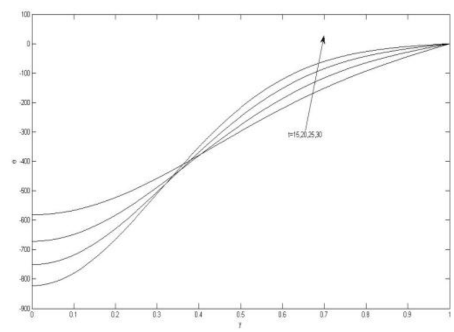

Figure 3: Variation of Temperature against y for different values of t.

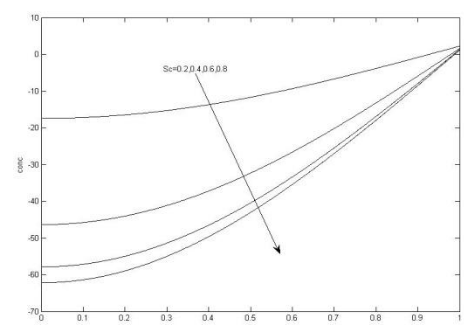

Figure 4: Variation of Concentration against y for different values of Sc.

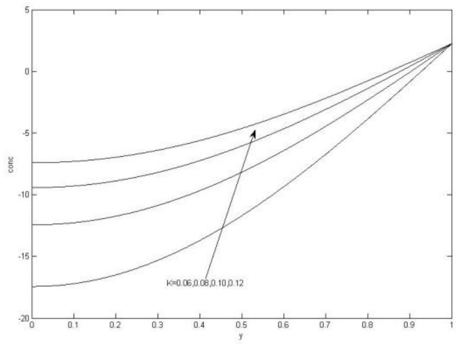

Figure 5: Variation of Concentration against y for different values of K.

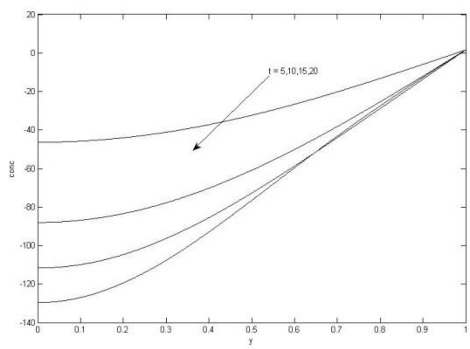

Figure 6: Variation of Concentration against y for different values of t.

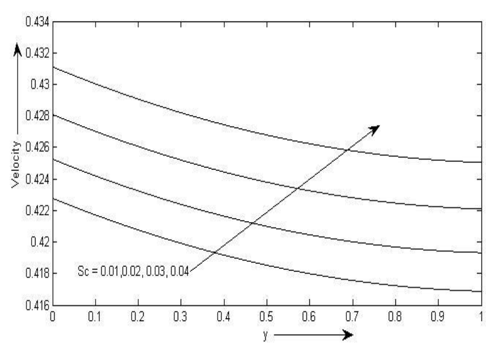

Figure 7: Variation of Velocity against y for different values of Sc.

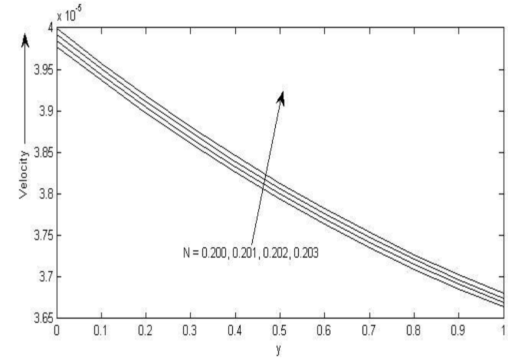

Figure 8: Variation of Velocity against y for different values of N.

N	0.1	0.11	0.21
a SF			
5	-14.9747	-15.0113	-15.3778
10	-156.6063	-156.7049	-157.6909
15	-378.1750	-378.3859	-380.4949

Table 1: Skin Friction for different values of N and a.

Pr	0.60	0.71	0.85
F SF			
0.02	6.5533	21.0164	131.9703
0.04	8.8542	31.3745	212.1228
0.06	7.4906	33.0496	251.5763

Table 2: Skin Friction for different values of Pr and F.

Sc	0.2	0.4	0.6
K SF			
1.0	-14.6135	-14.6283	-14.6636
1.1	-16.0222	-19.9407	-29.3027
1.2	-23.9491	-49.8349	-111.6810

Table 3: Skin Friction for different values of Sc and K.

Pr	0.60	0.71	0.85
F / Nu			
0.02	7.0539	5.1535	5.1396
0.04	3.5269	2.5767	2.5598
0.06	2.3513	1.7178	1.6932

Table 4: Nusselt number for different values of Pr and F.

Sc	0.2	0.4	0.6
K / Sh			
1.0	8.5083	2.1525	1.4108
1.1	7.7349	1.9568	1..2825
1.2	7.3621	1.7937	1.1756

Table 5: Sherwood number for different values of Sc and K.

4. CONCLUSION

This paper study the Transient convection fluid flow with heat and mass flux in a fixed vertical plate with radiation. The dimensionless governing equations were solved using the Laplace transform method. The effect of different parameters such as the Prandtl number, Schmidt number, radiation parameter and chemical reaction parameter were studied. The conclusion follows that

- The temperature increase with increasing Prandtl number, radiation parameter and time.
- The concentration increase with increase of the chemical reaction.
- The concentration decreases with increasing Schmidt number and time.
- The velocity increases with increasing Schmidt number, modified Grashof number and time.

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