Radiation-Convection Flow in Porous Medium with Chemical Reaction

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ABSTRACT
We study magnetohydrodynamic radiation-convection flow through a porous medium with mass transfer in the presence of a homogeneous first order chemical reaction and surface temperature oscillation. The governing equations are solved analytically using perturbation technique. Detailed computations of the influence of Hartmann number, frequency of the oscillation, radiation parameter, Prandtl number, Chemical reaction parameter, Grashof number, mass Grashof number and Schmidt number are discussed.

Keywords: Radiation-Convection, Porous Medium, Chemical Reaction

1. INTRODUCTION
Radiative transport in porous media has important engineering application in combustion heat exchangers for high temperature applications, including solar collectors, regenerators and recuperates, insulation systems, packed and circulating bed combustors and reactors manufacturing. Moreover, considerable interest has been shown in radiation with convection for heat and mass transfer in fluids. This is due to the significant role of thermal radiation in the surface heat transfer when convection heat transfer is small, particularly in free convection problems involving absorbing emitting fluids, Mohammed (2009). In convection heat and mass transfer process, it is known Muthucumaraswamy *et al* (2006) that diffusion is either heterogeneous if it takes place at the surface or homogeneous if in the solution. Fundamental studies of unsteady magneto hydrodynamic (MHD) flows continue to attract the attention of engineering science and applied mathematics researchers owing to extensive applications of such flow in the context of ionized aerodynamics, Soundalgekar and Takhar, (1977). Larson and Viskanta (1976), Helliwell and Mosa (1979), Alam *et al* (2006), Zueco (2007), Samad and Rahman (2006), and El-Hakiem (2000) have presented some works on thermal radiation with different conditions. The problem of radiation with convection flow through porous medium with different conditions can be found in Mohammed (2009), Mansour and Aly (2009), Ghosh and Pop (2007), and Larson and Viskanta (1976).
Recently Beg and Ghosh (2010) obtained analytical solution of MHD radiation convection with surface temperature oscillation and secondary flow effects. Chauhan and Rastogi (2010) studied radiation effects on natural convection MHD flow in a rotating vertical porous channel partially filled with a porous medium. Seddeek and Almushigeh (2010) investigated effects of radiation and variable viscosity on MHD free convective flow and mass transfer over a stretching sheet with chemical reaction. Muthucumaraswamy (2010) examined chemical reaction effects on vertical oscillating plate with variable temperature.

The aim of this paper is to investigate MHD radiation-convection flow through a porous medium with chemical reaction and surface temperature oscillation, and to discuss the effect of the parameters of the flow, and compute the skin frictions at the wall, rate of heat and mass transfer.

2. MATHEMATICAL ANALYSIS
Consider a two-dimensional unsteady MHD flow of a viscous, incompressible, electrically-conducting fluid in a channel filled with porous medium occupying a semi-infinite region of Space bounded by an infinite vertical plate moving with constant velocity, U, in the presence of a transverse magnetic field and uniform mass diffusion. It is assumed that the effect of viscous dissipation is negligible in the energy equation and there is a first order chemical reaction between the diffusing species and the fluid. The surface temperature and concentration of the plate oscillates with small amplitude about a non-uniform mean temperature and concentration. The co-ordinate system is such that the x-axis is taken along the plate and y-axis is normal to the plate. A uniform transverse magnetic field, B_o, is imposed parallel to y-direction. All the fluid properties are considered constant except the influence of the density variation in the buoyancy term, according to the classical Boussinesq approximation. The radiation heat flux in the x-direction is considered negligible in comparison to the y-direction. Then by usual Boussinesq approximation, the unsteady flow is governed by the following equations.

$$\frac{\partial u'}{\partial t'} = \nu \frac{\partial^2 u'}{\partial y'^2} + g\beta\left(T' - T'_\infty\right) + g\beta^*\left(C' - C'_\infty\right) - \frac{\nu}{K'}u' - \frac{\sigma B_0^2}{\rho}u' \tag{1}$$

$$\frac{\partial T'}{\partial t'} = \frac{k}{\rho C_p}\frac{\partial^2 T'}{\partial y'^2} - \frac{1}{\rho C_p}\frac{\partial q_r}{\partial y'} \tag{2}$$

$$\frac{\partial C'}{\partial t'} = D_1 \frac{\partial^2 C'}{\partial y'^2} - K^* C \tag{3}$$

With boundary conditions

$$u' = U, T' - T'_{\infty} = \theta_w(x)[1+\varepsilon e^{i\omega' t'}],$$
$$C' - C'_{\infty} = C_w[1+\varepsilon e^{i\omega' t'}], \qquad at \; y' = 0 \qquad (4)$$
$$u' \to \infty, T' \to \infty, C' \to \infty, \quad as \qquad y' \to \infty$$

Where U', ν, g, β, β^*, T', k, ρ, C_p, D_1, σ, B_o, K' C', K^*, u', ε, ω' and t' are stream velocity, kinematic viscosity coefficient, gravitational force, coefficient of volume expansion due to temperature, coefficient of volume expansion due to concentration, dimensional temperature, thermal conductivity, fluid density, specific heat at constant pressure, coefficient of diffusion, electrical conductivity, externally imposed magnetic field in the y-direction, dimensional concentration, dimensional chemical reaction parameter, axial velocity, a small parameter, dimensional frequency of the oscillation, and dimensional time.

For the case of an optically-thin gray gas, the thermal radiation flux gradient may be expressed as follows,

$$-\frac{\partial q_r}{\partial y'} = 4a\sigma^*\left(T_{\infty}'^4 - T'^4\right) \qquad (5)$$

Where q_r is the radiative heat flux, a is the absorption coefficient of the fluid, and σ^* is the Stefan-Boltzmann constant.

We assume that the temperature differences within the flow are sufficiently small such that T'^4 may be expressed as a linear function of the temperature. Expanding T'^4 using Taylor series about T'_{∞} and neglecting higher order terms, we have

$$\therefore T'^4 = 4T_{\infty}'^3 T' - 3T_{\infty}'^4 \qquad (6)$$

From (5) and (6) then (2) becomes

$$\frac{\partial T'}{\partial t'} = \frac{k}{\rho C_p}\frac{\partial^2 T'}{\partial y'^2} + \frac{16a\sigma^* T_{\infty}'^3(T_{\infty}' - T')}{\rho C_p} \qquad (7)$$

The following dimensionless variables and parameters of the problem are

$$\left.\begin{aligned}
&u_0 = \frac{u'_0}{U}, u_1 = \frac{u'_1 e}{U}, y = \frac{y'U}{\nu}, t = \frac{t'U^2}{\nu}, Da = \frac{K'}{U^2},\\
&Pr = \frac{\nu\rho C_p}{k}, K_1 = \frac{16a\sigma^*\nu^2 L T_{\infty}'^3}{kU^2}, \theta'_0 = \frac{\theta_0 \nu}{UL},\\
&C'_0 = \frac{C_0\nu}{UL}, \theta'_1 = \frac{\theta_1\nu}{ULe}, C'_1 = \frac{C_1\nu}{ULe}, \omega = \frac{\omega'\nu}{U^2},\\
&C_0 = C_1 = \frac{C' - C'_{\infty}}{C'_w - C'_{\infty}}, Gr = \frac{g\beta\nu^2\theta_w(x)}{U^4 L},\\
&Gc = \frac{g\beta^*\nu^2 C_w(x)}{U^4 L}, M^2 = \frac{\sigma B_0^2\nu}{\rho U^2},\\
&K = \frac{K^*\nu^2}{D_1 U^2}, \theta_0 = \theta_1 = \frac{T' - T'_{\infty}}{T'_w - T'_{\infty}}
\end{aligned}\right\} \qquad (8)$$

Using (8) into (1) to (4) yield the following

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} - (S^2 + M^2)u + Gr\theta + GcC \qquad (9)$$

(5)

$$\frac{\partial C}{\partial t} = \frac{1}{Sc}\frac{\partial^2 C}{\partial y^2} - KC \qquad (10)$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{\mathrm{Pr}}\frac{\partial^2 C}{\partial y^2} - \frac{K_1}{\mathrm{Pr}}\theta \qquad (11)$$

Where M, Gr, Gc, K, K_1, Sc, Pr and S are Hartmann number, the Grashof number, mass Grashof number, chemical reaction parameter, radiation-conduction parameter, Schimdt number, Prandtl number and porous medium shape factor respectively.

With the following boundary conditions

$$\left.\begin{aligned}
&u = U, \theta = 1 + \varepsilon e^{i\omega t}, C = 1 + \varepsilon e^{i\omega t} \; at \; y = 0\\
&u \to \infty, \theta \to \infty, C \to \infty \; as \; y \to \infty
\end{aligned}\right\}$$

(12)

3. METHOD OF SOLUTION

Assuming the solutions to equations (9) to (12) as follows

$$\left.\begin{aligned}
u(y,t) &= u_0(y) + \varepsilon u_1(y)e^{i\omega t}\\
\theta(y,t) &= \theta_0(y) + \varepsilon\theta_1(y)e^{i\omega t}\\
C(y,t) &= C_0(y) + \varepsilon C_1(y)e^{i\omega t}
\end{aligned}\right\} \qquad (13)$$

Where ω is the frequency of the oscillation, t is the time and u_o, u_1, θ_o, θ_1, C_o and C_1 are to be determined. Substituting (13) into (9) to (12), lead to the following

$$\frac{d^2 u_0}{dy^2} - \left(S^2 + M^2\right) = -Gr\theta_0 - GcC_0 \qquad (14)$$

$$\frac{d^2\theta_0}{dy^2} - K_1\theta_0 = 0 \qquad (15)$$

$$\frac{d^2C_0}{dy^2} - KC_0 = 0 \tag{16}$$

The imposed boundary conditions become

$$\left.\begin{aligned} &u_0 = 1, \theta_0 = 1, C_0 = 1 && at\ y = 0 \\ &u_0 = 0, \theta_0 = 0, C_0 = 0 && as\ y \to \infty \end{aligned}\right\} \tag{17}$$

and

$$\frac{d^2u_1}{dy^2} - \left(S^2 + M^2 + i\omega\right)u_1 = -Gr\theta_1 - GcC_1 \tag{18}$$

$$\frac{d^2\theta_1}{dy^2} - i\omega \Pr \theta_1 = 0 \tag{19}$$

$$\frac{d^2C_1}{dy^2} - \left(K + i\omega Sc\right)C_1 = 0 \tag{20}$$

and the boundary conditions are

$$\left.\begin{aligned} &u_1 = 1, \theta_1 = 1, C_1 = 1 && at\ y = 0 \\ &u_1 = 0, \theta_1 = 0, C_1 = 0 && as\ y \to \infty \end{aligned}\right\} \tag{21}$$

Equations (14) to (21) are solved and the solutions for temperature, concentration and velocity are given as follows.

$$\theta\left(y\right) = e^{-\sqrt{k_1}y} + \varepsilon\left(e^{-\sqrt{i\omega \Pr}y}\right)e^{i\omega t} \tag{22}$$

$$C\left(y\right) = e^{-\sqrt{K}y} + \varepsilon\left(e^{-\sqrt{K+i\omega Sc}y}\right)e^{i\omega t} \tag{23}$$

$$\begin{aligned} u\left(y\right) &= e^{-\sqrt{\alpha}y} + \frac{Gr}{k_1 - \alpha}\left(e^{\sqrt{\alpha}y} - e^{\sqrt{k_1}y}\right) \\ &+ \frac{Gc}{K - \alpha}\left(e^{-\sqrt{\alpha}y} - e^{-\sqrt{K}y}\right) \dots + \\ &\dots + \varepsilon\left(e^{-\sqrt{\alpha}y} + \frac{Gr}{l-\alpha}\left(e^{-\sqrt{\alpha}y} - e^{-\sqrt{l}y}\right)\right. \\ &\left. + \frac{Gc}{\xi - \alpha}\left(e^{-\sqrt{\alpha}y} - e^{-\sqrt{\xi}y}\right)\right)e^{i\omega t} \end{aligned} \tag{24}$$

The shear stress at the lower wall of the channel is given as

$$\begin{aligned} \left.\frac{\partial u}{\partial y}\right|_{y=0} &= -\sqrt{\alpha} + \frac{Gr}{k_1 - \alpha}\left(-\sqrt{\alpha} + \sqrt{k_1}\right) + \frac{Gr}{K - \alpha} \dots \times \\ &\left(-\sqrt{\alpha} + \sqrt{K}\right) + \varepsilon\begin{pmatrix} -\sqrt{\alpha} + \frac{Gr}{l - \alpha}\left(-\sqrt{\alpha} + \sqrt{l}\right) \\ + \frac{Gc}{\xi - \alpha}\left(-\sqrt{\alpha} + \sqrt{\xi}\right) \end{pmatrix} e^{i\omega t} \end{aligned} \tag{25}$$

The rate of heat transfer across the channel is given as

$$\left.\frac{\partial \theta}{\partial y}\right|_{y=0} = -\sqrt{k_1} + \varepsilon\left(-\sqrt{i\omega \Pr}\right)e^{i\omega t} \tag{26}$$

The rate of mass transfer across the channel is given as

$$\left.\frac{\partial C}{\partial y}\right|_{y=0} = -\sqrt{K} + \varepsilon\left(-\sqrt{K + i\omega Sc}\right)e^{i\omega t} \tag{27}$$

Where $\alpha = S^2 + M^2 + i\omega, l = i\omega \Pr, \xi = K + i\omega Sc$

4. RESULTS AND DISCUSSION

Selected computation for the velocity, temperature and concentration fields have been provided in Figures 1 to 7 and made use of the following parameter values except otherwise stated, Pr = 0.71, Gr = 0.2, Gc = 0.2, Sc = 0.6, K = 0.1, K_1 = 0.1, M = 1, S = 1, t = 0.1, ω = 2,and ε = 0.2. The velocity profiles have been studied and presented in Figures 1 to 3. The velocity profiles for different values of the Hartmann number (M = 1, 2, 3, 4), Grashof number (Gr = 0.2, 0.4, 0.6, 0.8) and mass Grashof number (Gc = 1, 3, 5, 7) are shown in Figures 1, 2, and 3 respectively. It is observed that the velocity increases with increasing Hartmann number, Grashof number and mass Grashof number respectively. The temperature profiles have been studied and presented in Figures 4 to 6. The temperature profiles for different values of the radiation parameter (K_1 = 0.1, 0.2, 0.3, 0.4), Prandtl number (Pr = 0.025, 0.71, 0.85, 3) and frequency of the oscillation (ω = 0.1, 0.3, 0.6, 0.9) are shown in Figure 4, 5 and 6 respectively. It is observed that the temperature decreases with increasing radiation parameter, Prandtl number and frequency of the oscillation respectively. The concentration profiles have been studied and presented in Figure 7 .The concentration profiles for different values of the chemical reaction parameter (K = 0.9, 0.6, 0.3, 0.1) . It is observed that the concentration decreases with increasing chemical reaction parameter. It is observed that the concentration increases with increasing epsilon. The variation of surface skin friction, Nusselt number, and Sherwood numbers are presented in Table 1. It is observed that increasing Hartmann number lead to the decrease in shear stress, increasing chemical reaction parameter causes a decrease in the Sherwood number and increasing radiation parameter causes a decrease in the Nusselt number.

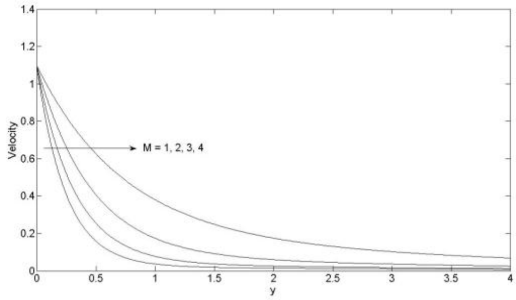

Figure 1: Velocity profiles for different values of the Hartmann number

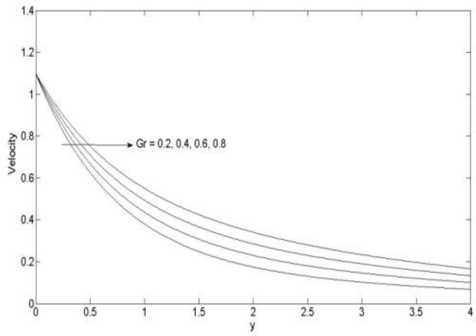

Figure 2: Velocity profiles for different values of the Grashof number

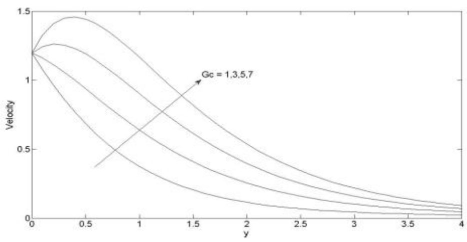

Figure 3: Velocity profiles for different values of the mass Grashof number

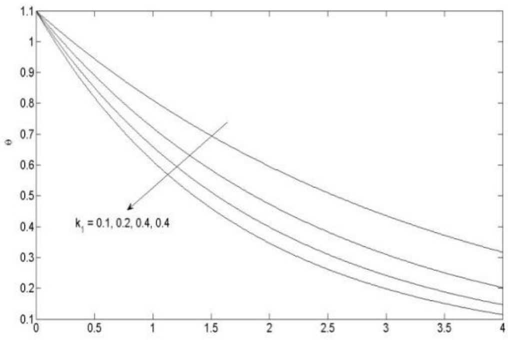

Figure 4: Temperature profiles for different values of the Radiation parameter

Figure 5: Temperature profiles for different values of the Prandtl number

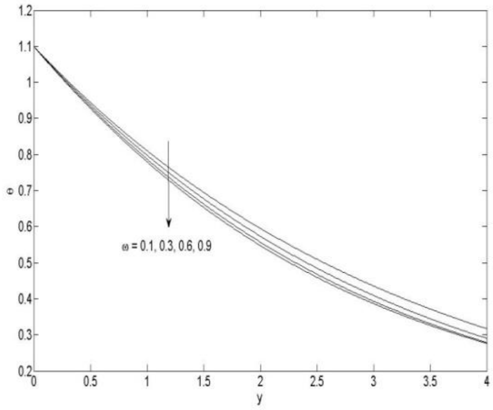

Figure 6: Temperature profiles for different values of the frequency of the oscillation

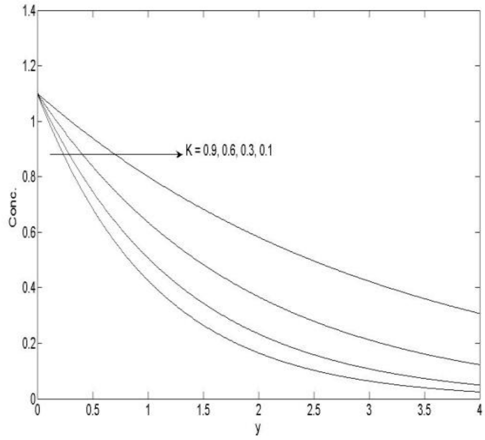

Figure 7: Concentration profiles for different values of the chemical reaction parameter

Table 1 Values of Shear Stress, Nussetl number and Sherwood number with different values of the material parameters

Pr	K	K_1	M	Sc	ω	T	τ	Nu	Sh
0.71	0.1	0.1	1.0	0.2	0.2	0.1	-1.6594	-0.4207	-0.4446
0.71	0.1	0.1	2.0	0.2	0.2	0.1	-2.9112	-0.4207	-0.4446
0.85	0.1	0.1	1.0	0.2	0.2	0.1	-1.6603	-0.4305	-0.4446
0.71	0.1	0.2	1.0	0.2	0.2	0.1	-1.6675	-0.5566	-0.4446
0.71	0.3	0.1	1.0	0.2	0.2	0.1	-1.6782	-0.4207	-0.7670
0.7	0.	0.	1.	0.	0.	0.	-	-	-

1	1	1	0	6	2	1	1.6609	0.4207	0.4580
0.71	0.1	0.1	1.0	0.2	0.3	0.1	-1.6653	-0.4428	-0.4467
0.71	0.6	0.1	1.0	0.2	0.2	0.1	-1.6929	-0.4207	-1.0843

4. CONCLUSIONS

Analytical solutions have been derived for transient MHD radiation-convection flow of an electrically-conducting, Newtonian, optically-thin fluid from a flat plate with thermal radiation and surface temperature oscillation in a porous medium with first order chemical reaction. Our analysis has shown that:

(i) It is observed that the velocity increases with increasing Hartmann number, Grashof number, or mass Grashof number

(ii) The temperature decreases with increasing radiation-conduction parameter, Prandtl number or frequency of the oscillation

(iii) The concentration decreases with increasing chemical reaction

(iv) An increase in magnetic field intensity leads to the decrease in the shear stress, while increasing radiation parameter causes a decrease in the Nussetl number and increasing chemical reaction parameter causes a decrease in the Sherwood number

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