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Strong Convergence and Stability results for Jungck-SP iterative scheme

Renu Chugh and Vivek Kumar
Department of Mathematics
Maharshi Dayanand University,
ROHTAK, INDIA

ABSTRACT
In this paper, we study strong convergence as well as stability results for a pair of nonself mappings using Jungck-SP iterative scheme and a certain contractive condition. Moreover, with the help of computer programs in C++, we show that Jungck-SP iterative scheme converges faster than Jungck-Noor, Jungck-Ishikawa and Jungck-Mann iterative schemes through example.

General Terms
Computational Mathematics
Keywords
Jungck-SP iteration, Jungck-Ishikawa iteration, Jungck-Noor iteration, Stability

1. INTRODUCTION
Most of the equations $f(x) = y$ arising in physical formulations can equivalently be transformed into a fixed point problem $x=Tx$ and then apply an approximate fixed point theorem to get information on the existence or existence and uniqueness of fixed point, that is, of the solution of the original equation.

Let (X, d) be a complete metric space and $T : X \longrightarrow X$ a selfmap of X. Suppose that $F(T) = \{ p \in X, Tp = p\}$ is the set of fixed points of T. There are several iterative processes in the literature for which the fixed points of operators have been approximated over the years by various authors. In a complete metric space, the Picard iterative process $\{x_n\}_{n=0}^{\infty}$ defined by

$$x_{n+1} = Tx_n \quad n = 0, 1, \ldots \tag{1.1}$$

is used to approximate the fixed points of mappings satisfying the inequality

$$d(Tx, Ty) \le \alpha\, d(x, y) \tag{1.2}$$

for all $x, y \in X$ and $\alpha \in [0, 1)$.
Condition (1.2) is called the Banach's contraction condition.
In 1953, W.R. Mann defined the Mann iteration [9] as

$$x_{n+1} = (1-\alpha_n)x_n + \alpha_n Tx_n\ , \tag{1.3}$$

where $\{ \alpha_n \}$ is a sequence of positive numbers in [0,1].
In 1974, S. Ishikawa defined the Ishikawa iteration [7] as

$$x_{n+1} = (1-\alpha_n)x_n + \alpha_n Ty_n$$
$$y_n = (1-\beta_n)x_n + \beta_n Tx_n\ , \tag{1.4}$$

where $\{ \alpha_n \}$ and $\{ \beta_n \}$ are sequences of positive numbers in [0,1].
In 2000, M. A. Noor defined the three step Noor iteration [10] as

$$x_{n+1} = (1-\alpha_n)x_n + \alpha_n Ty_n$$
$$y_n = (1-\beta_n)x_n + \beta_n Tz_n$$
$$z_n = (1-\gamma_n)x_n + \gamma_n Tx_n\ , \tag{1.5}$$

where $\{ \alpha_n \}$, $\{ \beta_n \}$ and $\{ \gamma_n \}$ are sequences of positive numbers in [0,1].
Recently, Phuengrattana and Suantai defined the SP iteration scheme [17] as

$$x_{n+1} = (1-\alpha_n)y_n + \alpha_n Ty_n$$
$$y_n = (1-\beta_n)z_n + \beta_n Tz_n$$
$$z_n = (1-\gamma_n)x_n + \gamma_n Tx_n\ , \tag{1.6}$$

where $\{ \alpha_n \}$, $\{ \beta_n \}$ and $\{ \gamma_n \}$ are sequences of positive numbers in [0,1].

Remarks:
1. If $\gamma_n = 0$, then Noor iteration (1.5) reduces to the Ishikawa iteration (1.4).
2. If $\beta_n = \gamma_n = 0$, then Noor iteration (1.5) reduces to the Mann iteration (1.3).
3. If $\beta_n = \gamma_n = 0$, then SP iteration (1.6) reduces to the Mann iteration (1.3).

In 1972, Zamfirescu [21] obtained the following interesting fixed point theorem:

Theorem 1.1. Let (X, d) be a complete metric space and $T : X \longrightarrow X$ a mapping for which there exists real numbers a, b and c satisfying $a \in (0,1)$, b, $c \in (0, \frac{1}{2})$ such that for each pair $x, y \in X$ at least one of the following conditions hold

(i) $d(Tx, Ty) \le a\, d(x, y)$
(ii) $d(Tx, Ty) \le b[d(x, Tx) + d(y, Ty)]$
(iii) $d(Tx, Ty) \le c[d(x, Ty) + d(y, Tx)]$ (1.7)

Then T has a unique fixed point p and the Picard iteration $\{x_n\}$ defined by

$$x_{n+1} = Tx_n\ , n = 0, 1, \ldots$$

converges to p for any arbitrary but fixed $x_0 \in X$.
The operators satisfying the condition (1.7) are called Zamfirescu operators.

Berinde[1] introduced a new class of operators on an arbitrary Banach space X satisfying

$$d(Tx,Ty) \leq 2\delta\, d(x,Tx) + \delta d(x,y) \quad (1.8)$$

$\forall\, x, y \in X$ and $\delta \in [0,1)$. He proved that this class is wider than the class of Zamfirescu operators and used the Ishikawa iteration process to approximate fixed points of this class of operators in an arbitrary Banach space.

The stability theory has extensively been studied by various authors due to its increasing importance in computational mathematics, especially due to revolution in computer programming.

The first stability result on T-stable mappings was due to Ostrowski [16] where he established the stability of the Picard iteration by using Banach contraction condition . Harder and Hicks [6], Rhoades [19], Osilike [14] and Singh et al. [20] used the method of the summability theory of infinite matrices to prove various stability results for certain contractive definitions. Harder [6] established applications of stability results to first order differential equations. Osilike and Udomene [15] introduced a shorter method of proof of stability results and this has also been employed by Berinde [2], Imoru and Olatinwo [11], Olatinwo [12] and some others.

2. PRELIMINARIES

Let X be a Banach space, Y an arbitrary set and $S, T : Y \rightarrow X$ such that $T(Y) \subseteq S(Y)$. For $x_0 \in Y$, consider the following iterative scheme :

$$Sx_{n+1} = Tx_n \; n = 0, 1, \ldots \quad (2.1)$$

This scheme is called Jungck iterative scheme and was essentially introduced by Jungck [8] in 1976 , and it becomes the Picard iterative scheme when $S=I_d$ (identity mapping) and $Y=X$.

For $\alpha_n \in [0,1]$, Singh et al. [20] defined the Jungck-Mann iterative scheme as

$$Sx_{n+1} = (1-\alpha_n)Sx_n + \alpha_n Tx_n \quad (2.2)$$

For $\alpha_n, \beta_n, \gamma_n \in [0,1]$, Olatinwo defined the Jungck-Ishikawa[12] and Jungck-Noor [13] iterative schemes as

$$\begin{aligned} Sx_{n+1} &= (1-\alpha_n)Sx_n + \alpha_n Ty_n \\ Sy_n &= (1-\beta_n)Sx_n + \beta_n Tx_n, \end{aligned} \quad (2.3)$$

and

$$\begin{aligned} Sx_{n+1} &= (1-\alpha_n)Sx_n + \alpha_n Ty_n \\ Sy_n &= (1-\beta_n)Sx_n + \beta_n Tz_n \\ Sz_n &= (1-\gamma_n)Sx_n + \gamma_n Tx_n \;, \end{aligned} \quad (2.4)$$

respectively.

Jungck[8] used the iterative scheme (2.1) to approximate the common fixed points of the mappings S and T satisfying the following Jungck-contraction

$$d(Tx,Ty) \leq \alpha\, d(Sx,Sy), \quad 0 \leq \alpha < 1 \; .$$

Olatinwo [12] used the following more general contractive definitions than (1.8) to prove the stability and strong convergence results for the Jungck-Ishikawa iteration process:

(a) there exists a real number $a \in [0, 1)$ and a monotone increasing function $\phi : R^+ \rightarrow R^+$ such that $\phi(0) = 0$ and $\forall x, y \in Y$, we have

$$\|Tx - Ty\| \leq \phi(\|Sx - Tx\|) + a\|Sx - Sy\| \quad (2.5)$$

(b) there exists real numbers $M \geq 0$, $a \in [0, 1)$ and a monotone increasing function $\phi : R^+ \rightarrow R^+$ such that $\phi(0) = 0$ and $\forall x, y \in Y$, we have

$$\|Tx - Ty\| \leq \frac{\phi(\|Sx - Tx\|) + a\|Sx - Sy\|}{1 + M\|Sx - Tx\|} \quad (2.6)$$

Olatinwo [13] used the convergences of Junck-Noor iterative scheme (2.4) to approximate the coincidence points (not common fixed points) of some pairs of generalized contractive-like operators satisfying (2.6) with the assumption that one of each of the pairs of maps is injective.

Bosede and Rhoades [4] proved the stability of Picard and Mann iterations for a general class of functions. In 2010, Bosede [3] also proved some strong convergence results for the Jungck-Ishikawa and Jungck-Mann iteration processes by using the following more general contractive condition than (1.8) :

$$\|Tx - Ty\| \leq e^{L\|Sx-Tx\|}\{2\delta\|Sx - Tx\| + \delta\|Sx - Sy\|\} \quad (2.7)$$

$\forall x, y \in Y$, where $L \geq 0$, δ is a real number $\in [0, 1)$.

Recently, Renu Chugh and Vivek Kumar[5] studied the strong convergence of SP iterative scheme (1.6) for quasi-contractive operators satisfying (1.8) in Banach spaces and showed its fastness as compared to Picard, Mann and Ishikawa iterative schemes.

Motivated by the above facts , we define the following Jungck-SP iterative scheme:

$$\begin{aligned} Sx_{n+1} &= (1-\alpha_n)Sy_n + \alpha_n Ty_n \\ Sy_n &= (1-\beta_n)Sz_n + \beta_n Tz_n \\ Sz_n &= (1-\gamma_n)Sx_n + \gamma_n Tx_n \;, \end{aligned} \quad (2.8)$$

where $\{\alpha_n\}$, $\{\beta_n\}$ and $\{\gamma_n\}$ are sequences of positive numbers in $[0,1]$.

Also, we employ the following contractive condition which is more general than (2.5) and (1.8):

$$\|Tx - Ty\| \leq e^{L\|Sx-Tx\|}\{\phi(\|Sx - Tx\|) + a\|Sx - Sy\|\} \quad (2.9)$$

$\forall x, y \in Y$, where $L \geq 0$, $\phi : R^+ \rightarrow R^+$ is a monotone increasing function such that $\phi(0) = 0$ and a is a real number $\in [0, 1)$.

We shall need the following Definition and Lemma:

Definition 2.1.[12] Let $S, T : Y \rightarrow X$ such that $T(Y) \subseteq S(Y)$ and z a coincidence point of S and T, that is, $Sz = Tz = p$(say). For any $x_0 \in Y$, let the sequence $\{Sx_n\}_{n=0}^{\infty}$ generated by the iteration procedure

$$Sx_n = f(T, x_n), n \geq 0 \quad (2.10)$$

converge to p. Let $\{Sy_n\}_{n=0}^{\infty} \subset X$ be an arbitrary sequence and set $\varepsilon_n = d(Sy_{n+1} - f(T, y_n))$, n = 0, 1, · · · .Then the iteration procedure (2.10) will be called (S,T)-stable if and only if $\lim_{n\to\infty} \epsilon_n = 0$ implies that $\lim_{n\to\infty} Sy_n = p$.

Lemma 2.1.[1] If δ is a real number such that $0 \leq \delta < 1$ and $\{\epsilon_n\}_{n=0}^{\infty}$ is a sequence of positive numbers such that $\lim_{n\to\infty} \epsilon_n = 0$, then for any sequence of positive numbers

$\{u_n\}_{n=0}^{\infty}$ satisfying

$$u_{n+1} \le \delta u_n + \epsilon_n, \ \mathrm{n} = 0,1,2,\ldots.$$

we have $\lim_{n\to\infty} u_n = 0$.

Remark 1. If $X=Y$ and $S=I_d$ (identity mapping), then the Jungck-SP (2.8), Jungck-Noor (2.4), Jungck-Ishikawa (2.3) and the Jungck-Mann (2.2) iterations, respectively, become the SP (1.6), Noor (1.5), Ishikawa (1.4) and the Mann (1.3) iterative procedures.

Remark 2. If $L= 0$ and $X=Y$, $S=I_d$, $L= 0$, $\phi(x) = x, a = \delta$ in (2.9), then we get contractive conditions (2.5) and (1.8) respectively.

The purpose of this paper is to study the strong convergence of Jungck-SP iterative scheme (2.8) for nonself mappings in an arbitrary Banach space and obtain some stability results for these nonself mappings in normed linear space by employing the contractive conditions (2.9). Moreover, with the help of C++ programming, we show that the Jungck-SP iterative scheme converges faster than the Jungck-Noor, Jungck-Ishikawa and Jungck-Mann iterative schemes by taking example of cubic equation.

3. STABILITY IN NORMED SPACE

Theorem 3.1. Let (X, ||.||) be a normed space and S, T : $Y\to X$ are nonself operators on an arbitrary set Y such that $T(Y) \subseteq S(Y)$, where $S(Y)$ is a complete subspace of X and S an injective operator. Let z be a coincidence point of S and T , i.e., $Sz = Tz = p$(say). Suppose that S and T satisfy condition (2.9) . For $x_0 \in Y$, let $\{Sx_n\}_{n=0}^{\infty}$ be the Jungck-SP iterative scheme (2.8) converging to p, where $\{\alpha_n\}$, $\{\beta_n\}, \{\gamma_n\}$ are sequences of positive numbers in [0,1] with $\{\alpha_n\}$ satisfying $0 < \alpha \le \alpha_n \forall n$ .Then, the Jungck-SP iterative scheme is (S, T)-stable.

Proof. Suppose that $\{Sy_n\}_{n=0}^{\infty} \subset E$,

$\varepsilon_n = \|Sy_{n+1} - (1-\alpha_n)Sb_n - \alpha_n Tb_n\|$, n= 0,1,2,3…,

where $Sb_n = (1-\beta_n)Sc_n + \beta_n Tc_n$, $Sc_n = (1-\gamma_n)Sy_n + \gamma_n Ty_n$

and let $\lim_{n\to\infty} \epsilon_n = 0$.

Then, it follows from (2.8) and (2.9) that

$$\begin{aligned}
\| Sy_{n+1} - p \| &\le \| Sy_{n+1} - (1-\alpha_n)Sb_n - \alpha_n Tb_n \| \\
&\quad + \| (1-\alpha_n)Sb_n + \alpha_n Tb_n - (1-\alpha_n+\alpha_n)p \| \\
&\le \varepsilon_n + (1-\alpha_n)\|Sb_n - p\| + \alpha_n\|Tb_n - p\| \\
&= \varepsilon_n + (1-\alpha_n)\|Sb_n - p\| + \alpha_n\|Tz - Tb_n\| \\
&\le \varepsilon_n + (1-\alpha_n)\|Sb_n - p\| \\
&\quad + \alpha_n e^{L\|Sz-Tz\|}\{\phi(\|Sz-Tz\|) + a\|Sz - Sb_n\|\} \\
&= \varepsilon_n + (1-\alpha_n)\|Sb_n - p\| \\
&\quad + \alpha_n e^{L\|0\|}\{\phi(\|0\|) + a\|Sz - Sb_n\|\} \\
&= [1-\alpha_n(1-a)]\|Sb_n - p\| + \varepsilon_n \qquad (3.1)
\end{aligned}$$

Now, we have the following estimates:

$$\begin{aligned}
\| Sb_n - p \| &= \| (1-\beta_n)Sc_n + \beta_n Tc_n - (1-\beta_n+\beta_n)p \| \\
&\le (1-\beta_n)\|Sc_n - p\| + \beta_n\|Tc_n - p\| \\
&= \| (1-\beta_n)\|Sc_n - p\| + \beta_n\|Tz - Tc_n\| \\
&\le (1-\beta_n)\|Sc_n - p\| \\
&\quad + e^{L\|Sz-Tz\|}\beta_n\{\phi(\|Sz-Tz\|) + a\|Sz - Sc_n\|\} \\
&= (1-\beta_n(1-a))\|Sc_n - p\| \qquad (3.2)
\end{aligned}$$

and

$$\begin{aligned}
\| Sc_n - p \| &= \| (1-\gamma_n)Sy_n + \gamma_n Ty_n - (1-\gamma_n+\gamma_n)p \| \\
&\le (1-\gamma_n)\|Sy_n - p\| + \gamma_n\|Ty_n - p\| \\
&= (1-\gamma_n)\|Sy_n - p\| + \gamma_n\|Tz - Ty_n\| \\
&\le (1-\gamma_n)\|Sy_n - p\| \\
&\quad + \gamma_n e^{L\|Sz-Tz\|}\{\phi(\|Sz-Tz\|) + a\|Sz - Sy_n\|\} \\
&= (1-\gamma_n(1-a))\|Sy_n - p\| \qquad (3.3)
\end{aligned}$$

It follows from (3.1),(3.2) and (3.3)that

$$\begin{aligned}
&\| Sy_{n+1} - p \| \\
&\le [1-\alpha_n(1-a)][1-\beta_n(1-a)][1-\gamma_n(1-a)] \| Sy_n - p \| \\
&\qquad + \varepsilon_n \qquad (3.4)
\end{aligned}$$

Using $0 < \alpha \le \alpha_n$ and $a \in [0,1)$ we have

$$[1-\alpha_n(1-a)][1-\beta_n(1-a)][1-\gamma_n(1-a)] < 1.$$

Hence using Lemma (2.1), (3.4) yields $\lim_{n\to\infty} Sy_{n+1} = p$.

Conversely, let $\lim_{n\to\infty} Sy_{n+1} = p$ .Then using contractive condition (2.9) and the triangle inequality , we have

$$\begin{aligned}
\varepsilon_n &= \| Sy_{n+1} - (1-\alpha_n)Sb_n - \alpha_n Tb_n \| \\
&\le \| Sy_{n+1} - p \| + \| (1-\alpha_n+\alpha_n)p - (1-\alpha_n)Sb_n - \alpha_n Tb_n \| \\
&\le \| Sy_{n+1} - p \| + (1-\alpha_n)\|p - Sb_n\| + \alpha_n\|p - Tb_n\| \\
&= \| Sy_{n+1} - p \| + (1-\alpha_n)\|Sb_n - p\| + \alpha_n\|Tz - Tb_n\| \\
&\le \| Sy_{n+1} - p \| + (1-\alpha_n)\|Sb_n - p\| + a\alpha_n\|Sz - Sb_n\| \\
&= \| Sy_{n+1} - p \| + [1-\alpha_n(1-a)]\|Sb_n - p\| \qquad (3.5)
\end{aligned}$$

Using estimates (3.2) and (3.3), (3.5) yields

$$\begin{aligned}
\varepsilon_n &\le [1-\alpha_n(1-a)][1-\beta_n(1-a)][1-\gamma_n(1-a)] \| Sy_{n+1} - p \| \\
&\qquad + \| Sy_{n+1} - p \|
\end{aligned}$$

Hence , $\lim_{n\to\infty} \varepsilon_n = 0$

Therefore, the SP iterative scheme (2.8) is (S,T) stable.

4. STRONG CONVERGENCE IN AN ARBITRARY BANACH SPACE

Theorem 4.1. Let (X, ||.||) be an arbitrary Banach space and $S, T : Y\to X$ are nonself operators on an arbitrary set Y such that $T(Y) \subseteq S(Y)$, where $S(Y)$ is a complete subspace of X and S is an injective operator. Let z be a coincidence point of S and T , i.e., $Sz = Tz = p$(say). Suppose that S and T satisfy condition (2.9) . For $x_0 \in Y$, let $\{Sx_n\}_{n=0}^{\infty}$ be the Jungck-SP iterative scheme defined by (2.8) , where $\{\alpha_n\}$, $\{\beta_n\}$, $\{\gamma_n\}$ are sequences of positive numbers in [0,1] with $\{\alpha_n\}$ satisfying $\sum_{n=0}^{\infty}\alpha_n = \infty$ .Then the Jungck-SP iterative scheme $\{Sx_n\}_{n=0}^{\infty}$ converges strongly to p .

Proof . Let C(S,T) be the set of the coincidence points of S and T. We use condition (2.9) to establish that S and T have a unique coincidence point z, i.e. $Sz = Tz = p$ (say). Suppose that there exists z_1, $z_2 \in$ C(S, T) such that $Sz_1 = Tz_1 = p_1$ and $Sz_2 = Tz_2 = p_2$. If $p_1 = p_2$, then $Sz_1 = Sz_2$ and since S is injective, it follows that z_1=z_2 .

If $p_1 \neq p_2$, then using contractive condition (2.9) we have

$$0 \le \|p_1 - p_2\| = \|Tz_1 - Tz_2\| \le e^{\|Sz_1 - Tz_1\|}\{\phi(\|Sz_1 - Tz_1\|) + a\|Sz_1 - Sz_2\|\}$$
$$= a\|p_1 - p_2\|,$$

which leads to $a\|p_1 - p_2\| \le 0$, from which it follows that $\|p_1 - p_2\| \le 0$ since $a \in [0, 1)$, which is a contradiction since norm is nonnegative.

Therefore, we have that $\|p_1 - p_2\| = 0$, that is, $p_1 = p_2 = p$.

Again $p_1 = p_2$ implies $p_1 = Sz_1 = Tz_1 = Sz_2 = Tz_2 = p_2$, leading to $Sz_1 = Sz_2$ which again yields $z_1 = z_2 = z$ (since S is injective). Hence $z \in$ C(S,T), that is, z is a unique coincidence point of S and T.

We now prove that $\{Sx_n\}_{n=0}^{\infty}$ converges strongly to p

It follows from (2.8) and (2.9) that

$$\begin{aligned}
\| Sx_{n+1} - p \| &= \| (1-\alpha_n)Sy_n + \alpha_n Ty_n - (1-\alpha_n+\alpha_n)p \| \\
&\le (1-\alpha_n)\|Sy_n - p\| + \alpha_n\|Ty_n - p\| \\
&= (1-\alpha_n)\|Sy_n - p\| + \alpha_n\|Tz - Ty_n\| \\
&\le (1-\alpha_n)\|Sy_n - p\| \\
&\quad + \alpha_n e^{\|Sz-Tz\|}\{\phi(\|Sz - Tz\|) + a\|Sz - Sy_n\|\} \\
&= (1-\alpha_n)\|Sy_n - p\| + a\alpha_n\|Sy_n - p\|\} \\
&= [1-\alpha_n(1-a)]\|Sy_n - p\| \qquad (4.1)
\end{aligned}$$

Now, we have the following estimates:

$$\begin{aligned}
\| Sy_n - p \| &= \| (1-\beta_n)Sz_n + \beta_n Tz_n - (1-\beta_n+\beta_n)p \| \\
&\le (1-\beta_n)\|Sz_n - p\| + \beta_n\|Tz_n - p\| \\
&\le (1-\beta_n)\|Sz_n - p\| + \beta_n\|Tz_n - Tz\| \\
&\le (1-\beta_n)\|Sz_n - p\| \\
&\quad + \beta_n e^{\|Sz-Tz\|}\{\phi(\|Sz - Tz\|) + a\|Sz_n - Sz\|\} \\
&= (1-\beta_n(1-a))\|Sz_n - p\| \qquad (4.2)
\end{aligned}$$

and

$$\begin{aligned}
\| Sz_n - p \| &= \| (1-\gamma_n)Sx_n + \gamma_n Tx_n - (1-\gamma_n+\gamma_n)p \| \\
&\le \| (1-\gamma_n)\|Sx_n - p\| + \gamma_n\|Tx_n - Tz\| \\
&\le (1-\gamma_n)\|Sx_n - p\| \\
&\quad + \gamma_n e^{\|Sz-Tz\|}\{\phi(\|Sz - Tz\|) + a\|Sx_n - Sz\|\} \\
&= (1-\gamma_n(1-a))\|Sx_n - p\| \qquad (4.3)
\end{aligned}$$

It follows from (4.1), (4.2) and (4.3) that

$$\begin{aligned}
&\| Sx_{n+1} - p \| \\
&\le [1-\alpha_n(1-a)][1-\beta_n(1-a)][1-\gamma_n(1-a)]\| Sx_n - p \| \\
&\le [1-\alpha_n(1-a)]\| Sx_n - p \| \\
&\le \prod_{k=0}^{n}[1-\alpha_k(1-a)]\| Sx_0 - p \| \\
&\le e^{-(1-a)\sum_{k=0}^{\infty}\alpha_k}\|Sx_0 - p\| \qquad (4.4)
\end{aligned}$$

Since $0 \le a < 1$, $\alpha_k \in [0,1]$ and $\sum_{n=0}^{\infty}\alpha_n = \infty$, so $e^{-(1-a)\sum_{k=0}^{n}\alpha_k} \to 0$ as $n \to \infty$.

Hence, it follows from (4.4) that $\lim_{n\to\infty}\|Sx_{n+1} - p\|$=0.

Therefore $\{Sx_n\}_{n=0}^{\infty}$ converges strongly to p.

5. EXPERIMENTS

Recently , Bhagwati and Ritu[18] solved the cubic equation

$x^3+4x^2-5x-10 = 0$ (5.1)

using Jungck-Ishikawa iterative scheme. Rewrting (5.1) as

$Sx = Tx$, (5.2)

where Sx=5x and Tx= x^3+4x^2-10, they observed that neither Picard iteration nor Jungck-Picard iteration scheme converges toward the solution of cubic equtaion (5.1).

But the claim made by the authors regarding the use of Jungck-Ishikawa itérative scheme in Table on page 32 is false and it is actually Jungck-Mann iterative scheme.

Now, to solve cubic equation $x^3+4x^2-5x-10$ using Jungck type iterative schemes (Jungck-SP, Jungck-Noor, Jungck-Mann, Jungck-Ishikawa), we rewrite the equation (5.1) as in (5.2).

To solve cubic equation (5.1) using simple iterative schemes (Noor, SP, Mann, Ishikawa), we rewrite this equation as follows :

x=Tx=(x^3+4x^2-10)/5. (5.3)

Then, coincidence point of S and T in (5.2) and fixed point of T in (5.3) leads to the solution of (5.1).

We develope and execute seprate programs in C++ for Jungck type iterative schemes and the corresponding simple iterative schemes. The outcome is listed in the form of Tables1 and 2 for Jungck type and the corresponding simple iterative schemes, respectively, by taking intial approximation x_0=1 and α_n=β_n=γ_n=0.9 for both type of iterative schemes.

6. CONCLUSION

Keeping in mind Bhagwati and Ritu's result's [i. e. Table in [9]] and Table1 we observe that the decreasing order of convergence of Jungck type iterative schemes is as follows :Jungck-SP, Jungck-Noor, Jungck-Mann and Jungck-Ishikawa iterative scheme.

From Table 2, we observe that the decreasing order of convergence of simple iterative schemes is as follows : SP, Noor , Mann and Ishikawa iterative scheme .

Also, from Table 1 and 2 we conclude that SP, Noor and Mann iterative schemes converges fast as compared to corresponding Jungck type iterative schemes .

Table 1

No. of Iterations	Jungck-SP iteration			Jungck-Noor iteration			Jungck-Ishikawa iteration		
n	Sx_{n+1}	Tx_n	x_{n+1}	Sx_{n+1}	Tx_n	x_{n+1}	Sx_{n+1n}	Tx_n	x_{n+1}
0	-7.5568	-5	-0.927913	-6.6568	-5	-0.848561	-6.6568	-5	-1.33136
1	-5.042186	-7.354865	-1.353238	-4.473539	-7.730788	-1.437413	-6.592527	-5.269784	-1.318505
2	-6.54513	-5.153111	-1.100902	-7.060505	-4.705293	-1.014879	-6.534847	-5.338338	-1.306969
3	-5.639365	-6.486337	-1.255135	-5.1701	-6.925386	-1.330669	-6.482958	-5.399849	-1.296592
4	-6.194718	-5.67584	-1.161024	-6.589429	-5.273471	-1.094563	-6.436189	-5.455166	-1.287238
5	-5.855019	-6.173127	-1.218834	-5.522614	-6.519091	-1.273625	-6.39397	-5.505004	-1.278794
-	-	-	-	-	-	-	-	-	-
30	-5.984435	-5.984433	-1.196887	-5.984943	-5.983841	-1.196801	-6.020619	-5.942323	-1.204124
31	-5.984433	-5.984434	-1.196887	-5.984046	-5.984885	-1.196952	-6.017273	-5.946219	-1.203455
32	**-5.984434**	**-5.984434**	**-1.196887**	-5.98473	-5.98409	-1.196837	-6.014235	-5.949755	-1.202847
33	**-5.984434**	**-5.984434**	**-1.196887**	-5.984209	-5.984696	-1.196925	-6.011478	-5.952965	-1.202296
-	-	-	-	-	-	-	-	-	-
55	**-5.984434**	**-5.984434**	**-1.196887**	-5.984433	-5.984435	-1.196887	-5.987952	-5.980342	-1.19759
56	**-5.984434**	**-5.984434**	**-1.196887**	-5.984434	-5.984433	-1.196887	-5.987626	-5.980721	-1.197525
57	**-5.984434**	**-5.984434**	**-1.196887**	**-5.984434**	**-5.984434**	**-1.196887**	-5.98733	-5.981064	-1.197466
58	**-5.984434**	**-5.984434**	**-1.196887**	**-5.984434**	**-5.984434**	**-1.196887**	-5.987062	-5.981376	-1.197412
-	-	-	-	-	-	-	-	-	-
149	**-5.984434**	**-5.984434**	**-1.196887**	**-5.984434**	**-5.984434**	**-1.196887**	-5.984434	-5.984433	-1.196887
150	**-5.984434**	**-5.984434**	**-1.196887**	**-5.984434**	**-5.984434**	**-1.196887**	**-5.984434**	**-5.984434**	**-1.196887**
151	**-5.984434**	**-5.984434**	**-1.196887**	**-5.984434**	**-5.984434**	**-1.196887**	**-5.984434**	**-5.984434**	**-1.196887**
152	**-5.984434**	**-5.984434**	**-1.196887**	**-5.984434**	**-5.984434**	**-1.196887**	**-5.984434**	**-5.984434**	**-1.196887**

Table 2

No. of Iterations	SP Iteration		Noor Iteration		Ishikawa Iteration		Mann iteration	
n	Tx_n	x_{n+1}	Tx_n	x_{n+1}	Tx_n	x_{n+1}	Tx_n	x_{n+1}
0	-1.419125	-1.216246	0	-1	-1	-1.33136	-1	-0.8
1	-1.176424	-1.19676	-1	-1.546158	-1.053957	-1.318505	-1.5904	-1.51136
2	-1.19702	-1.196887	-2	-0.941059	-1.067668	-1.306969	-0.863085	-0.927913
-	-	-	-	-	-	-	-	-
28	-1.196886	-1.196886	-1.196683	-1.196739	-1.18666	-1.205674	-1.192326	-1.193214
29	**-1.196887**	**-1.196887**	-1.197042	-1.196999	-1.187606	-1.204861	-1.200763	-1.200008
30	**-1.196887**	**-1.196887**	-1.196768	-1.196801	-1.188465	-1.204124	-1.193592	-1.194233
-	-	-	-	-	-	-	-	-
52	**-1.196887**	**-1.196887**	-1.196886	-1.196887	-1.195893	-1.197741	-1.196795	-1.196813
53	**-1.196887**	**-1.196887**	**-1.196887**	**-1.196887**	-1.195985	-1.197662	-1.196965	-1.19695
54	**-1.196887**	**-1.196887**	**-1.196887**	**-1.196887**	-1.196068	-1.19759	-1.19682	-1.196833
-	-	-	-	-	-	-	-	-
86	**-1.196887**	**-1.196887**	**-1.196887**	**-1.196887**	-1.19685	-1.196918	-1.196886	-1.196886
87	**-1.196887**	**-1.196887**	**-1.196887**	**-1.196887**	-1.196854	-1.196915	**-1.196887**	**-1.196887**
88	**-1.196887**	**-1.196887**	**-1.196887**	**-1.196887**	-1.196857	-1.196913	**-1.196887**	**-1.196887**
-	-	-	-	-	-	-	-	-
136	**-1.196887**	**-1.196887**	**-1.196887**	**-1.196887**	-1.196886	-1.196887	**-1.196887**	**-1.196887**
137	**-1.196887**	**-1.196887**	**-1.196887**	**-1.196887**	**-1.196887**	**-1.196887**	**-1.196887**	**-1.196887**
138	**-1.196887**	**-1.196887**	**-1.196887**	**-1.196887**	**-1.196887**	**-1.196887**	**-1.196887**	**-1.196887**

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