Effects of Radiation on Free Convection Flow in a vertical Channel Embedded in Porous Media

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ABSTRACT
The effects of radiative heat transfer on the fully developed free convection flow of a viscous incompressible fluid-saturated porous medium between two vertical walls in the presence of a uniform gravitational field have been studied. An exact solution of the governing equations has been obtained. Radiation is found to have significant effects on the velocity field and temperature distribution. It is observed that the fluid velocity decreases with an increase in either radiation parameter or porosity parameter. It is also observed that the velocity at any point in the flow region increases with an increase in Grashof number. The effect of thermal radiation on temperature field is also analyzed. The fluid temperature increases with an increase in either radiation parameter or temperature parameter.

Keywords: Free convection, radiation, Grashof number, porous media and heat transfer.

1. INTRODUCTION
The radiative effects have important applications in physics and engineering. The radiative heat transfer effects on different flows are very important in space technology and high temperature processes. Thermal radiation effects may play an important role in controlling heat transfer in industry where the quality of the final product depends on the heat controlling factors to some extent. High temperature plasmas, cooling of nuclear reactors, liquid metal fluids, power generation systems are some important applications of radiative heat transfer from a vertical wall to conductive gray fluids. Heat transfer phenomenon in porous media is of considerable interest due to its ever increasing industrial applications and important bearings on several technological processes. Processes involving heat and mass transfer in porous media are frequently encountered in the chemical industry, in reservoir engineering in connection with thermal recovery process etc.. A better understanding of convection through porous medium can benefit several areas like insulation design, grain storage, geothermal systems, heat exchangers, filtering devices, metal processing, catalytic reactors etc. In recent years, in high-temperature applications, a great deal of attention has been focused on the usage of porous media. Porous media have been utilized for enhancement of heat transfer in coolant passages and in thermal insulation systems. The subject of porous media and its applications have reviewed by Nield and Bejan(1971), Bejan(1994), Pop and Ingham(2001), Kaviany(1995), Ingham and Pop(2005), Vafai(2005), Ingham et al.(2004). Radiation heat transfer in porous media has been studied by many researchers. Thermal radiation effect on mixed convection from horizontal surfaces in saturated porous media has been investigated by Bakier and Gorla(1996). Raptis(1998) has studied the effect of radiation on free convection flow through a porous medium. The MHD unsteady free convection flow over an infinite vertical plate in the presence of radiation has been studied by Perdikis and Raptis(2006). Radiation effects on the free convection over a vertical flat plate embedded in porous medium with high porosity have been studied by Hossain and Pop(2001). Thermal dispersion-radiation effects on non-Darcy natural convection in a fluid saturated porous medium have been investigated by Mohammadein and El-Amin(2000). Raptis and Perdikis(2004) have studied the unsteady flow through a highly porous medium in the presence of radiation. Sharma et al.(2007) have investigated the radiation effect on temperature distribution in three-dimensional Couette flow suction and injection. Weidman and Medina(2008) have studied the convective flow between vertical walls embedded in porous medium. Effect of radiation on forced convective flow and heat transfer over a porous plate in a porous medium has been investigated by Mukhopadhyay and Layek(2009).

The aim of the present paper is to study the effects of radiative heat transfer on the fully developed free convection flow of a viscous incompressible fluid-saturated porous medium between vertical walls in the presence of a uniform gravitational field. The governing equations are solved analytically. The effects of the permeability of the porous medium and the influence of radiation parameter on the velocity and temperature fields are investigated and analyzed with the help of their graphical representations. It is observed that the velocity field decreases with an increase in either radiation parameter Ra or porosity parameter α. The temperature distribution decreases with an increase in either radiation parameter Ra or temperature parameter r_T. It is found that the critical wall temperature $(\theta_0)_c$ at the wall $(\eta = -1/2)$ decreases with an increase in either radiation parameter Ra or porosity parameter α.

2. FORMULATION OF THE PROBLEM AND ITS SOLUTIONS
We consider a fully developed flow of a viscus incompressible fluid-saturated porous medium between vertical walls in the presence of a uniform gravitational field. The distance between the channel walls is d. Employ a Cartesian coordinates system with x - axis vertically upwards along the direction of flow and

y-axis perpendicular to it. The origin of the axes is such that the channel walls are at positions $y=-d/2$ and $y=d/2$ (see Figure 1). The velocity components are (u,v) relative to the Cartesian frame of reference. We do not model the pressure drop across the end caps and only consider the fully-developed flow far from the end caps.

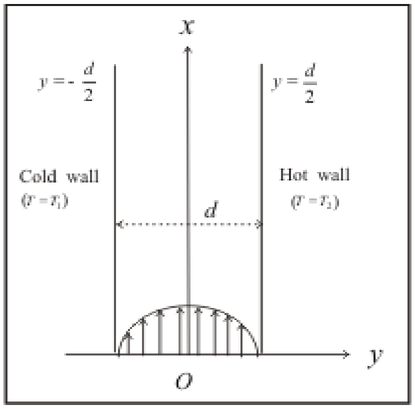

Figure 1: Geometry of the problem

The Boussinesq approximation is assumed to hold and for the evaluation of the gravitational body force, the density is assumed to be dependent on the temperature according to the equation of state

$$\rho=\rho_0[1-\beta(T-T_0)], \tag{1}$$

where T, ρ, β, T_0 and ρ_0 are respectively, the fluid temperature, the fluid density, thermal expansion coefficient, the reference temperature and the density in the reference state.

Flow away from the top and bottom ends of the cavity is rectilinear so that $u=u(y),\ v=0$. In this case the equation of continuity is satisfied identically. On using Boussinesq approximation (1), the momentum and energy equations are simplified to the following form

$$\bar{\mu}\frac{d^2u}{dy^2}-\frac{\mu}{k^*}u+\rho_0 g\beta(T-T_0)=0, \tag{2}$$

$$0=k\frac{d^2T}{dy^2}-\frac{\partial q_r}{\partial y}, \tag{3}$$

where μ is the fluid viscosity, $\bar{\mu}$ the effective viscosity of the fluid-saturated porous medium, g the acceleration due to gravity, k^* permeability of the porous medium and k the thermal conductivity.

The equation of conservation of radiative heat transfer per unit volume for all wave length is

$$\nabla.q_r=\int_0^\infty K_\lambda(T)(4e_{\lambda h}(T)-G_\lambda)d\lambda, \tag{4}$$

where $e_{\lambda h}$ is the Plank's function and the incident radiation G_λ is defined as

$$G_\lambda=\frac{1}{\pi}\int_{\Omega=4\pi}e_\lambda(\Omega)d\Omega, \tag{5}$$

$\nabla.q_r$ is the radiative flux divergence and Ω is the solid angle. Now, for an optically thin fluid exchanging radiation with an isothermal flat walls at temperature T_0 and according to the above definition for the radiative flux divergence and Kirchhoffs law, the incident radiation is given by $G_\lambda=4e_{\lambda h}(T_0)$ then,

$$\nabla.q_r=4\int_0^\infty K_\lambda(T)(e_{\lambda h}(T)-e_{\lambda h}(T_0))d\lambda, \tag{6}$$

Expanding $K_\lambda(T)$ and $e_{\lambda h}(T_0)$ in a Taylor series around T_0, for small $(T-T_0)$, we can rewrite the radiative flux divergence as

$$\nabla.q_r=4(T-T_0)\int_0^\infty K_{\lambda_0}\left(\frac{\partial e_{\lambda h}}{\partial T}\right)_0 d\lambda, \tag{7}$$

where $K_{\lambda_0}=K_{\lambda(T_0)}$.

Hence an optically thin limit for a non-gray gas near equilibrium, the following relation holds

$$\nabla.q_r=4(T-T_0)I, \tag{8}$$

and hence

$$\frac{\partial q_r}{\partial y}=4(T-T_0)I, \tag{9}$$

where

$$I=\int_0^\infty K_{\lambda_0}\left(\frac{\partial e_{\lambda h}}{\partial T}\right)_0 d\lambda. \tag{10}$$

The velocity and temperature boundary conditions are

$$u=0 \text{ at } y=\pm\frac{d}{2},$$

$$T=T_1 \text{ at } y=-\frac{d}{2} \text{ and } T=T_2 \text{ at } y=\frac{d}{2}. \tag{11}$$

Introducing the non-dimensional variables

$$\eta=\frac{y}{d},\ u_1=\frac{ud}{\nu},\ \theta=\frac{T-T_0}{T_2-T_1}, \tag{12}$$

and on using (9) and (12), equations (2) and (3) become

$$\frac{d^2u_1}{d\eta^2}-\alpha^2u_1=-Gr\theta, \tag{13}$$

$$\frac{d^2\theta}{d\eta^2}-Ra\theta=0, \tag{14}$$

where $\alpha^2=\frac{1}{M\,Da}$, $M=\frac{\bar{\mu}}{\mu}$ is viscosity ratio, $Da=\frac{k^*}{d^2}$ the Darcy number, $Gr=\frac{g\beta(T_2-T_1)d^3}{\nu^2}$ the Grashof number and $Ra=\frac{4Id^2}{k}$ the radiation parameter. It may be noted that the limit $\alpha\to 0$ gives a clear fluid and the limit $\alpha\to\infty$ gives unmitigated Darcy flow.

On the use of (12), the velocity and the temperature boundary conditions (11) become

$$u_1=0 \text{ at } \eta=\pm\frac{1}{2},$$

$$\theta=-\theta_0 \text{ at } \eta=-\frac{1}{2} \text{ and } \theta=1-\theta_0 \text{ at } \eta=\frac{1}{2}, \tag{15}$$

where the parameter θ_0 measures the continuous cross-channel variation of the reference temperature T_0.

The solutions of (13) and (14) subject to the boundary conditions (15) are

$$\theta(\eta) = \left(\frac{1}{2} - \theta_0\right)\frac{\cosh\sqrt{Ra}\,\eta}{\cosh\frac{\sqrt{Ra}}{2}} + \frac{1}{2}\frac{\sinh\sqrt{Ra}\,\eta}{\sinh\frac{\sqrt{Ra}}{2}}, \qquad (16)$$

$$u_1(\eta) = \begin{cases} \dfrac{Gr}{Ra-\alpha^2}\left[\left(\dfrac{1}{2}-\theta_0\right)\dfrac{\cosh\alpha\eta}{\cosh\frac{\alpha}{2}} + \dfrac{1}{2}\dfrac{\sinh\alpha\eta}{\sinh\frac{\alpha}{2}} - \left\{\left(\dfrac{1}{2}-\theta_0\right)\dfrac{\cosh\sqrt{Ra}\,\eta}{\cosh\frac{\sqrt{Ra}}{2}} + \dfrac{1}{2}\dfrac{\sinh\sqrt{Ra}\,\eta}{\sinh\frac{\sqrt{Ra}}{2}}\right\}\right] & \text{for } Ra \neq \alpha^2, \\ \dfrac{Gr}{4\alpha}\left[\left(\dfrac{1}{2}-\theta_0\right)\dfrac{\cosh\alpha\eta}{\cosh\frac{\alpha}{2}}\tanh\dfrac{\alpha}{2} + \dfrac{1}{2}\dfrac{\sinh\alpha\eta}{\sinh\frac{\alpha}{2}}\coth\dfrac{\alpha}{2} - 2\eta\left\{\left(\dfrac{1}{2}-\theta_0\right)\dfrac{\sinh\alpha\eta}{\cosh\frac{\alpha}{2}} + \dfrac{1}{2}\dfrac{\cosh\alpha\eta}{\sinh\frac{\alpha}{2}}\right\}\right] & \text{for } Ra = \alpha^2, \end{cases} \qquad (17)$$

It is observed from the equations (16) and (17) that the velocity field depends on the Grashof number Gr as well as porosity parameter α while the temperature distribution does not depend on the Grashof number Gr and porosity parameter α.

3. RESULTS AND DISCUSSION

To study the effects of radiation, buoyancy force, porosity of the medium and temperature parameter on the velocity field u_1 and temperature distribution θ, we have presented the non-dimensional velocity u_1 and the temperature θ against η for various values of radiation parameter Ra, Grashof number Gr, porosity parameter α and the temperature parameter θ_0 in Figures 2-7. It is observed from Figure 2, that the velocity u_1 decreases with an increase in radiation parameter Ra. It is observed from Figure 3, that the velocity u_1 decreases with an increase in porosity parameter α. It is seen from Figure 4 that the velocity decreases with an increase in θ_0. This implies that the radiation, porosity of the medium and wall temperature have a retarding influence on the free convection flow. Figure 5 shows that the velocity at any point in the flow region increases with an increase in Grashof number Gr. This means that buoyancy force accelerating the velocity field. Figures 6 and 7 reveal that the temperature θ decreases with an increase in either radiation parameter Ra or temperature parameter θ_0. The effect of radiation parameter Ra is to reduce the temperature significantly in the flow region. The increase in radiation parameter means the release of heat energy from the flow region and so the fluid temperature decreases. The incipient flow reversal will occur only for those values of θ_0 which are greater than the critical values of θ_0 at the cold wall.

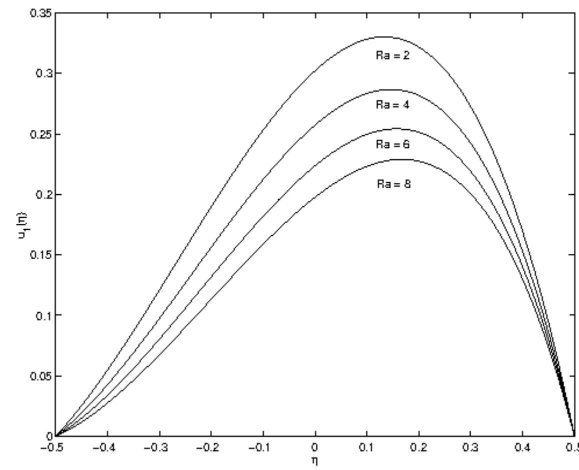

Figure 2: Variations of velocity u_1 for $\theta_0 = 0.2$, $\alpha = 0.5$ and $Gr = 10$

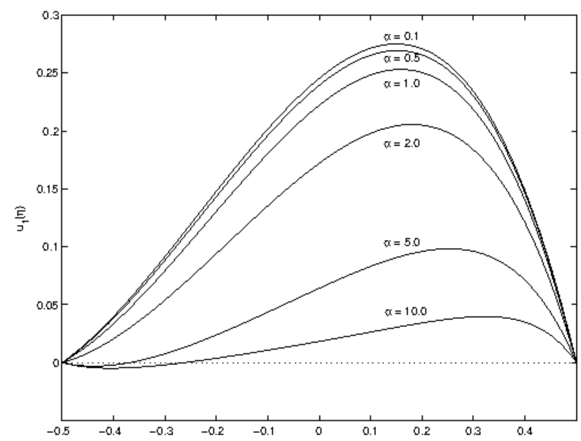

Figure 3: Variations of velocity u_1 for $Ra = 5$, $\theta_0 = 0.2$ and $Gr = 10$.

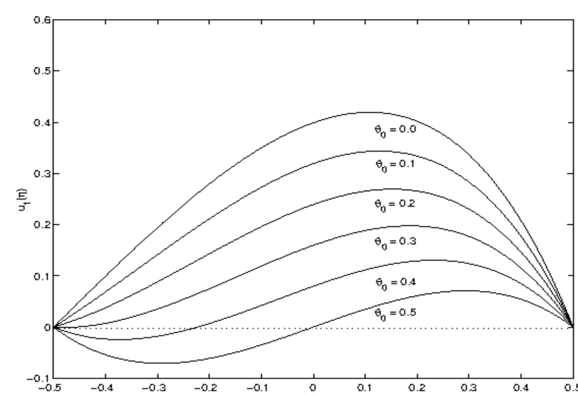

Figure 4: Variations of velocity u_1 for $\theta_0 = 0.2$, $Ra = 5$ and $Gr = 10$.

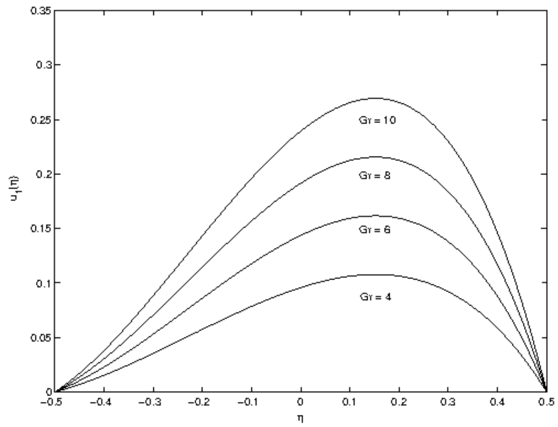

Figure 5: Variations of velocity u_1 for $Ra = 4$, $\theta_0 = 0.2$ and $\alpha = 0.5$.

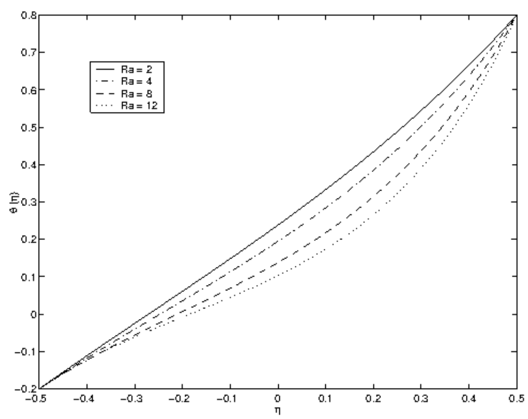

Figure 6: Variations of temperature θ for $\theta_0 = 0.2$.

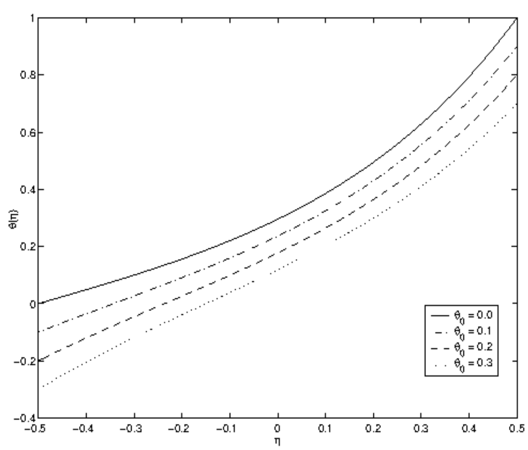

Figure 7: Variations of temperature θ for $Ra = 5$.

The non-dimensional shear stresses at the cold wall $(\eta = -1/2)$ and hot wall $(\eta = 1/2)$ are given by $\tau_{x_1} = \left(\frac{du_1}{d\eta}\right)_{\eta=-\frac{1}{2}}$ and $\tau_{x_2} = \left(\frac{du_1}{d\eta}\right)_{\eta=\frac{1}{2}}$, where

$$\left(\frac{du_1}{d\eta}\right)_{\eta=-\frac{1}{2}} = \begin{cases} \frac{Gr}{Ra-\alpha^2}\left[\sqrt{Ra}\left\{\left(\frac{1}{2}-\theta_0\right)\tanh\frac{\sqrt{Ra}}{2} - \frac{1}{2}\coth\frac{\sqrt{Ra}}{2}\right\}\right. \\ \left. -\alpha\left\{\left(\frac{1}{2}-\theta_0\right)\tanh\frac{\alpha}{2} - \frac{1}{2}\coth\frac{\alpha}{2}\right\}\right] \quad \text{for } Ra \neq \alpha^2, \\ \frac{Gr}{4\alpha}\left[\frac{1}{2}\left(\alpha\coth^2\frac{\alpha}{2} - 2\coth\frac{\alpha}{2} - \alpha\right)\right. \\ \left. -\left(\frac{1}{2}-\theta_0\right)\left(\alpha\tanh^2\frac{\alpha}{2} - 2\tanh\frac{\alpha}{2} - \alpha\right)\right] \quad \text{for } Ra = \alpha^2, \end{cases} \tag{18}$$

$$\left(\frac{du_1}{d\eta}\right)_{\eta=\frac{1}{2}} = \begin{cases} \frac{Gr}{Ra-\alpha^2}\left[\alpha\left\{\left(\frac{1}{2}-\theta_0\right)\tanh\frac{\alpha}{2} + \frac{1}{2}\coth\frac{\alpha}{2}\right\}\right. \\ \left. -\sqrt{Ra}\left\{\left(\frac{1}{2}-\theta_0\right)\tanh\frac{\sqrt{Ra}}{2} + \frac{1}{2}\coth\frac{\sqrt{Ra}}{2}\right\}\right] \text{for } Ra \neq \alpha^2, \\ \frac{Gr}{4\alpha}\left[\frac{1}{2}\left(\alpha\coth^2\frac{\alpha}{2} - 2\coth\frac{\alpha}{2} - \alpha\right)\right. \\ \left. +\left(\frac{1}{2}-\theta_0\right)\left(\alpha\tanh^2\frac{\alpha}{2} - 2\tanh\frac{\alpha}{2} - \alpha\right)\right] \text{ for } Ra = \alpha^2, \end{cases} \tag{19}$$

Numerical values of shear stresses at the cold wall $(\eta = -1/2)$ and hot wall $(\eta = 1/2)$ are shown graphically against Ra for different values of α in Figure 8. It is observed from Figure 8 that for fixed values of Ra and Gr both the magnitude of the shear stress τ_{x_1} at the cold wall and the shear stress τ_{x_2} at the hot wall decrease with an increase in porosity parameter α. On the other hand, it is seen that for fixed values of α and Gr, the magnitude of τ_{x_1} and τ_{x_2} decrease with an increase in radiation parameter Ra.

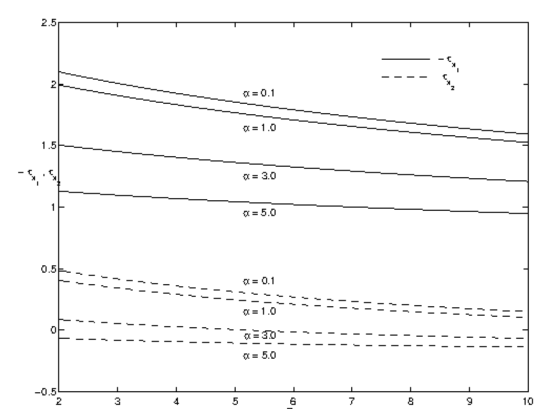

Figure 8: Variation of shear stresses $-\tau_{x_1}$ and τ_{x_2} for $\theta_0 = 0.2$ and $Gr = 10$

The rate of heat transfer at the cold wall $(\eta = -1/2)$ and hot wall $(\eta = 1/2)$ are respectively given by

$$\left(\frac{d\theta}{d\eta}\right)_{\eta=-\frac{1}{2}} = \sqrt{Ra}\left[\frac{1}{2}\coth\frac{\sqrt{Ra}}{2} - \left(\frac{1}{2}-\theta_0\right)\tanh\frac{\sqrt{Ra}}{2}\right], \quad (20)$$

$$\left(\frac{d\theta}{d\eta}\right)_{\eta=\frac{1}{2}} = \sqrt{Ra}\left[\frac{1}{2}\coth\frac{\sqrt{Ra}}{2} + \left(\frac{1}{2}-\theta_0\right)\tanh\frac{\sqrt{Ra}}{2}\right]. \quad (21)$$

The values of $\left(\frac{d\theta}{d\eta}\right)_{\eta=-\frac{1}{2}}$ and $\left(\frac{d\theta}{d\eta}\right)_{\eta=\frac{1}{2}}$ against Ra are plotted in Figure 9 for different values of θ_0. It is seen from Figure 9 that for fixed value of Ra, the rate of heat transfer at the cold wall $(\eta = -1/2)$ decreases while the rate of heat transfer at the hot wall $(\eta = 1/2)$ increases with an increase in θ_0. On the other hand, the rate of heat transfer at the cold wall $(\eta = -1/2)$ increases and that at the hot wall $(\eta = 1/2)$ decreases with an increase in radiation parameter Ra for fixed value of θ_0.

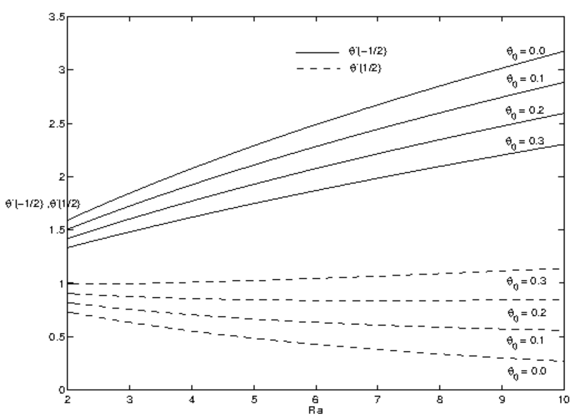

Figure 9: Variation of rate of heat transfer $\theta'(-1/2)$ and $\theta'(1/2)$.

As θ_0 decreases from the maximum value $\theta_0 = \frac{1}{2}$, we arrive to the value $\theta_0 = (\theta_0)_c$ for which there is an incipient flow reversal near the cold wall. The condition for incipient flow reversal can be obtained by letting $\left(\frac{du_1}{d\eta}\right)_{\eta=-\frac{1}{2}} = 0$ which in term gives the critical value of θ_0 as

$$(\theta_0)_c = \begin{cases} \frac{1}{2} - \dfrac{\alpha\coth\frac{\alpha}{2} - \sqrt{Ra}\coth\frac{\sqrt{Ra}}{2}}{2\left(\alpha\tanh\frac{\alpha}{2} - \sqrt{Ra}\tanh\frac{\sqrt{Ra}}{2}\right)}, & \text{for } Ra \neq \alpha^2 \\ \frac{1}{2} - \dfrac{\alpha\coth^2\frac{\alpha}{2} - 2\coth\frac{\alpha}{2} - \alpha}{2\left(\alpha\tanh^2\frac{\alpha}{2} - 2\tanh\frac{\alpha}{2} - \alpha\right)} & \text{for } Ra = \alpha^2. \end{cases} \quad (22)$$

The values of $(\theta_0)_c$ are entered in the Table 1 for different values of Ra and α. It is observed that the critical $(\theta_0)_c$ decreases with increase in either Ra or α.

Table 1 : Critical value $(\theta_0)_c$ of θ_0 at the cold wall $(\eta = -1/2)$

$Ra \setminus \alpha$	0.1	0.5	3	5	10
2	0.31248	0.30962	0.23262	0.16293	0.07988
4	0.29436	0.29133	0.21134	0.14196	0.06495
6	0.25630	0.27526	0.19331	0.12488	0.05356
8	0.26431	0.26105	0.17789	0.11079	0.04470
10	0.25173	0.24841	0.16459	0.09904	0.03768

The rate of volume flux is given by

$$Q=\begin{cases}\dfrac{2Gr}{Ra-\alpha^2}\left[\left(\dfrac{1}{2}-\theta_0\right)\left(\dfrac{\tanh\frac{\alpha}{2}}{\alpha}-\dfrac{\tanh\frac{\sqrt{Ra}}{2}}{\sqrt{Ra}}\right)\right] & \text{for } Ra\neq\alpha^2\\[2ex] \dfrac{Gr}{2\alpha^2}\left[\left(\dfrac{1}{2}-\theta_0\right)\left(\tanh^2\dfrac{\alpha}{2}+\dfrac{2}{\alpha}\tanh\dfrac{\alpha}{2}-1\right)\right] & \text{for } Ra=\alpha^2.\end{cases}\tag{23}$$

It is interesting to note that the volume flux comes only from the symmetric portion of the velocity field. The numerical values of the rate of flow Q are entered in the Table 2 for different values of the radiation parameter Ra, the Grashof number Gr and the temperature parameter θ_0. Table 2 shows that the magnitude of the rate of flow decreases with increase in either Ra or θ_0. On the other hand, the rate of flow increases with an increase in Grashof number Gr as expected since the velocity increases with increase in Gr.

Table 2: The rate of volume flux $-10\times Q$ for $\alpha = 0.5$

	Gr and $\theta_0 = 0.2$			θ_0 and $Gr = 10$		
Ra	2	6	10	0	0.2	0.4
2	0.20334	0.61003	1.01672	1.69454	1.01672	0.33891
4	0.17446	0.52339	0.87232	1.45387	0.87232	0.29077
6	0.15285	0.45855	0.76425	1.27375	0.76425	0.25475
8	0.13606	0.40818	0.68031	1.13384	0.68031	0.22677

Equation (23) shows that if $\theta_0 = \frac{1}{2}$ then the rate of flow $Q = 0$, which means that the cavity is closed. On the other hand, the maximum rate of flow occurs at $\theta_0 = 0$ and is given by

$$Q=\begin{cases}\dfrac{Gr}{Ra-\alpha^2}\left(\dfrac{\tanh\frac{\alpha}{2}}{\alpha}-\dfrac{\tanh\frac{\sqrt{Ra}}{2}}{\sqrt{Ra}}\right) & \text{for } Ra\neq\alpha^2\\[2ex] \dfrac{Gr}{4\alpha^2}\left(\tanh^2\dfrac{\alpha}{2}+\dfrac{2}{\alpha}\tanh\dfrac{\alpha}{2}-1\right) & \text{for } Ra=\alpha^2\end{cases}\tag{24}$$

We shall now discuss the case when $\alpha = 1$ and $Ra = 1$. In this case the velocity field, temperature distribution, the flow rate and $(\theta_0)_c$ become

$$\theta(\eta)=\eta+\left(\frac{1}{2}-\theta_0\right)+\frac{1}{6}Ra\left(\eta^2-\frac{1}{4}\right)\left\{\eta+3\left(\frac{1}{2}-\theta_0\right)\right\}\tag{25}$$

$$u_1(\eta)=\begin{cases}Gr\Bigg[\dfrac{1}{6}\left(\dfrac{1}{4}-\eta^2\right)\left\{\eta+3\left(\dfrac{1}{2}-\theta_0\right)\right\}\\ -(\alpha^2+Ra)\left\{\dfrac{1}{384}\left(\dfrac{1}{2}-\theta_0\right)\left(16\eta^4-24\eta^2+5\right)\right.\\ \left.+\dfrac{1}{5760}\left(48\eta^5-40\eta^3+7\eta\right)\right\}\Bigg] \text{for } Ra\neq\alpha^2,\\[2ex] Gr\Bigg[\dfrac{1}{6}\left(\dfrac{1}{4}-\eta^2\right)\left\{\eta+3\left(\dfrac{1}{2}-\theta_0\right)\right\}\\ -\dfrac{\alpha^2}{5760}\left\{30\left(\dfrac{1}{2}-\theta_0\right)\left(16\eta^4-24\eta^2+5\right)\right.\\ \left.+2\left(48\eta^5-40\eta^3+7\eta\right)\right\}\Bigg] \text{ for } Ra=\alpha^2,\end{cases}\tag{26}$$

$$Q=\begin{cases}Gr\left(\dfrac{1}{2}-\theta_0\right)\left(\dfrac{1}{12}-\dfrac{Ra+\alpha^2}{120}\right) & \text{for } Ra\neq\alpha^2,\\[2ex] Gr\left(\dfrac{1}{2}-\theta_0\right)\left(\dfrac{1}{12}-\dfrac{\alpha^2}{60}\right) & \text{for } Ra=\alpha^2,\end{cases}\tag{27}$$

$$(\theta_0)_c=\begin{cases}\dfrac{1}{3}-\dfrac{Ra+\alpha^2}{90} & \text{for } Ra\neq\alpha^2\\[2ex] \dfrac{1}{3}-\dfrac{\alpha^2}{45} & \text{for } Ra=\alpha^2.\end{cases}\tag{28}$$

Taking the limit $Ra \to 0$, the equations (25) - (28) become

$$\theta(\eta) = \eta + \left(\frac{1}{2} - \theta_0\right), \quad (29)$$

$$u_1(\eta) = \frac{Gr}{\alpha^2}\left[\eta + \left(\frac{1}{2} - \theta_0\right)\left(1 - \frac{\cosh\alpha\eta}{\cosh\frac{\alpha}{2}}\right) - \frac{1}{2}\frac{\sinh\alpha\eta}{\sinh\frac{\alpha}{2}}\right], \quad (30)$$

$$Q = \frac{Gr}{\alpha^2}\left(\frac{1}{2} - \theta_0\right)\left(1 - \frac{2\tanh\frac{\alpha}{2}}{\alpha}\right), \quad (31)$$

$$\left(\theta_0\right)_c = \frac{1}{2} + \frac{2 - \alpha\coth\frac{\alpha}{2}}{2\alpha\tanh\frac{\alpha}{2}}. \quad (32)$$

Equations (29) - (32) are identical with the equations (11) - (13) and (15) of Weidman [17].

In the limit $\alpha \to 0$ corresponding to clear fluid and $Ra \to 0$, the equations (22) - (25) become

$$\theta(\eta) = \eta + \left(\frac{1}{2} - \theta_0\right), \quad (33)$$

$$u_1(\eta) = Gr\left[\frac{1}{6}\left(\frac{1}{4} - \eta^2\right)\left\{\eta + 3\left(\frac{1}{2} - \theta_0\right)\right\}\right], \quad (34)$$

$$Q = \frac{Gr}{12}\left(\frac{1}{2} - \theta_0\right), \quad (35)$$

$$\left(\theta_0\right)_c = \frac{1}{3}. \quad (36)$$

Equations (33) - (36) are the conduction regime solutions as reported by B ü hler [3].

4. CONCLUSION

The effects of radiative heat transfer on the fully developed free convection flow of a viscous incompressible fluid-saturated porous medium between vertical channel have been analyzed in the presence of a uniform gravitational field. Radiation is found to have significant effects on the velocity field and temperature distribution. It is found that the velocity decreases with an increase in porosity of the medium. It is found that the critical wall temperature $\left(\theta_0\right)_c$ at the cold wall decreases with increase in either radiation parameter Ra or porosity parameter α.

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