A Study on Anti-Fuzzy Subsemiring of a Semiring

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ABSTRACT
In this paper, we made an attempt to study the algebraic nature of anti- fuzzy subsemiring of a semiring and we introduce the some theorems in anti-fuzzy subsemiring of a semiring.

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KEY WORDS
fuzzy set, fuzzy subsemiring, anti-fuzzy subsemiring, anti-fuzzy normal subsemiring, homomorphism, anti-homomorphism, isomorphism, anti-isomorphism.

1. INTRODUCTION
There are many concepts of universal algebras generalizing an associative ring (*R ; + ; .*). Some of them in particular, nearrings and several kinds of semirings have been proven very useful. An algebra (R ; +, .) is said to be a semiring if (R ; +) and (R ; .) are semigroups satisfying a. (b+c) = a. b+a. c and (b+c) . a = b. a+c. a for all a, b and c in R. A semiring R is said to be additively commutative if a+b = b+a for all a, b in R. A semiring R may have an identity 1, defined by 1. a = a = a. 1 and a zero 0, defined by 0+a = a = a+0 and a.0 = 0 = 0.a for all a in R. After the introduction of fuzzy sets by L.A.Zadeh[9], several researchers explored on the generalization of the concept of fuzzy sets. The notion of fuzzy subnearrings and ideals was introduced by S.Abou Zaid [1]. In this paper, we introduce the some theorems in anti-fuzzy subsemiring of a semiring.

2. PRELIMINARIES

2.1 Definition
Let X be a non–empty set. A **fuzzy subset A** of X is a function A : X → [0, 1].

2.2 Definition:
Let R be a semiring. A fuzzy subset A of R is said to be a **fuzzy subsemiring** (FSSR) of R if it satisfies the following conditions:

(i) $\mu_A(x + y) \geq \min\{\mu_A(x), \mu_A(y)\}$,

(ii) $\mu_A(xy) \geq \min\{ \mu_A(x), \mu_A(y) \}$, for all x and y in R.

2.3 Definition
Let R be a semiring. A fuzzy subset A of R is said to be an **anti-fuzzy subsemiring** (AFSSR) of R if it satisfies the following conditions:

(i) $\mu_A(x + y) \leq \max\{ \mu_A(x), \mu_A(y) \}$,

(ii) $\mu_A(xy) \leq \max\{ \mu_A(x), \mu_A(y) \}$, for all x and y in R.

2.4 Definition
Let R be a semiring. An anti-fuzzy subsemiring A of R is said to be an **anti-fuzzy normal subsemiring** (AFNSSR) of R if it satisfies the following conditions:

(i) $\mu_A(x+y) = \mu_A(y+x)$,

(ii) $\mu_A(xy) = \mu_A(yx)$, for all x and y in R.

2.5 Definition
Let A and B be fuzzy subsets of sets G and H, respectively. The anti-product of A and B, denoted by AxB, is defined as AxB $=\{\langle(x, y), \mu_{AxB}(x,y)\rangle$/for all x in G and y in H}, where $\mu_{AxB}(x, y) = \max\{ \mu_A(x), \mu_B(y) \}$.

2.6 Definition:
Let A be a fuzzy subset in a set S, the anti-strongest fuzzy relation on S, that is a fuzzy relation on A is V given by $\mu_V(x, y) = \max \{ \mu_A(x), \mu_A(y) \}$, for all x and y in S.

2.7 Definition:
Let (R, +, ·) and (R^{l}, +, ·) be any two semirings. Let f : R → R^{l} be any function and A be an anti-fuzzy subsemiring in R, V be an anti-fuzzy subsemiring in f(R) = R^{l}, defined by $\mu_V(y) = \inf\limits_{x\in f^{-1}(y)} \mu_A(x)$, for all x in R and y in R^{l}. Then A is called a preimage of V under f and is denoted by $f^{-1}(V)$.

2.8 Definition:
Let (R, +, **.**) and (R^{l}, +, **.**) be any two semirings. Then the function f : R → R^{l} is called a **semiring homomorphism** if f(x+y) = f(x) + f(y), f(xy) = f(x) f(y), for all x and y in R.

2.9 Definition:
Let (R, +, .) and (R^{l}, +, .) be any two semirings. Then the function f : R → R^{l} is called a **semiring anti-homomorphism** if f(x + y) = f(y) + f(x), f(xy) = f(y) f(x), for all x and y in R.

2.10 Definition:
Let (R, +, **.**) and (R^{l}, +, **.**) be any two semirings. Then the function f : R → R^{l} be a semiring homomorphism. If f is one-to-one and onto, then f is called a **semiring isomorphism**.

2.11 Definition:
Let (R, +, **.**) and (R^{l}, +, **.**) be any two semirings. Then the function f : R → R^{l} be a semiring anti-homomorphism. If f is one-to-one and onto, then f is called a **semiring anti-isomorphism**.

2.12 Definition:
Let A be an anti-fuzzy subsemiring of a semiring (R, +, ·) and a in R. Then the **pseudo anti-fuzzy coset** $(aA)^p$ is defined by $((a\mu_A)^p)(x) = p(a)\mu_A(x)$, for every x in R and for some p in P.

3. PROPERTIES OF ANTI-FUZZY SUBSEMIRING OF A SEMIRING

3.1. Theorem:
Union of any two anti-fuzzy subsemiring of a semiring R is an anti-fuzzy subsemiring of R.

Proof: Let A and B be any two anti-fuzzy subsemirings of a semiring R and x and y in R.Let A={(x, $\mu_A(x)$)/x∈R}and B={(x,$\mu_B(x)$) / x∈R}and also let C = A ∪ B = { (x, $\mu_C(x)$) / x∈R}, where max{ $\mu_A(x)$, $\mu_B(x)$} = $\mu_C(x)$. Now, $\mu_C(x+y)$ = max{$\mu_A(x + y)$,$\mu_B(x+ y)$} ≤ max{ max { $\mu_A(x)$, $\mu_A(y)$ }, max{$\mu_B(x)$, $\mu_B(y)$}} = max{max{ $\mu_A(x)$, $\mu_B(x)$ }, max{ $\mu_A(y)$, $\mu_B(y)$ } } = max{ $\mu_C(x)$, $\mu_C(y)$ }. Therefore, $\mu_C(x + y)$ ≤max{$\mu_C(x)$,$\mu_C(y)$},for all x and y in R. And, $\mu_C(xy)$ = max {$\mu_A(xy)$, $\mu_B(xy)$}≤max{max{$\mu_A(x)$, $\mu_A(y)$}, max{$\mu_B(x)$, $\mu_B(y)$ } }=max{max{$\mu_A(x)$,$\mu_B(x)$ }, max{$\mu_A(y)$,$\mu_B(y)$}}= max{ $\mu_C(x)$, $\mu_C(y)$}.Therefore, $\mu_C(xy) \le$ max{ $\mu_C(x)$, $\mu_C(y)$ }, for all x and y in R. Therefore C is an anti-fuzzy subsemiring of a semiring R. Hence the union of any two anti-fuzzy subsemirings of a semiring R is an anti-fuzzy subsemiring of R.

3.2. Theorem:
The union of a family of anti-fuzzy subsemirings of semiring R is an anti-fuzzy subsemiring of R.

Proof: Let {V_i : i∈I} be a family of anti-fuzzy subsemirings of a semiring R and let A = $\bigcup_{i\in I} V_i$. Let x and y in R. Then,

$\mu_A(x + y) = \sup_{i\in I} \mu_{Vi}(x + y) \le \sup_{i\in I} \max\{ \mu_{Vi}(x), \mu_{Vi}(y)\} = \max\{ \sup_{i\in I} \mu_{Vi}(x), \sup_{i\in I} \mu_{Vi}(y) \} = \max\{ \mu_A(x), \mu_A(y) \}$.Therefore, $\mu_A(x+y) \le \max\{ \mu_A(x), \mu_A(y) \}$, for all x and y in R.

And,$\mu_A(xy) = \sup_{i\in I} \mu_{Vi}(xy) \le \sup_{i\in I} \max\{\mu_{Vi}(x),\mu_{Vi}(y)\} = \max\{ \sup_{i\in I} \mu_{Vi}(x), \sup_{i\in I} \mu_{Vi}(y)\} = \max\{\mu_A(x),\mu_A(y)\}$. Therefore, $\mu_A(xy) \le \max\{ \mu_A(x), \mu_A(y) \}$, for all x and y in R. That is, A is an anti-fuzzy subsemiring of a semiring R. Hence, the union of a family of anti-fuzzy subsemirings of R is an anti-fuzzy subsemiring of R.

3.3. Theorem:
If A and B are any two anti-fuzzy subsemirings of the semirings R_1 and R_2 respectively, then anti-product AxB is an anti-fuzzy subsemiring of R_1xR_2.

Proof: Let A and B be two anti-fuzzy subsemirings of the semirings R_1 and R_2 respectively. Let x_1 and x_2 be in R_1, y_1 and y_2 be in R_2. Then (x_1, y_1) and (x_2, y_2) are in R_1xR_2.Now, μ_{AxB} [(x_1, y_1)+(x_2, y_2)] =$\mu_{AxB}(x_1+x_2,y_1+y_2)$=max{$\mu_A(x_1+x_2)$,$\mu_B(y_1+y_2)$} ≤max{max{$\mu_A(x_1)$,$\mu_A(x_2)$},max{$\mu_B(y_1)$,$\mu_B(y_2)$}} =max{max{$\mu_A(x_1)$,$\mu_B(y_1)$},max{$\mu_A(x_2)$,$\mu_B(y_2)$}} = max{$\mu_{AxB}$ (x_1, y_1), μ_{AxB} (x_2, y_2)}. Therefore, μ_{AxB} [(x_1, y_1) + (x_2, y_2)] ≤ max{ μ_{AxB} (x_1, y_1), μ_{AxB} (x_2, y_2) }. Also, μ_{AxB} [(x_1, y_1)(x_2, y_2)] = μ_{AxB} (x_1x_2, y_1y_2) =max{$\mu_A(x_1x_2)$, $\mu_B(y_1y_2)$ } ≤ max{max{$\mu_A(x_1)$,$\mu_A(x_2)$ },max{ $\mu_B(y_1)$, $\mu_B(y_2)$ }} = max{max{$\mu_A(x_1)$, $\mu_B(y_1)$}, max{$\mu_A(x_2)$, $\mu_B(y_2)$}} = max{μ_{AxB} (x_1, y_1), μ_{AxB} (x_2, y_2) }. Therefore, μ_{AxB} [(x_1, y_1)(x_2, y_2)] ≤ max{ μ_{AxB} (x_1, y_1), μ_{AxB} (x_2, y_2)}. Hence AxB is an anti-fuzzy subsemiring of semiring of R_1xR_2.

3.4. Theorem:
Let A be a fuzzy subset of a semiring R and V be the strongest anti-fuzzy relation of R. Then A is an anti-fuzzy subsemiring of R if and only if V is an anti-fuzzy subsemiring of RxR.

Proof: Suppose that A is an anti-fuzzy subsemiring of a semiring R.Then for any x = (x_1, x_2) and y = (y_1, y_2) are in RxR. We have, μ_V (x+ y) =μ_V[(x_1,x_2)+(y_1,y_2)] =$\mu_V(x_1+y_1,x_2+y_2)$= max{$\mu_A(x_1+y_1)$,$\mu_A(x_2+y_2)$}≤max{max{$\mu_A(x_1)$, $\mu_A(y_1)$},max{$\mu_A(x_2)$,$\mu_A(y_2)$}}=max {max{$\mu_A(x_1)$,$\mu_A(x_2)$},max{$\mu_A(y_1)$,$\mu_A(y_2)$}} =max{$\mu_V(x_1,x_2)$,$\mu_V(y_1,y_2)$} = max{ μ_V (x), μ_V (y) }. Therefore, μ_V (x + y) ≤ max{ μ_V (x), μ_V (y) }, for all x and y in RxR. And,$\mu_V(xy)$=μ_V [(x_1, x_2) (y_1, y_2)] = μ_V(x_1y_1 , x_2y_2) = max { $\mu_A(x_1y_1)$, $\mu_A(x_2y_2)$} ≤ max {max{ $\mu_A(x_1)$, $\mu_A(y_1)$ }, max{$\mu_A(x_2)$, $\mu_A(y_2)$ }} = max{max {$\mu_A(x_1)$, $\mu_A(x_2)$}, max{$\mu_A(y_1)$, $\mu_A(y_2)$ }} = max{$\mu_V(x_1, x_2)$,$\mu_V(y_1, y_2)$}= max{ μ_V (x), μ_V (y) }. Therefore, μ_V (xy) ≤ max {μ_V (x), μ_V (y)}, for all x and y in RxR. This proves that V is an anti-fuzzy subsemiring of RxR. Conversely assume that V is an anti-fuzzy subsemiring of RxR, then for any x = (x_1, x_2) and y = (y_1, y_2) are in RxR, we have max{ $\mu_A(x_1+ y_1)$, $\mu_A(x_2+ y_2)$ } = μ_V(x_1+ y_1 , x_2+ y_2) =μ_V[(x_1,x_2)+(y_1, y_2)]=μ_V (x+ y)≤ max{μ_V(x),μ_V (y)} = max{ μ_V (x_1, x_2), μ_V (y_1, y_2) }= max{max{ $\mu_A(x_1)$, $\mu_A(x_2)$ }, max{$\mu_A(y_1)$, $\mu_A(y_2)$} }. If $\mu_A(x_1+ y_1) \ge \mu_A(x_2+ y_2)$, $\mu_A(x_1) \ge \mu_A(x_2)$, $\mu_A(y_1) \ge \mu_A(y_2)$, we get, $\mu_A(x_1+ y_1)$ ≤ max{ $\mu_A(x_1)$, $\mu_A(y_1)$}, for all x_1 and y_1 in R. And, max{$\mu_A(x_1y_1)$, $\mu_A(x_2y_2)$ } = μ_V(x_1y_1 , x_2y_2) = μ_V[(x_1, x_2) (y_1, y_2)] = μ_V (x y) ≤max{μ_V (x), μ_V (y)} = max{μ_V (x_1, x_2), μ_V (y_1, y_2)} = max{max{$\mu_A(x_1)$, $\mu_A(x_2)$}, max {$\mu_A(y_1)$, $\mu_A(y_2)$}}. If $\mu_A(x_1y_1) \ge \mu_A(x_2y_2)$,$\mu_A(x_1) \ge \mu_A(x_2)$, $\mu_A(y_1) \ge \mu_A(y_2)$, we get $\mu_A(x_1y_1)$≤ max{ $\mu_A(x_1)$, $\mu_A(y_1)$ }, for all x_1, y_1 in R. Therefore A is an anti-fuzzy subsemiring of R.

3.5. Theorem:
A is an anti-fuzzy subsemiring of a semiring (R, +, ·) if and only if $\mu_A(x+y)$ ≤ max{$\mu_A(x)$, $\mu_A(y)$}, $\mu_A(xy)$ ≤ max{$\mu_A(x)$, $\mu_A(y)$ }, for all x and y in R.

Proof: It is trivial.

3.6. Theorem:
If A is an anti-fuzzy subsemiring of a semiring (R, +, ·), then H = { x / x∈R: $\mu_A(x)$ =0} is either empty or is a subsemiring of R.

Proof: If no element satisfies this condition, then H is empty. If x and y in H, then $\mu_A(x+y)$ ≤ max {$\mu_A(x)$, $\mu_A(y)$}= max {0, 0}=0. Therefore, $\mu_A(x+y)$ = 0. And, μ_A (xy) ≤ max {$\mu_A(x)$, $\mu_A(y)$} = max{0,0} = 0. Therefore, μ_A(xy)=0. We get x+y, xy in H. Therefore, H is a subsemiring of R. Hence H is either empty or is a subsemiring of R.

3.7. Theorem:
If A be an anti-fuzzy subsemiring of a semiring (R, +, ·), then if $\mu_A(x+ y)$ = 1, then either $\mu_A(x)$ =1 or $\mu_A(y)$ = 1, for all x and y in R.

Proof: Let x and y in R. By the definition $\mu_A(x+ y)$ ≤ max { μ_A(x), μ_A(y) }, which implies that 1 ≤ max {$\mu_A(x)$, $\mu_A(y)$}. Therefore, either $\mu_A(x)$ = 1 or $\mu_A(y)$ = 1.

In the following Theorem ◦ is the composition operation of functions :

3.8. Theorem:
Let A be an anti-fuzzy subsemiring of a semiring H and f is an isomorphism from a semiring R onto H. Then A◦f is an anti-fuzzy subsemiring of R.

Proof: Let x and y in R and A be an anti-fuzzy subsemiring of a semiring H.

Then we have, $(\mu_A \circ f)(x+y)=\mu_A(f(x+y)) = \mu_A(f(x)+f(y)) \leq \max\{\mu_A(f(x)), \mu_A(f(y))\} \leq \max\{(\mu_A \circ f)(x), (\mu_A \circ f)(y)\}$, which implies that $(\mu_A \circ f)(x+y) \leq \max\{(\mu_A \circ f)(x), (\mu_A \circ f)(y)\}$. And $(\mu_A \circ f)(xy) = \mu_A(f(xy)) = \mu_A(f(x)f(y)) \leq \max\{\mu_A(f(x)), \mu_A(f(y))\} \leq \max\{(\mu_A \circ f)(x), (\mu_A \circ f)(y)\}$, which implies that $(\mu_A \circ f)(xy) \leq \max\{(\mu_A \circ f)(x), (\mu_A \circ f)(y)\}$. Therefore (A◦f) is an anti-fuzzy subsemiring of a semiring R.

3.9. Theorem:
Let A be an anti-fuzzy subsemiring of a semiring H and f is an anti-isomorphism from a semiring R onto H. Then A◦f is an anti-fuzzy subsemiring of R.

Proof: Let x and y in R and A be an anti-fuzzy subsemiring of a semiring H.

Then we have, $(\mu_A \circ f)(x+y) = \mu_A(f(x+y)) = \mu_A(f(y)+f(x)) \leq \max\{\mu_A(f(x)), \mu_A(f(y))\} \leq \max\{(\mu_A \circ f)(x), (\mu_A \circ f)(y)\}$, which implies that $(\mu_A \circ f)(x+y) \leq \max\{(\mu_A \circ f)(x), (\mu_A \circ f)(y)\}$. And $(\mu_A \circ f)(xy) = \mu_A(f(xy)) = \mu_A(f(y)f(x)) \leq \max\{\mu_A(f(x)), \mu_A(f(y))\} \leq \max\{(\mu_A \circ f)(x), (\mu_A \circ f)(y)\}$, which implies that $(\mu_A \circ f)(xy) \leq \max\{(\mu_A \circ f)(x), (\mu_A \circ f)(y)\}$. Therefore A◦f is an anti-fuzzy subsemiring of a semiring R.

3.10. Theorem:
Let A be an anti-fuzzy subsemiring of a semiring (R, +, .), then the pseudo anti-fuzzy coset $(aA)^p$ is an anti-fuzzy subsemiring of a semiring R, for a in R.

Proof: Let A be an anti-fuzzy subsemiring of a semiring R. For every x and y in R, we have, $((a\mu_A)^p)(x+y) = p(a)\mu_A(x+y) \leq p(a)\max\{\mu_A(x), \mu_A(y)\} = \max\{p(a)\mu_A(x), p(a)\mu_A(y)\} = \max\{((a\mu_A)^p)(x), ((a\mu_A)^p)(y)\}$. Therefore, $((a\mu_A)^p)(x+y) \leq \max\{((a\mu_A)^p)(x), ((a\mu_A)^p)(y)\}$. Now, $((a\mu_A)^p)(xy) = p(a)\mu_A(xy) \leq p(a)\max\{\mu_A(x), \mu_A(y)\} = \max\{p(a)\mu_A(x), p(a)\mu_A(y)\} = \max\{((a\mu_A)^p)(x), ((a\mu_A)^p)(y)\}$. Therefore, $((a\mu_A)^p)(xy) \leq \max\{((a\mu_A)^p)(x), ((a\mu_A)^p)(y)\}$. Hence $(aA)^p$ is an anti-fuzzy subsemiring of a semiring R.

3.11. Theorem:
Let (R, +, .) and (R^l, +, .) be any two semirings. The homomorphic image of an anti-fuzzy subsemiring of R is an anti-fuzzy subsemiring of R^l.

Proof: Let (R, +, .) and (R^l, +, .) be any two semirings. Let f: $R \to R^l$ be a homomorphism. Then, f(x+y) = f(x) + f(y) and f(xy) = f(x) f(y), for all x and y in R. Let V = f(A), where A is an anti-fuzzy subsemiring of R. We have to prove that V is an anti-fuzzy subsemiring of R^l. Now, for f(x), f(y) in R^l, $\mu_v(f(x)+f(y)) = \mu_v(f(x+y)) \leq \mu_A(x+y) \leq \max\{\mu_A(x), \mu_A(y)\}$ which implies that $\mu_v(f(x)+f(y)) \leq \max\{\mu_v(f(x)), \mu_v(f(y))\}$. Again, $\mu_v(f(x)f(y)) = \mu_v(f(xy)) \leq \mu_A(xy) \leq \max\{\mu_A(x), \mu_A(y)\}$ which implies that $\mu_v(f(x)f(y)) \leq \max\{\mu_v(f(x)), \mu_v(f(y))\}$. Hence V is an anti-fuzzy subsemiring of R^l.

3.12. Theorem:
Let (R, +, .) and (R^l, +, .) be any two semirings. The homomorphic preimage of an anti-fuzzy subsemiring of R^l is an anti-fuzzy subsemiring of R.

Proof: Let (R, +, .) and (R^l, +, .) be any two semirings. Let f : $R \to R^l$ be a homomorphism. Then, f(x+y) = f(x) + f(y) and f(xy) = f(x) f(y), for all x and y in R. Let V = f(A), where V is an anti-fuzzy subsemiring of R^l. We have to prove that A is an anti- fuzzy subsemiring of R. Let x and y in R. Then, $\mu_A(x+y) = \mu_v(f(x+y)) = \mu_v(f(x)+f(y)) \leq \max\{\mu_v(f(x)), \mu_v(f(y))\} = \max\{\mu_A(x), \mu_A(y)\}$ which implies that $\mu_A(x+y) \leq \max\{\mu_A(x), \mu_A(y)\}$. Again, $\mu_A(xy) = \mu_v(f(xy)) = \mu_v(f(x)f(y)) \leq \max\{\mu_v(f(x)), \mu_v(f(y))\} = \max\{\mu_A(x), \mu_A(y)\}$ which implies that $\mu_A(xy) \leq \max\{\mu_A(x), \mu_A(y)\}$. Hence A is an anti-fuzzy subsemiring of R.

3.13. Theorem:
Let (R, +, .) and (R^l, +, .) be any two semirings. The anti-homomorphic image of an anti-fuzzy subsemiring of R is an anti-fuzzy subsemiring of R^l.

Proof: Let (R, +, .) and (R^l, +, .) be any two semirings. Let f: $R \to R^l$ be an anti-homomorphism. Then, f(x+y) = f(y) + f(x) and f(xy) = f(y) f(x), for all x, y ∈ R. Let V = f(A), where A is an anti-fuzzy subsemiring of R. We have to prove that V is an anti-fuzzy subsemiring of R^l. Now, for f(x), f(y) in R^l, $\mu_v(f(x)+f(y)) = \mu_v(f(y+x)) \leq \mu_A(y+x) \leq \max\{\mu_A(y), \mu_A(x)\} = \max\{\mu_A(x), \mu_A(y)\}$ which implies that $\mu_v(f(x)+f(y)) \leq \max\{\mu_v(f(x)), \mu_v(f(y))\}$. Again, $\mu_v(f(x)f(y)) = \mu_v(f(yx)) \leq \mu_A(yx) \leq \max\{\mu_A(y), \mu_A(x)\} = \max\{\mu_A(x), \mu_A(y)\}$, which implies that $\mu_v(f(x)f(y)) \leq \max\{\mu_v(f(x)), \mu_v(f(y))\}$. Hence V is an anti-fuzzy subsemiring of R^l.

3.14. Theorem:
Let (R, +, .) and (R^l, +, .) be any two semirings. The anti-homomorphic preimage of an anti-fuzzy subsemiring of R^l is an anti-fuzzy subsemiring of R.

Proof: Let (R, +, .) and (R^l, +, .) be any two semirings. Let f: $R \to R^l$ be an anti-homomorphism. Then, f(x+y) = f(y) + f(x) and f(xy) = f(y) f(x), for all x and y in R. Let V = f(A), where V is an anti-fuzzy subsemiring of R^l. We have to prove that A is an anti-fuzzy subsemiring of R. Let x and y in R. Then $\mu_A(x+y) = \mu_v(f(x+y)) = \mu_v(f(y)+f(x)) \leq \max\{\mu_v(f(y)), \mu_v(f(x))\} = \max\{\mu_v(f(x)), \mu_v(f(y))\} = \max\{\mu_A(x), \mu_A(y)\}$, which implies that $\mu_A(x+y) \leq \max\{\mu_A(x), \mu_A(y)\}$. Again, $\mu_A(xy) = \mu_v(f(xy)) = \mu_v(f(y)f(x)) \leq \max\{\mu_v(f(y)), \mu_v(f(x))\} = \max\{\mu_v(f(x)), \mu_v(f(y))\} = \max\{\mu_A(x), \mu_A(y)\}$ which implies that $\mu_A(xy) \leq \max\{\mu_A(x), \mu_A(y)\}$. Hence A is an anti-fuzzy subsemiring of R.

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