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Transient Analysis of an Interdependent Forked Tandem Queuing Model with Load Dependent Service Rate

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ABSTRACT

In this paper, we develop and analyze an interdependent forked queueing model with state dependent service times. Here, it is assumed that the arrival and service processes are correlated and follows a multivariate poisson process. Using the difference-differential equations, the joint probability generating function of the number of customers in each queue is derived. The system performance like the average number of customers in each queue, the average waiting time of a customer, the throughput of each service station, the idleness of the servers are derived explicitly. The sensitivity analysis of the model reveals that the dependency parameter and state dependent service rates can reduce congestion in queues and average waiting time of the customer. This model also includes some of the earlier models as particular cases for specific values of the parameters. The forked queueing models are much useful for analyzing and monitoring several communication networks and production processes.

Keywords

Forked Queueing Model, Multivariate Poisson Process, Transient analysis, Performance measures.

1. INTRODUCTION

Congestion is a natural phenomenon in every day life. Queueing is a mechanism that is used to handle congestion. A system consisting of a service facility, a process of arrival of customers who wish to be served by the facility and the process of service is called a queueing system. Starting from pioneering work by ERLANG, A.K (1909) [1], remarkable progress has been made in queueing theory and its applications. In many of the realistic situations, the output from one queueing process serves as the input to another. This sort of queueing system is known as tandem or serial queues.

Tandem queueing models provide the basic framework for analyzing many of the realistic situations arising at places like manufacturing, production scheduling, data or voice transmission, transportation, communication networks, biological studies, machine repairing, assembly line scheduling, reliability analysis, neuro-physiological problems etc,. Various queueing models have been developed for analyzing many of the realistic situations with wide variety of assumptions. Relaxation of some of these assumptions brings the queueing models much closer to reality.

Due to the stochastic nature of the constituent processes namely arrival and service processes of the queueing systems, these models gained a lot of importance in stochastic modeling. Along with other assumptions it is customary to assume that these two processes are interdependent of each other. The assumption is valid only if these two processes are studied separately. However, in several situations like store and forward data communication network, the standard type of independent assumptions is realistically in approximate due to the fact that messages generally preserve their length as they traverse the network (S E Raphin B.Calo) [2]. In these types of networks, arrival and service processes are to be viewed as interdependent in order to have optimal operation policies and to predict the performance measures more effectively.

B.W.Conolly(1968)[3], N.S.Kambo and H.S.Bhalaik (1982)[4], F.W.Kerry et.al (1991)[5], David Lucatoni (1998)[6] have developed some queueing models with indirect dependence between arrival and service processes. They analyzed the models via simulation or by theory of approximation. Srinivas Rao et.al (2000)[7], Aftab Begum et.al (2002)[8], Pal.S(2003)[9],Prasad Reddy and K.Srinivas Rao(2006)[10], S.S.Mishra(2009)[11], Maurya(2010)[12] have developed interdependent queueing models with assumption that arrival and service processes are correlated and follow a Bivariate poisson process. In all these models, they assumed that there is only one service station. Srinivas Rao et.al (2006)[10] developed an interdependent queueing model with two queues in tandem using Bi- variate poisson process. But in many practical situations, the arrival of the system will join the first queue and after getting service at first service station, the customer may join either second or third queues which are in parallel. This sort of queueing network is generally known as forked queue. Very little work has been reported regarding interdependent forked queueing models which are very much useful for analyzing performance of several communications systems and production processes. Hence in this paper, we develop and analyze an interdependent forked queue model with state dependent service rates. In addition to interdependence between arrival and service processes, the state dependent service rates can further reduce congestion in queues and average waiting time of the customer. Here it is assumed that number of service completions in each node and the number of arrivals are correlated and follows a trivariate poisson process.

Using the difference differential equations, the transient behavior of the model is analyzed by deriving the joint probability generating function of the number of customers in each queue. The performance measures of the system are derived explicitly. This model also includes several of the earlier models as particular cases for specific and limiting values of the parameters.

2. INTERDEPENDENT FORKED QUEUEING MODEL WITH STATE DEPENDENT SERVICE RATES

Consider three queues Q_1, Q_2, Q_3 and three service stations S_1, S_2, S_3 connected as forked queueing model. Let the capacity of the queue be infinite. We assume that the arrivals after getting service through first node may join either S_2 or S_3, which are parallel and connected to the first node in tandem. That is, the unit after getting served at S_1 may join either in the second queue Q_2 with probability π or in the third queue Q_3 with probability $\theta = (1 - \pi)$. We further assume that the number of arrivals and service completions in each service station are correlated and follows a correlated poisson process, with the probability mass function with parameters $\lambda, \mu_1, \mu_2, \mu_3$ and $\in$. It is further assumed that the service rate in each service station is linearly dependent on the content of the queue connected to it.

Let n_1, n_2 and n_3 denote the number of customers in the first queue, second queue and third queue respectively. The schematic diagram of the interdependent forked queueing model is shown in figure (1).

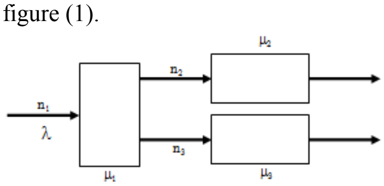

Figure 1: Interdependent Forked Queueing Model

Let $P_{n_1,n_2,n_3}(t)$ be the probability that there are n_1 customers in the first queue, n_2 customers in the second queue and n_3 customers in the third queue of the system at time t.

The difference-differential equations of the model are

$$\frac{\partial P_{n_1,n_2,n_3}(t)}{\partial t} = -\left((\lambda-\in)+n_1(\mu_1-\in)+n_2(\mu_2-\in)+n_3(\mu_3-\in)\right)P_{n_1,n_2,n_3}(t)+(\lambda-\in)P_{n_1-1,n_2,n_3}(t)$$
$$+(n_1+1)\,\Pi(\mu_1-\in)P_{n_1+1,\,n_2-1,\,n_3}(t)+(n_1+1)\,(\mu_1-\in)\theta P_{n_1+1,n_2,n_3-1}(t)$$
$$+(n_2+1)(\mu_2-\in)P_{n_1,n_2+1,n_3}(t)+(n_3+1)(\mu_3-\in)P_{n_1,n_2,n_3+1}(t), \quad n_1, n_2, n_3 > 0$$

$$\frac{\partial P_{0,n_2,n_3}(t)}{\partial t} = -\left((\lambda-\in)+n_2(\mu_2-\in)+n_3(\mu_3-\in)\right)P_{0,n_2,n_3}(t)+\Pi(\mu_1-\in)P_{1,\,n_2-1,\,n_3}(t)$$
$$+(\mu_1-\in)\theta P_{1,n_2,n_3-1}(t)+(n_2+1)(\mu_2-\in)P_{0,n_2+1,n_3}(t)$$
$$+(n_3+1)(\mu_3-\in)P_{0,n_2,n_3+1}(t), \quad n_1=0, n_2, n_3 > 0$$

$$\frac{\partial P_{n_1,0,n_3}(t)}{\partial t} = -\left((\lambda-\in)+n_1(\mu_1-\in)+n_3(\mu_3-\in)\right)P_{n_1,0,n_3}(t)+(\lambda-\in)P_{n_1-1,0,\,n_3}(t)$$
$$+(n_1+1)(\mu_1-\in)\theta P_{n_1+1,0,n_3-1}(t)+(\mu_2-\in)P_{n_1,1,n_3}(t)$$
$$+(n_3+1)(\mu_3-\in)P_{n_1,0,n_3+1}(t), \quad n_1, n_3 > 0, n_2 = 0$$

$$\frac{\partial P_{n_1,n_2,0}(t)}{\partial t} = -\left((\lambda-\in)+n_1(\mu_1-\in)+n_2(\mu_2-\in)\right)P_{n_1,n_2,0}(t)+(\lambda-\in)P_{n_1-1,\,n_2,\,0}(t)$$
$$+(n_2+1)\Pi(\mu_1-\in)P_{n_1+1,n_2-1,0}(t)$$
$$+(n_2+1)(\mu_2-\in)P_{n_1,\,n_2+1,\,0}(t)+(\mu_3-\in)P_{n_1,n_2,1}(t) \quad n_1, n_2 > 0, n_3 = 0$$

$$\frac{\partial P_{0,0,n_3}(t)}{\partial t} = -\left((\lambda-\in)+n_3(\mu_3-\in)\right)P_{0,0,n_3}(t)+(\mu_1-\in)\theta P_{1,0,n_3-1}(t)$$
$$+(\mu_2-\in)P_{0,1,\,n_3}(t)+(n_3+1)(\mu_3-\in)P_{0,0,n_3+1}(t) \quad n_1, n_2 = 0, n_3 > 0$$

$$\frac{\partial P_{n_1,0,0}(t)}{\partial t} = -\left((\lambda-\in)+n_1(\mu_1-\in)\right)P_{n_1,0,0}(t)+(\lambda-\in)P_{n_1-1,\,0,0}(t)$$
$$+(\mu_2-\in)P_{n_1,1,0}(t)+(\mu_3-\in)P_{n_1,0,1}(t), \quad n_1 > 0, n_2\, n_3 = 0$$

$$\frac{\partial P_{0,n_2,0}(t)}{\partial t} = -\left((\lambda-\in)+n_2(\mu_2-\in)\right)P_{0,n_2,0}(t)+\Pi(\mu_1-\in)P_{1,\,n_2-1,0}(t)$$
$$+(n_2+1)(\mu_2-\in)P_{0,n_2+1,0}(t)+(\mu_3-\in)P_{0,n_2,1}(t), \quad n_1, n_3 = 0, n_2 > 0$$

$$\frac{\partial P_{0,0,0}(t)}{\partial t} = -(\lambda-\in)P_{0,0,0}(t)+(\mu_2-\in)P_{0,1,0}(t)+(\mu_3-\in)P_{0,0,1}(t) \quad n_1, n_2, n_3 = 0 \tag{1}$$

Let $P(S_1, S_2, S_3, t) = \sum_{n_1=0}^{\infty}\sum_{n_2=0}^{\infty}\sum_{n_3=0}^{\infty} P_{n_1,n_2,n_3}(t) S_1^{n_1}, S_2^{n_2}, S_3^{n_3}$ (2)

be the Joint Probability generating function of $P_{n_1,n_2,n_3}(t)$

Multiplying the Equation (1) with $S_1^{n_1} S_2^{n_2} S_3^{n_3}$ and summing over n_1, n_2, n_3 and simplifying, we get,

$$\frac{\partial P}{\partial t} = -(\mu_1-\in)\,(\Pi S_2+\theta S_3-S_1)\frac{\partial P}{\partial S_1}+(\mu_2-\in)(1-S_2)\frac{\partial P}{\partial S_2}+(\mu_3-\in)\,(1-S_3)\frac{\partial P}{\partial S_3}-(\lambda-\in)\,P(1-S_1) \tag{3}$$

Solving the Equation (3) by Lagrangian's method, the auxiliary equations are

$$\frac{dt}{1} = \frac{ds_1}{(\mu_1-\in)(S_1-\Pi S_2-\theta S_3)} = \frac{ds_2}{(\mu_2-\in)(S_2-1)} = \frac{ds_3}{(\mu_3-\in)(S_3-1)} = \frac{dp}{(\lambda-\in)P(S_1-1)} \tag{4}$$

Solving these equations, we get

$$a = (S_3-1)e^{-(\mu_3-\in)t} \qquad b = (S_2-1)e^{-(\mu_2-\in)t}$$
$$C = (S_1-1)e^{-(\mu_1-\in)t}+\frac{(\mu_1-\in)\Pi}{(\mu_2-\mu_1)}(S_2-1)\,e^{-(\mu_1-\in)t}+\frac{(\mu_1-t)\theta}{(\mu_3-\mu_1)}(S_3-1)e^{-(\mu_1-\in)t}$$
$$d = P(s_1, s_2, s_3 t)\exp\left[-(\lambda-\in)\left(\frac{(S_1-1)}{\mu_1-\in}+\frac{\Pi(S_2-1)}{\mu_2-\in}+\frac{\theta(S_3-1)}{\mu_3-\in}\right)\right] \tag{5}$$

where, a, b, c and d are arbitrary constants with the initial conditions $P_{000}(0) = 1$, $P_{000}(t) = 0 \quad \forall, t > 0$

The general solution of (3) gives the Probability generating function of the number of customers in the first queue, second queue and third queues respectively at time t, as $P(S_1, S_2, S_3, t)$.

Using the initial conditions, we get the Joint Probability generating function of the number of customers in the first queue, second queue and third queues respectively can be obtained as

$$P(S_1,S_2.S_3,t) = \exp\left\{(\lambda-\in)\left[\frac{1}{\mu_1-\in}(S_1-1)(1-e^{-(\mu_1-\in)t})+(S_2-1)\frac{\Pi}{\mu_2-\in}(1-e^{-(\mu_2-\in)t})+(S_2-1)\frac{\Pi}{(\mu_2-\mu_1)}(e^{-(\mu_2-\in)t}-e^{-(\mu_1-\in)t})\right.\right.$$
$$\left.\left.+(S_3-1)\frac{\theta}{(\mu_3-\in)}(1-e^{-(\mu_3-\in)t}+(S_3-1)\frac{\theta}{(\mu_3-\mu_1)}(e^{-(\mu_3-\in)t}-e^{-(\mu_1-\in)t})\right]\right\}$$

$$P(S_1,S_2.S_3,t) = \exp\left\{(\lambda-\in)\left[\frac{1}{\mu_1-\in}(S_1-1)(1-e^{-(\mu_1-\in)t})+(S_2-1)\frac{\Pi}{\mu_2-\in}(1-e^{-(\mu_2-\in)t}+(S_2-1)\frac{\Pi}{(\mu_2-\mu_1)}(e^{-(\mu_2-\in)t}-e^{-(\mu_1-\in)t})\right.\right.$$
$$\left.\left.+(S_3-1)\frac{\theta}{(\mu_3-\in)}(1-e^{-(\mu_3-\in)t}+(S_3-1)\frac{\theta}{(\mu_3-\mu_1)}(e^{-(\mu_3-\in)t}-e^{-(\mu_1-\in)t})\right]\right\}$$
$$e < 1 < \text{Min}\{m_1, m_2, m_3\} \tag{6}$$

3 .MODEL CHARACTERISTICS

Expanding $P(S_1, S_2, S_3, t)$ given in equation (6) and collecting the constant terms, we get the probability that the system is empty as

$$P_{000}(t) = \exp \Big\{ -(\lambda-\in) \Big[\frac{1}{(\mu_1-\in)}\left(1-e^{-(\mu_1-\in)t}\right) + \frac{\Pi}{(\mu_2-\in)}\left(1-e^{-(\mu_2-\in)t}\right) + \frac{\Pi}{(\mu_2-\mu_1)}\left(e^{-(\mu_2-\in)t} - e^{-(\mu_1-\in)t}\right) + \frac{\theta}{(\mu_3-\in)}\left(1-e^{-(\mu_3-\in)t}\right) + \frac{\theta}{(\mu_3-\mu_1)}\left(e^{-(\mu_3-\in)t} - e^{-(\mu_1-\in)t}\right) \Big] \Big\} \tag{7}$$

Taking $S_2=1$ and $S_3 = 1$ we get the Probability generating function of the first queue size distribution as

$$P(S_1,t) = \exp\left(\frac{(\lambda-\in)}{(\mu_1-\in)}(S_1-1)(1-e^{-(\mu_1-\in)t}) \right) \tag{8}$$

Expanding $P(S_1, t)$ and collecting constant terms, we get the probability that the emptiness of the first queue as

$$P_{0..}(t) = \exp\left(-\frac{(\lambda-\in)}{(\mu_1-\in)}(1-e^{-(\mu_1-\in)t}) \right) \tag{9}$$

Taking $S_1=1$ and $S_3=1$, we get the Probability generating function of the second queue size distribution as

$$P(S_2,t) = \exp \Big\{ (\lambda-\in) \Big[(S_2-1)\frac{\Pi}{\mu_2-\in}(1-e^{-(\mu_2-\in)t}) + (S_2-1)\frac{\Pi}{(\mu_2-\mu_1)}(e^{-(\mu_2-\in)t} - e^{-(\mu_1-\in)t}) \Big] \Big\} \tag{10}$$

Expanding $P(S_2, t)$ and collecting constant terms, we get the probability that the emptiness of the second queue as

$$P_{.0.}(t) = \exp \Big\{ (\lambda-\in) \Big[-\frac{\Pi}{\mu_2-\in}(1-e^{-(\mu_2-\in)t}) - \frac{\Pi}{(\mu_2-\mu_1)}(e^{-(\mu_2-\in)t} - e^{-(\mu_1-\in)t}) \Big] \Big\} \tag{11}$$

Taking $S_1=1$ and $S_2 =1$, we get the Probability generating function of the third queue size distribution as

$$P(S_3,t) = \exp \Big\{ (\lambda-\in) \Big[(S_3-1)\frac{\theta}{(\mu_3-\in)}(1-e^{-(\mu_3-\in)t}) + (S_3-1)\frac{\theta}{(\mu_3-\mu_1)}(e^{-(\mu_3-\in)t} - e^{-(\mu_1-\in)t}) \Big] \Big\} \tag{12}$$

Expanding P (S_3, t) and collecting constant terms, we get the probability that the emptiness of the third queue as

$$P_{..0}(t) = \exp \Big\{ (\lambda-\in) \Big[\frac{\theta}{\mu_3-\in}(1-e^{-(\mu_3-\in)t}) - \frac{\theta}{(\mu_3-\mu_1)}(e^{-(\mu_3-\in)t} - e^{-(\mu_1-\in)t}) \Big] \Big\} \tag{13}$$

The mean number of customers in the first queue is

$$L_1 = E(N_1) = \frac{(\lambda-\in)}{(\mu_1-\in)}(1-e^{-(\mu_1-\in)t}) \tag{14}$$

The utilization of first service station is

$$U_1 = 1 - P_{0..}(t) = 1 - \exp\left(-\frac{(\lambda-\in)}{(\mu_1-\in)}\left(1-e^{-(\mu_1-\in)t}\right) \right) \tag{15}$$

The variance of the number of customers in the first queue is

$$Var(N_1) = \left(\frac{\lambda-\in}{\mu_1-\in} \right)\left(1-e^{-(\mu_1-\in)t}\right) \tag{16}$$

The throughput of the first node is

$$(\mu_1-\in)(1-P_{0..}(t)) = (\mu_1-\in)\left[1 - \exp\left(-\frac{(\lambda-\in)}{\mu_1-\in}(1-e^{-(\mu_1-\in)t}) \right) \right] \tag{17}$$

The average waiting time in the first queue is

$$W_1 = \frac{L_1}{(\mu_1-\in)(1-P_{0..}(t))} = \frac{\frac{(\lambda-\in)}{(\mu_1-\in)}(1-e^{-(\mu_1-\in)t})}{(\mu_1-\in)\left[1-\exp\left(-\frac{(\lambda-\in)}{(\mu_1-\in)}(1-e^{-(\mu_1-\in)t})\right)\right]} \tag{18}$$

The mean number of customers in the second queue is

$$L_2 = E(N_2) = \left[\frac{(\lambda-\in)\Pi}{(\mu_2-\in)}(1-e^{-(\mu_2-\in)t}) + \frac{(\lambda-\in)\Pi}{(\mu_2-\mu_1)}(e^{-(\mu_2-\in)t} - e^{-(\mu_1-\in)t}) \right] \tag{19}$$

The utilization of second service station is

$$U_2 = 1 - P_{.0.}(t) = 1 - \exp \Big\{ (\lambda-\in) \Big[-\frac{\Pi}{\mu_2-\in}(1-e^{-(\mu_2-\in)t}) - \frac{\Pi}{(\mu_2-\mu_1)}(e^{-(\mu_2-\in)t} - e^{-(\mu_1-\in)t}) \Big] \Big\} \tag{20}$$

The variance of the number of customers in the second queue is

$$Var(N_2) = \left[\frac{(\lambda-\in)\pi}{(\mu_2-\in)}(1-e^{-(\mu_2-\in)t}) + \frac{(\lambda-\in)\pi}{(\mu_2-\mu_1)}(e^{-(\mu_2-\in)t)} - e^{-(\mu_1-\in)t}) \right] \tag{21}$$

The throughput of the second node is

$$(\mu_2-\in)(1-P_{.0.}(t)) = (\mu_2-\in)[1-\exp \Big\{ (\lambda-\in) \Big[-\frac{\Pi}{\mu_2-\in}(1-e^{-(\mu_2-\in)t}) - \frac{\Pi}{(\mu_2-\mu_1)}(e^{-(\mu_2-\in)t} - e^{-(\mu_1-\in)t}) \Big] \Big\} \tag{22}$$

The average waiting time in the second queue is

$$W_2 = \frac{L_2}{(\mu_2-\in)(1-P_0(t))}$$

$$W_2 = \frac{\frac{(\lambda-\in)}{(\mu_2-\in)}(1-e^{-(\mu_2-\in)} + \frac{(\lambda-\in)\Pi}{(\mu_2-\mu_1)}\left(e^{-(\mu_2-\in)t} - e^{-(\mu_1-\in)t}\right)}{(\mu_2-\in)\Big[1-\exp \Big\{ (\lambda-\in)\Big[-\frac{\Pi}{(\mu_2-\in)}(1-e^{-(\mu_2-\in)t})} - \frac{\Pi}{(\mu_2-\mu_1)}(e^{-(\mu_2-\in)t} - e^{-(\mu_1-\in)t}) \Big] \Big\} \Big] \tag{23}$$

The mean number of customers in the third queue is

$$L_3 = E(N_3) = \frac{(\lambda-\in)\theta}{\mu_3-\in}(1-e^{-(\mu_3-\in)t}) + \frac{(\lambda-\in)\theta}{(\mu_3-\mu_1)}(e^{-(\mu_3-\in)t} - e^{-(\mu_1-\in)t}) \tag{24}$$

The utilization of third service station is

$$U_3 = 1 - P_{..0}(t) = 1 - \exp \Big\{ (\lambda-\in) \Big[-\frac{\theta}{\mu_3-\in}1-e^{-(\mu_3-\in)t}) - \frac{\theta}{(\mu_3-\mu_1)}(e^{-(\mu_3-\in)t} - e^{-(\mu_1-\in)t}) \Big] \Big\} \tag{25}$$

The throughput of the third node is

$$(\mu_3-\in)(1-P_{..0}(t)) = (\mu_3-\in)\Big[1-\exp \Big\{ (\lambda-\in) \Big[-\frac{\theta}{\mu_3-\in}(1-e^{-(\mu_3-\in)t}) - \frac{\theta}{(\mu_3-\mu_1)}(e^{-(\mu_3-\in)t} - e^{-(\mu_1-\in)t}) \Big] \Big\} \Big] \tag{26}$$

The average waiting time in the third queue is

$$W_3 = \frac{L_3}{(\mu_3-\in)(1-P_{..0}(t))}$$

$$W_3 = \frac{\frac{(\lambda-\in)\theta}{(\mu_3-\in)}(1-e^{-(\mu_3-\in)t}) + \frac{(\lambda-\in)\theta}{(\mu_3-\mu_1)}\left(e^{-(\mu_3-\in)t} - e^{-(\mu_1-\in)t}\right)}{(\mu_3-\in)\Big[1-\exp \Big\{ (\lambda-\in)\Big[-\frac{\theta}{(\mu_3-\in)}(1-e^{-(\mu_3-\in)t})} - \frac{\theta}{(\mu_3-\mu_1)}(e^{-(\mu_3-\in)t} - e^{-(\mu_1-\in)t}) \Big] \Big\} \Big] \tag{27}$$

The variance of the number of customers in the third queue is

$$Var(N_3) = \left[\frac{(\lambda-\in)\theta}{(\mu_3-\in)}(1-e^{-(\mu_3-\in)t}) + \frac{(\lambda-\in)\theta}{(\mu_3-\mu_1)}(e^{-(\mu_3-\in)t} - e^{-(\mu_1-\in)t}) \right] \tag{28}$$

The mean number of customers in the entire system at time t, is

$$L(t) = E(N_1) + E(N_2) + E(N_3) \quad (29)$$

The variability of the number of customers in the system is

$$Var(N) = Var(N_1) + Var(N_2) + Var(N_3) \quad (30)$$

4. NUMERICAL ILLUSTRATION

To study the performance of the queueing model the following numerical values of the parameters are considered.
t=1,2,………,15;
$\lambda = 1.5, 3.5, 5.5; \mu_1 = 2.5, 3.5, 5.5, 7.5; \mu_2 = 24.5, 29.5, 39.5, 49.5;$
$\mu_3 = 29.5, 39.5, 49.5; \varepsilon = 0.5; \Pi = 0.1, 0.4, 0.6, 0.8; \theta = 0.2, 0.4, 0.6$

For the above values of t, λ, μ_1, μ_2, μ_3, ε, θ and Π the probability of the system is empty and the probabilities of emptiness of the three queues, the mean number of customers in queues, the utilization of service stations, the variance of the content of queues, throughput of nodes and mean waiting time in the queues are computed and given in Table 1and shown in figure 2.

From the equations (7) to (30), Table 1 and figure 2, it is observed that as the time increases, the probability of the system emptiness, the emptiness of the queues are decreasing, the mean number of customers in the queues are increasing up to a point and thereafter stabilized, the mean number of customers in the queues are increasing, the utilization of service stations are increasing, the mean number of customers in the system is increasing, the variance of the number of customers in the queues are increasing, the throughput of the service station are increasing, the mean waiting time in the queues are increasing, the total number of customers in the system is increasing, variance of number of customers in the system is increasing, when other parameters are fixed.

It is further observed that as the mean arrival rate increases, the probability of the system emptiness and queues emptiness are decreasing, the mean number of customers in the queues are increasing, the mean number of customers in the queues are increasing, the utilization of the service station are increasing, the mean number of customers in the system is increasing, and the variance of the number of customers in the queues are increasing, the throughput of the nodes are increasing, the mean waiting time in the queues are increasing, the mean number of customers in the system is increasing, the variance of number of the customers in the network is increasing, when other parameters are fixed.

It is also observed that as the service rate of the first service station increases, the probability of the system emptiness is increasing, the probability of the first queue emptiness is increasing, the probability of second and third queues are unchanged, the mean number of customers in the first queue is decreasing, the mean number of customers in the second and third queues are unchanged, the mean number of customers in the first queue is decreasing, the mean number customers in the second and third queues are unchanged, the utilization of the first service station is decreasing, the utilization of second and third service stations are unchanged, the mean number of customers in the system is decreasing and variance of the first queue is decreasing, the variance of number of customers in the second and third queues are unchanged, the throughput of the first service station is increasing, the throughput of the second and third service stations are unchanged, the mean waiting time in the first queue is decreasing, mean waiting time in the second and third queues are unchanged, the mean number of customers in the network is decreasing, the variance of the number of customers in the system is decreasing, when other parameters are fixed.

It is also observed that as the service rate of the second service station increases, the probability of the system emptiness is increasing, the probability of the first and third queues emptiness are unchanged, the probability of second queue emptiness is increasing, the mean number of customers in the first and third queues are unchanged, the mean number of customers in the second queue is decreasing, the mean number of customers in the first and third queue are unchanged, the mean number of customers in the second queue is decreasing, the utilization of the first and third service stations are unchanged, the utilization of second node is decreasing, the mean number of customers in the network is decreasing and variance of number of customers in the first and third queues are unchanged, the variance of number of customers in the second queue is decreasing, the throughput of the first and third nodes are unchanged, the throughput of the second service station is increasing, the mean waiting time in the first and third queues are unchanged, mean waiting time in the second queue is decreasing, the total number of customers in the system is decreasing, the variability of the number of customers in the system is decreasing, when other parameters are fixed.

It is also observed that as the service rate of the third service station increases, the probability of the system emptiness is increasing, the probability of the first and second queues emptiness are unchanged, the probability of third queue emptiness is increasing, the mean number of customers in the first and second queues are unchanged, the mean number of customers in the third queue is decreasing, the mean number of customers in the first and second service stations are unchanged, the mean number of customers in the third router are decreasing, the utilization of the first and third service station are unchanged, the utilization of third node is decreasing, the mean number of customers in the system is decreasing and variance of number of customers in the first and second queues are unchanged, the variance of number of customers in the third queue is decreasing, the throughput of the first and second service stations are unchanged, the throughput of the third service station is increasing, the mean waiting time in the first and second queues are unchanged, mean waiting time in the third queue is decreasing, the mean number of customers in the system is decreasing, the variance of the number of customers in the system is decreasing, when other parameters are fixed.

As the parameter θ increases, the probability of the first queue emptiness is unchanged, the probability of second queue emptiness is increasing and the probability of third queue emptiness is decreasing, the mean number of customers in the first queue is unchanged, the mean number of customers in the second queue is decreasing, the mean number of customers in the third queue is increasing, the utilization of the first service station is unchanged, the utilization of second service station is decreasing, the utilization of third service station is increasing, the mean number of customers in the system is decreasing and variance of number of customers in the first queue is unchanged, the variance of number of customers in the second queue is decreasing, the variance of number of customers in the third queue is increasing, the throughput of the first node is unchanged, the throughput of the second node is decreasing, the throughput of the third node is increasing, the mean waiting time in the first queue is unchanged, the mean waiting time in the second queue is decreasing, the mean waiting time in the third queue is increasing, the mean number of customers in the system is decreasing, variance of number of customers in the system is decreasing, when other parameters are fixed.

5. SENSITIVITY ANALYSIS

In this section we considered the sensitivity analysis of the model with the values of the parameters as t=2, λ=1.5, μ_1=2.5, μ_2 = 3.5, μ_3 = 4.5 θ = 0.1, ε = 0.5 and π = 0.9. The effect of variation of -15%, -10%, -5%, 0%, 5%, 10%, 15% on the performance measures L_1, L_2, L_3, L_s, W_1, W_2, W_3 were computed and are given in Table 3.

Form table 3, it is observed that as time increases the system characteristics L_1, L_2, L_3, W_1, W_2, W_3, are increasing and as time decreases they are decreasing. Similar phenomenon is observed with respect to λ. But as μ_1 increases, L_1, W_1 are decreasing and L_2, L_3, W_2, W_3 are increasing. When μ_2 increases L_2, W_2 are decreasing and there is no change in L_1, L_2, W_1 and W_2. It is also observed that as θ increases L_2, W_2 are increasing, L_3, W_3 are decreasing and there is no change in L_1 and W_1. It is also observed that as π increases L_3, W_3 are increasing, L_2, W_2 are decreasing and there is no change in L_1 and W_1. It is also observed that all parameters are increases L_1, L_2 and L_3 are increasing and W_1, W_2, W_3 are decreasing.

The developed model performs faster than the traditional model without dependences. In many of the communication systems state dependence is an optimal strategy which is possible through bit dropping or flow control mechanism.

6. STEADY STATE ANALYSIS OF THE MODEL

In this section we study the steady state behaviour of the interdependent forked queueing model with state dependent. The steady state analysis of the model can be done by assuming that the system is stable and under equilibrium.

i.e.

$$\lim_{t\to\infty} P(S_1,S_2,S_3,t) = P(S_1,S_2,S_3) \text{ and } \lim_{t\to\infty} P_{n_1,n_2,n_3}(t) = P_{n_1,n_2 n_3}$$

The probability generating function of $P_{n_1,n_2 n_3}$ at any arbitrary time is obtaining by taking limit t→∞ in the equation (6).

This implies

$$P(S_1,S_2,S_3) = \exp\left\{(\lambda-\epsilon)\left[\frac{(S_1-1)}{(\mu_1-\epsilon)} + \frac{(S_2-1)\Pi}{(\mu_2-\epsilon)} + \frac{(S_3-1)\theta}{(\mu_3-\epsilon)}\right]\right\},$$

for $\varepsilon < \lambda < \min\{\mu_1, \mu_2, \mu_3\}$ (31)

Expanding $P(S_1, S_2, S_3)$ given in equation 31 and collecting the constant terms, we get the probability that the system is empty as

$$P_{000} = \exp\left[-(\lambda-\epsilon)\left(\frac{1}{(\mu_1-\epsilon)} + \frac{\pi}{(\mu_2-\epsilon)} + \frac{\theta}{(\mu_3-\epsilon)}\right)\right] \quad (32)$$

Taking S_2 = 1 and S_3 = 1, we get the probability generating function of the first queue size distribution as

$$P(S_1) = \exp\left\{(\lambda-\epsilon)\left[\frac{(S_1-1)}{(\mu_1-\epsilon)}\right]\right\}, \ \varepsilon < \lambda < \mu_1 \quad (33)$$

Expanding P (S_1) and collecting the constant terms, we get the probability that the first queue is empty as

$$P_{0..} = \exp\left\{-\left[\frac{\lambda-\epsilon}{\mu_1-\epsilon}\right]\right\} \quad (34)$$

Similarly S_1 = 1 and S_3 = 1 we get the probability generating function of the second queue size distribution as

$$P(S_2) = \exp\left\{(\lambda-\epsilon)\left[\frac{(S_2-1)\pi}{\mu_2-\epsilon}\right]\right\}, \varepsilon < \lambda < \mu_2 \quad (35)$$

Expanding $P(S_2)$ and collecting the constant terms, we get the probability that the second queue is empty as

$$P_{.0.} = \exp\left\{-\left[\frac{(\lambda-\epsilon)\pi}{\mu_2-\epsilon}\right]\right\} \quad (36)$$

Similarly S_1 = 1 and S_2 = 1 we get the probability generating function of the third queue size distribution as

$$P(S_3) = \exp\left\{(\lambda-\epsilon)\left[\frac{(S_3-1)\theta}{\mu_3-\epsilon}\right]\right\}, \quad \varepsilon < \lambda < \mu_3 \quad (37)$$

Expanding $P(S_3)$ and collecting the constant terms, we get the probability that the third queue is empty as

$$P_{..0} = \exp\left\{-\left[\frac{(\lambda-\epsilon)\theta}{\mu_3-\epsilon}\right]\right\} \quad (38)$$

The mean number of customers in the first queue is

$$L_1 = E(N_1) = \frac{(\lambda-\epsilon)}{(\mu_1-\epsilon)} \quad (39)$$

The utilization of the first service station is

$$U_1 = 1 - P_{0..} = 1 - \exp\left\{\left[-\frac{\lambda-\epsilon}{\mu_1-\epsilon}\right]\right\} \quad (40)$$

The variance of the number of customers in the first queue is

$$Var(N_1) = E[N_1^2 - N_1] + E[N_1] - (E[N_1])^2 = \left(\frac{\lambda-\epsilon}{\mu_1-\epsilon}\right) \quad (45)$$

The throughput of the first node is

$$(\mu_1-\epsilon)(1-P_{0..}) = (\mu_1-\epsilon)\left[1-\exp\left(-\frac{(\lambda-\epsilon)}{(\mu_1-\epsilon)}\right)\right] \quad (48)$$

The mean waiting time in the first queue is W_1

$$\frac{L_1}{(\mu_1-\epsilon)(1-P_{0..})} = \frac{\frac{(\lambda-\epsilon)}{(\mu_1-\epsilon)}}{(\mu_1-\epsilon)\left[1-\exp\left(-\frac{(\lambda-\epsilon)}{(\mu_1-\epsilon)}\right)\right]} \quad (49)$$

(53)

The total number of customers in the system at time t is, L then

$$L = E[N_1] + E[N_2] + E[N_3] = \left(\frac{\lambda-\epsilon}{\mu_1-\epsilon}\right) + \left(\frac{(\lambda-\epsilon)\pi}{\mu_2-\epsilon}\right) + \left(\frac{(\lambda-\epsilon)\theta}{\mu_3-\epsilon}\right)$$

(54)

The variability of the number of customers in the system is

$$Var(N) = \left(\frac{\lambda-\epsilon}{\mu_1-\epsilon}\right) + \left(\frac{(\lambda-\epsilon)\pi}{\mu_2-\epsilon}\right) + \left(\frac{(\lambda-\epsilon)\theta}{\mu_3-\epsilon}\right) \quad (55)$$

For different values of t, λ, μ_1, μ_2, μ_3, θ, π and ε, the probability of the emptiness, the probability of first queue emptiness, the mean number of customers in the first queue, the utilization of the first service station, the variance of number of customers in the first queue, the throughput of the first service station, mean waiting time of customers in first queue are computed and are given in Table 2.

For different values of t, λ, μ_1, μ_2, μ_3, θ, π and ε, the mean number of customers in the second queue, the probability of second queue emptiness, the utilization of the second node, the variance of number of customers in the second queue, the throughput of the second service station, mean waiting time of customers in second queue are computed and given in Table 2.

For different values of t, λ, μ_1, μ_2, μ_3, θ, π and ε, the mean number of customers in the third queue, the probability of third queue emptiness, the utilization of the third service station, the variance of number of customers in the third queue, the throughput of the third service station, mean waiting time of customers in third queue, the probability of number of customers in the system , and the variance of the number of the customers in the system are computed and are given in Table 2.

From the table 2, it is also observed that the parameters λ, μ_1, μ_2, θ and ε, have similar influence on the performance measures like, mean number of customers in the queue, the utilization of service stations, the variance of number of customers in the queues, the throughput of the nodes , the mean waiting time in queues.

7. COMPARATIVE STUDY

It is interesting to note that the time, t has significant influence on all performance measures. The difference in performance between steady state and transient conditions for t = 1, 3, 5 are computed and given in Table 4.

From the table 4, it is observed that there is significant difference between steady state behavior and transient behavior of the model. At t=1, the variation in various measures is highly significant. The difference can be seen from the last column of the table. There is 3.69% decrease in the throughput of first service station, 5.97% decrease in throughput with second service station and 5.68% decrease in throughput with the third service station of the model under transient condition at t = 1. Similarly the mean waiting times and content in the queue are also having difference between transient and steady state conditions. It is also observed that as t increases, the difference between transient and steady state become negligible in performance measures which is natural phenomena of equilibriumstate.

8. CONCLUSION

This paper deals with development and analysis of interdependent forked queueing model with state dependent service rates. The forked queueing models play a dominant role in performance evaluation of several communication networks, ATM scheduling, production processes etc,. The transient analysis of the model gives better insights in predicting the performance measure of the system. The explicit expressions for performance measure like the average number of the customers in each queue, the mean waiting time of customers in each queue, the throughput of the service station, the idleness of the server ,etc,. The sensitivity analysis performed reveals that the time and the covariance between arrival and service processes have tremendous influence on the performance of the system like the congestion, burstness of the buffers and mean delay. Since it includes many of the earlier models as particular cases under Markovian environment, this can be viewed as a generalized model. The model can be extended to multiple models under non Markovian set up which require further investigations.

9. REFERENCES

[1] ERLANG,A.K.,"Probability and Telephone calls", Nyt.Tidsskr.B-20,(1909).

[2] S E Raphin, B.Calo., " Message delays in repeated service tandem connection", IEEE Trans.Comm, 29(5)(1981).

[3] Conolly.B., "The waiting time process for a certain correlated Queue" , Operation Research,16(1968).

[4] Kambo.N.S. and Bhalaik.H.S., "Bulk arrival and heterogeneous queueing systems",Opsearch,vol.19,pp.97-105(1982).

[5] F.W.Kerry, et.al., "Investigating dependence in packet queues with the index of dispersion for work", IEEE Trans. Comm., 38,pp.121-125(1991).

[6] David Lucantoni, "Further transient analysis of the BMAP/G/1 queue",S.M.Vol.14,No's 1&2,pp.461-478 (1998).

[7] K.Srinivasa Rao, et.al., " On an interdependent communication networks",Opsearch,37(2),pp.134-143 (2000).

[8] Aftab Begum, et.al, "The M/M/C interdependent queueing model with controllable arrival rates", Opsearch, 39(2),pp.89-110 (2002).

[9] Pal.S., "The cost analysis of M/M/1/N interdependent queueing model with controllable arrival rates" ,Vol 17(2), 13-20(2003).

[10] Prasad Reddy and K.Srinivasa Rao, et.al, " Interdependent queueing model with jockeying" , Ultra Science ,Vol.18(1) M, pp.87-98(2006).

[11] S.S.Mishra., "Optimum performance measures of interdependent queueing system with controllable arrival rates", Intl. Journ. Mathematical, Physical and Engineering Sciences, pp.72-75(2009).

[12] V.N.Maurya, "On the expected busy period of an interdependent M/M/1:(∞;Gd) Queueing model using Bi-variate Poisson process and controllable arrival rates", IEEE Trans.,pp.243-246(2010).

Table 1: Probability of emptiness, mean no. of customers, utilization of service stations, variance, throughput and waiting times in queues for transient state

t	λ	μ_1	μ_2	μ_3	ε	π	θ	$P_{000}(t)$	L_1	$P_{0..}(t)$	U_1	$Var(N_1)$	Th_1	W_1	L_2	$P_{.0.}(t)$	U_2	$Var(N_2)$	Th_2	W_2	L_3	$P_{.0.}(t)$	U_3	$Var(N_3)$	Th_3	W_3
1	1.5	2.5	24.5	29.5	0.5	0.1	0.9	0.498	0.633	0.531	0.469	0.633	1.408	0.450	0.008	0.992	0.008	0.008	0.187	0.040	0.057	0.945	0.055	0.057	1.653	0.034
2	1.5	2.5	24.5	29.5	0.5	0.1	0.9	0.481	0.665	0.514	0.486	0.665	1.457	0.456	0.008	0.992	0.008	0.008	0.199	0.040	0.060	0.942	0.058	0.060	1.742	0.034
3	1.5	2.5	24.5	29.5	0.5	0.1	0.9	0.480	0.667	0.513	0.487	0.667	1.460	0.457	0.008	0.920	0.008	0.008	0.199	0.040	0.060	0.942	0.058	0.060	1.747	0.034
5	1.5	2.5	24.5	29.5	0.5	0.1	0.9	0.480	0.667	0.513	0.487	0.667	1.460	0.457	0.008	0.992	0.008	0.008	0.199	0.040	0.060	0.942	0.058	0.060	1.748	0.034
10	1.5	2.5	24.5	29.5	0.5	0.1	0.9	0.480	0.667	0.513	0.487	0.667	1.460	0.457	0.008	0.992	0.008	0.008	0.199	0.040	0.060	0.942	0.058	0.060	1.747	0.034
15	1.5	2.5	24.5	29.5	0.5	0.1	0.9	0.480	0.667	0.513	0.487	0.667	1.460	0.457	0.008	0.992	0.008	0.008	0.199	0.040	0.060	0.942	0.058	0.060	1.747	0.034
5	1.5	7.5	24.5	29.5	0.5	0.1	0.9	0.728	0.250	0.779	0.221	0.250	1.770	0.141	0.008	0.992	0.008	0.008	0.199	0.040	0.060	0.942	0.058	0.060	1.747	0.034
5	3.5	7.5	24.5	29.5	0.5	0.1	0.9	0.529	0.500	0.607	0.393	0.500	3.148	0.159	0.016	0.984	0.016	0.016	0.397	0.040	0.120	0.887	0.113	0.120	3.392	0.034
5	5.5	7.5	24.5	29.5	0.5	0.1	0.9	0.385	0.750	0.472	0.528	0.750	4.221	0.178	0.024	0.976	0.024	0.024	0.593	0.040	0.180	0.835	0.165	0.180	4.942	0.034
5	1.5	3.5	24.5	29.5	0.5	0.1	0.9	0.567	0.500	0.607	0.393	0.500	1.574	0.318	0.008	0.992	0.008	0.008	0.199	0.040	0.060	0.942	0.058	0.060	1.747	0.034
5	1.5	5.5	24.5	29.5	0.5	0.1	0.9	0.669	0.333	0.717	0.283	0.333	1.701	0.196	0.008	0.992	0.008	0.008	0.199	0.040	0.060	0.942	0.058	0.060	1.747	0.034
5	1.5	7.5	24.5	29.5	0.5	0.1	0.9	0.728	0.250	0.779	0.221	0.250	1.770	0.141	0.008	0.992	0.008	0.008	0.199	0.040	0.060	0.942	0.058	0.060	1.747	0.034
5	1.5	2.5	29.5	29.5	0.5	0.1	0.9	0.480	0.667	0.513	0.487	0.667	1.460	0.457	0.007	0.993	0.007	0.007	0.199	0.033	0.060	0.942	0.058	0.060	1.747	0.034
5	1.5	2.5	39.5	29.5	0.5	0.1	0.9	0.481	0.667	0.513	0.487	0.667	1.460	0.457	0.005	0.995	0.005	0.005	0.200	0.025	0.060	0.942	0.058	0.060	1.747	0.034
5	1.5	2.5	49.5	29.5	0.5	0.1	0.9	0.482	0.667	0.513	0.487	0.667	1.460	0.457	0.004	0.996	0.004	0.004	0.200	0.020	0.060	0.942	0.058	0.060	1.747	0.034
5	1.5	2.5	24.5	29.5	0.5	0.1	0.9	0.480	0.667	0.513	0.487	0.667	1.460	0.457	0.008	0.992	0.008	0.008	0.199	0.040	0.060	0.942	0.058	0.060	1.747	0.034
5	1.5	2.5	24.5	39.5	0.5	0.1	0.9	0.487	0.667	0.513	0.487	0.667	1.460	0.457	0.008	0.992	0.008	0.008	0.199	0.040	0.045	0.956	0.044	0.045	1.760	0.026
5	1.5	2.5	24.5	49.5	0.5	0.1	0.9	0.491	0.667	0.513	0.487	0.667	1.460	0.457	0.008	0.992	0.008	0.008	0.199	0.040	0.036	0.965	0.035	0.036	1.768	0.020
5	1.5	2.5	24.5	29.5	0.5	0.8	0.2	0.475	0.667	0.513	0.487	0.667	1.460	0.457	0.064	0.938	0.062	0.064	1.550	0.041	0.013	0.987	0.013	0.013	0.397	0.034
5	1.5	2.5	24.5	29.5	0.5	0.6	0.4	0.476	0.667	0.513	0.487	0.667	1.460	0.457	0.048	0.953	0.047	0.048	1.172	0.041	0.027	0.974	0.026	0.027	0.789	0.034
5	1.5	2.5	24.5	29.5	0.5	0.4	0.6	0.478	0.667	0.513	0.487	0.667	1.460	0.457	0.032	0.969	0.031	0.032	0.787	0.041	0.040	0.961	0.039	0.040	1.176	0.034

Table 2: Probability of emptiness, mean no. of customers, utilization of service stations, variance, throughput and waiting times in queues for steady state

λ	μ_1	μ_2	μ_3	π	θ	ε	$P_{000}(t)$	$P_{0..}(t)$	L_1	U_1	$Var(N_1)$	Th_1	W_1	L2	$P_{.0.}(t)$	U_2	$Var(N_2)$	Th_2	W_2	L(t)	Var(N)	L_3	$P_{..0}(t)$	U_3	$Var(N_3)$	Th_3	W_3
1.5	7.5	24.5	29.5	0.1	0.9	0.5	0.728	0.779	0.250	0.221	0.250	1.770	0.141	0.060	0.942	0.058	0.060	1.747	0.034	0.605	0.605	0.008	0.992	0.008	0.008	0.199	0.040
2.5	7.5	24.5	29.5	0.1	0.9	0.5	0.621	0.687	0.375	0.313	0.375	2.502	0.150	0.090	0.914	0.086	0.090	2.582	0.035	0.888	0.888	0.012	0.988	0.012	0.012	0.298	0.040
3.5	7.5	24.5	29.5	0.1	0.9	0.5	0.529	0.607	0.500	0.393	0.500	3.148	0.159	0.120	0.887	0.113	0.120	3.392	0.035	1.158	1.158	0.016	0.984	0.016	0.016	0.397	0.040
4.5	7.5	24.5	29.5	0.1	0.9	0.5	0.452	0.535	0.625	0.465	0.625	3.718	0.168	0.150	0.861	0.139	0.150	4.179	0.036	1.419	1.419	0.020	0.980	0.020	0.020	0.495	0.040
5.5	7.5	24.5	29.5	0.1	0.9	0.5	0.385	0.472	0.750	0.528	0.750	4.221	0.178	0.180	0.835	0.165	0.180	4.942	0.036	1.670	1.670	0.024	0.976	0.024	0.024	0.593	0.040
6.5	7.5	24.5	29.5	0.1	0.9	0.5	0.329	0.417	0.875	0.583	0.875	4.665	0.188	0.210	0.811	0.189	0.210	5.682	0.037	1.913	1.913	0.028	0.972	0.028	0.028	0.690	0.041
1.5	2.5	24.5	29.5	0.1	0.9	0.5	0.480	0.513	0.667	0.487	0.667	1.460	0.457	0.060	0.942	0.058	0.060	1.747	0.034	1.287	1.287	0.008	0.992	0.008	0.008	0.199	0.040
1.5	3.5	24.5	29.5	0.1	0.9	0.5	0.567	0.607	0.500	0.393	0.500	1.574	0.318	0.060	0.942	0.058	0.060	1.747	0.034	1.028	1.028	0.008	0.992	0.008	0.008	0.199	0.040
1.5	4.5	24.5	29.5	0.1	0.9	0.5	0.626	0.670	0.400	0.330	0.400	1.648	0.243	0.060	0.942	0.058	0.060	1.747	0.034	0.864	0.864	0.008	0.992	0.008	0.008	0.199	0.040
1.5	5.5	24.5	29.5	0.1	0.9	0.5	0.669	0.717	0.333	0.283	0.333	1.701	0.196	0.060	0.942	0.058	0.060	1.747	0.034	0.751	0.751	0.008	0.992	0.008	0.008	0.199	0.040
1.5	6.5	24.5	29.5	0.1	0.9	0.5	0.702	0.751	0.286	0.249	0.286	1.740	0.164	0.060	0.942	0.058	0.060	1.747	0.034	0.668	0.668	0.008	0.992	0.008	0.008	0.199	0.040
1.5	7.5	24.5	29.5	0.1	0.9	0.5	0.728	0.779	0.250	0.221	0.250	1.770	0.141	0.060	0.942	0.058	0.060	1.747	0.034	0.605	0.605	0.008	0.992	0.008	0.008	0.199	0.040
1.5	2.5	24.5	29.5	0.1	0.9	0.5	0.480	0.513	0.667	0.487	0.667	1.460	0.457	0.060	0.942	0.058	0.060	1.747	0.034	1.287	1.287	0.008	0.992	0.008	0.008	0.199	0.040
1.5	2.5	29.5	29.5	0.1	0.9	0.5	0.480	0.513	0.667	0.487	0.667	1.460	0.457	0.060	0.942	0.058	0.060	1.747	0.034	1.285	1.285	0.006	0.993	0.007	0.006	0.199	0.033
1.5	2.5	34.5	29.5	0.1	0.9	0.5	0.481	0.513	0.667	0.487	0.667	1.460	0.457	0.060	0.942	0.058	0.060	1.747	0.034	1.283	1.283	0.005	0.994	0.006	0.005	0.199	0.029
1.5	2.5	39.5	29.5	0.1	0.9	0.5	0.481	0.513	0.667	0.487	0.667	1.460	0.457	0.060	0.942	0.058	0.060	1.747	0.034	1.281	1.281	0.005	0.995	0.005	0.005	0.200	0.025
1.5	2.5	44.5	29.5	0.1	0.9	0.5	0.481	0.513	0.667	0.487	0.667	1.460	0.457	0.060	0.942	0.058	0.060	1.747	0.034	1.280	1.280	0.004	0.996	0.004	0.004	0.200	0.022
1.5	2.5	49.5	29.5	0.1	0.9	0.5	0.482	0.513	0.667	0.487	0.667	1.460	0.457	0.060	0.942	0.058	0.060	1.747	0.034	1.279	1.279	0.004	0.996	0.004	0.004	0.200	0.020
1.5	2.5	24.5	29.5	0.1	0.9	0.5	0.474	0.513	0.667	0.487	0.667	1.460	0.457	0.072	0.931	0.069	0.072	1.737	0.041	1.311	1.311	0.008	0.992	0.008	0.008	0.199	0.040
1.5	2.5	24.5	29.5	0.1	0.9	0.5	0.480	0.513	0.667	0.487	0.667	1.460	0.457	0.060	0.942	0.058	0.060	1.747	0.034	1.287	1.287	0.008	0.992	0.008	0.008	0.199	0.040
1.5	2.5	24.5	29.5	0.1	0.9	0.5	0.484	0.513	0.667	0.487	0.667	1.460	0.457	0.051	0.950	0.050	0.051	1.754	0.029	1.271	1.271	0.008	0.992	0.008	0.008	0.199	0.040
1.5	2.5	24.5	29.5	0.1	0.9	0.5	0.487	0.513	0.667	0.487	0.667	1.460	0.457	0.045	0.956	0.044	0.045	1.760	0.256	1.258	1.258	0.008	0.992	0.008	0.008	0.199	0.040
1.5	2.5	24.5	29.5	0.1	0.9	0.5	0.489	0.513	0.667	0.487	0.667	1.460	0.457	0.040	0.961	0.039	0.040	1.764	0.023	1.248	1.248	0.008	0.992	0.008	0.008	0.199	0.040
1.5	2.5	24.5	29.5	0.1	0.9	0.5	0.491	0.513	0.667	0.487	0.667	1.460	0.457	0.036	0.965	0.035	0.036	1.768	0.020	1.241	1.241	0.008	0.992	0.008	0.008	0.199	0.040
1.5	2.5	24.5	29.5	0.9	0.1	0.5	0.475	0.513	0.667	0.487	0.667	1.460	0.457	0.006	0.993	0.007	0.007	0.199	0.033	1.308	1.308	0.072	0.931	0.069	0.072	1.737	0.041
1.5	2.5	24.5	29.5	0.8	0.2	0.5	0.475	0.513	0.667	0.487	0.667	1.460	0.457	0.013	0.987	0.013	0.013	0.397	0.034	1.306	1.306	0.064	0.938	0.062	0.064	1.550	0.041
1.5	2.5	24.5	29.5	0.7	0.3	0.5	0.476	0.513	0.667	0.487	0.667	1.460	0.457	0.020	0.980	0.020	0.020	0.594	0.034	1.304	1.304	0.056	0.946	0.054	0.056	1.362	0.041
1.5	2.5	24.5	29.5	0.6	0.4	0.5	0.476	0.513	0.667	0.487	0.667	1.460	0.457	0.026	0.974	0.026	0.027	0.789	0.034	1.301	1.301	0.048	0.953	0.047	0.048	1.172	0.041
1.5	2.5	24.5	29.5	0.4	0.6	0.5	0.478	0.513	0.667	0.487	0.667	1.460	0.457	0.033	0.967	0.033	0.033	0.984	0.034	1.299	1.299	0.040	0.961	0.039	0.040	0.980	0.041
1.5	2.5	24.5	29.5	0.4	0.6	0.5	0.478	0.513	0.667	0.487	0.667	1.460	0.457	0.040	0.961	0.039	0.040	1.176	0.034	1.296	1.296	0.032	0.969	0.031	0.032	0.787	0.041

Figure 2: Relation between Performance measures and Time

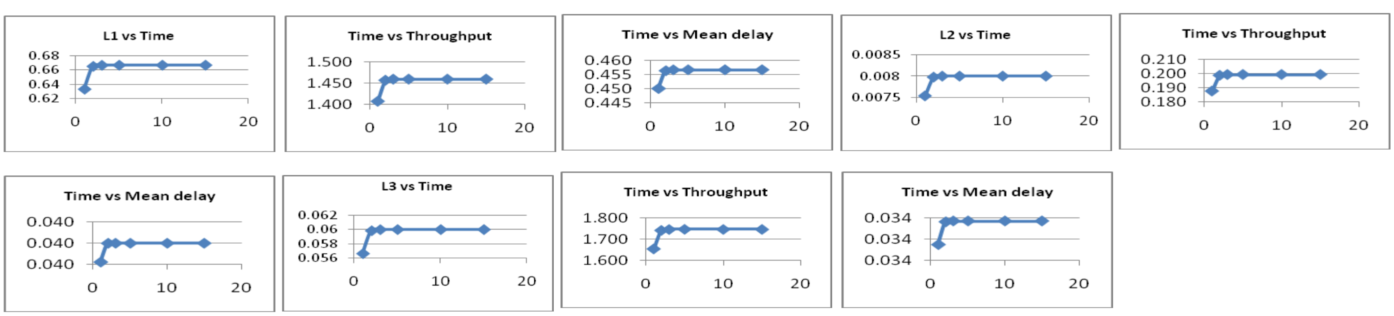

Table 3: Values of the L_1, L_2, L_3, L_s, L_3, W_1, W_2, W_3 for different values of t, λ, μ_1, μ_2, μ_3, θ and π. (SENSITIVITY ANALYSIS)

Variation of parameters	Performance measures	Percentage change in parameters						
		-15%	-10%	-5%	0	5%	10%	15%
t	L_1	0.66	0.66	0.66	0.67	0.67	0.67	0.67
	L_2	0.01	0.01	0.01	0.01	0.01	0.01	0.01
	L_3	0.06	0.06	0.06	0.06	0.06	0.06	0.06
	W_1	0.46	0.46	0.46	0.46	0.46	0.46	0.46
	W_2	0.04	0.04	0.04	0.04	0.04	0.04	0.04
	W_3	0.03	0.03	0.03	0.03	0.03	0.03	0.03
λ	L_1	0.57	0.60	0.63	0.67	0.70	0.73	0.76
	L_2	0.01	0.01	0.01	0.01	0.01	0.01	0.01
	L_3	0.05	0.05	0.06	0.06	0.06	0.07	0.07
	W_1	0.44	0.44	0.45	0.46	0.46	0.47	0.48
	W_2	0.04	0.04	0.04	0.04	0.04	0.04	0.04
	W_3	0.03	0.03	0.03	0.03	0.03	0.03	0.03
μ_1	L_1	0.78	0.74	0.70	0.67	0.61	0.61	0.58
	L_2	0.01	0.01	0.01	0.01	0.01	0.01	0.01
	L_3	0.06	0.06	0.06	0.06	0.06	0.06	0.06
	W_1	0.56	0.52	0.49	0.46	0.40	0.40	0.38
	W_2	0.04	0.04	0.04	0.04	0.04	0.04	0.04
	W_3	0.03	0.03	0.03	0.03	0.03	0.03	0.03
μ_2	L_1	0.67	0.67	0.67	0.67	0.67	0.67	0.67
	L_2	0.01	0.01	0.01	0.01	0.01	0.01	0.01
	L_3	0.06	0.06	0.06	0.06	0.06	0.06	0.06
	W_1	0.46	0.46	0.46	0.46	0.46	0.46	0.46
	W_2	0.05	0.04	0.04	0.04	0.04	0.04	0.03
	W_3	0.03	0.03	0.34	0.03	0.03	0.03	0.03
μ_3	L_1	0.67	0.67	0.67	0.67	0.67	0.67	0.67
	L_2	0.01	0.01	0.01	0.01	0.01	0.01	0.01
	L_3	0.07	0.06	0.06	0.06	0.06	0.05	0.05
	W_1	0.46	0.46	0.46	0.46	0.46	0.46	0.46
	W_2	0.04	0.04	0.04	0.04	0.04	0.04	0.04
	W_3	0.04	0.04	0.04	0.33	0.03	0.03	0.03
θ	L_1	0.67	0.67	0.67	0.67	0.67	0.67	0.67
	L_2	0.01	0.01	0.01	0.01	0.01	0.01	0.01
	L_3	0.06	0.06	0.06	0.06	0.06	0.06	0.06
	W_1	0.46	0.46	0.46	0.46	0.46	0.46	0.46
	W_2	0.04	0.04	0.04	0.04	0.04	0.04	0.04
	W_3	0.03	0.03	0.03	0.34	0.03	0.03	0.03
π	L_1	0.67	0.67	0.67	0.67	0.67	0.67	0.67
	L_2	0.02	0.02	0.01	0.00	0.00	0.00	0.00
	L_3	0.05	0.05	0.06	0.06	0.06	0.07	0.07
	W_1	0.46	0.46	0.46	0.46	0.46	0.46	0.46
	W_2	0.04	0.04	0.04	0.04	0.04	0.04	0.04
	W_3	0.03	0.03	0.03	0.03	0.03	0.34	0.03
All Parameters	L_1	0.66	0.66	0.66	0.67	0.67	0.67	0.67
	L_2	0.01	0.01	0.01	0.01	0.01	0.01	0.01
	L_3	0.06	0.06	0.06	0.06	0.06	0.06	0.06
	W_1	0.54	0.51	0.48	0.46	0.43	0.42	0.40
	W_2	0.05	0.04	0.04	0.04	0.04	0.04	0.03
	W_3	0.04	0.04	0.04	0.03	0.03	0.03	0.03

Table 4: Comparative Study Of Under Transient and Steady State Conditions

Time t	Performance	Transient State	Steady state	Difference	Percentage of variation
t = 1	L_1	0.633	0.667	0.033191	5.2396
	L_2	0.008	0.008	0.000453	5.9969
	L_3	0.057	0.060	0.003319	5.8558
	Th_1	1.408	1.460	0.051981	3.6924
	Th_2	0.188	0.199	0.011228	5.9729
	Th_3	1.653	1.747	0.093931	5.6820
	W_1	0.450	0.457	0.006714	1.4920
	W_2	0.040	0.040	0.000009	0.0226
	W_3	0.034	0.034	0.000056	0.1645
t = 3	L1	0.667	0.667	0.000082	0.0123
	L2	0.008	0.008	0.000001	0.0140
	L3	0.060	0.060	0.000008	0.0137
	Th1	1.460	1.460	0.000127	0.0087
	Th2	0.199	0.199	0.000028	0.0140
	Th3	1.747	1.747	0.000232	0.0133
	W1	0.457	0.457	0.000017	0.0037
	W2	0.040	0.040	0.000000	0.0001
	W3	0.034	0.034	0.000000	0.0004
t = 5	L_1	0.667	0.667	0.000000	0.0000
	L_2	0.008	0.008	0.000000	0.0000
	L_3	0.060	0.060	0.000000	0.0000
	Th_1	1.460	1.460	0.000000	0.0000
	Th_2	0.199	0.199	0.000000	0.0000
	Th_3	1.747	1.747	0.000001	0.0000
	W_1	0.457	0.457	0.000000	0.0000
	W_2	0.040	0.040	0.000000	0.0000
	W_3	0.034	0.034	0.000000	0.0000