A Novel Signal Segmentation Method based on Standard Deviation and Variable Threshold

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ABSTRACT
Decomposition of non-stationary signals such as electroencephalogram (EEG) and electrocardiogram (ECG) into stationary or quasi-stationary, signal segmentation, is a well-known problem in many signal processing applications. Previous methods for segmenting a signal had problems such as slow speed, low performance, and several parameters which must be defined experimentally. In this paper a new method based on standard deviation and variable threshold has been suggested. The standard deviation can indicate changes in the amplitude and/or frequency that it is the purpose of the signal segmentation. Since the standard deviation isn't able to indicate the effect of the shift in a signal, the proposed method utilizes the integral as a pre-processing level. Also, to improve the efficiency of the proposed method we use variable threshold. In order to evaluate the performance of this method, we use synthetic and real EEG signals. In EEG signals to remove destructive noises like EMG and EOG, we propose to use discrete wavelet transform (DWT). The obtained results indicate superiority of the proposed method in signal segmentation.

General Terms

Signal Segmentation, Non-stationary Signal, Adaptive Segmentation, Integral, Discrete Wavelet Transform, and EEG Signal.

Keywords
Non-stationary Signal, Adaptive Segmentation, Standard Deviation, Integral, Discrete Wavelet Transform, and Variable Threshold.

1. INTRODUCTION
In a general manner signals are categorized into two basic types: determined and random or statistical signals. Statistical signals are created by recorded random data, such as EEG and ECG signals. Depending on the behavior of signals, statistical signals are divided into stationary and non-stationary signals [1]. In many applications, it often need the non-stationary signal split into several pieces which each piece has approximately the same statistical properties such as amplitude and frequency. This act is named signal segmentation [1].

There are two major kinds for segmenting a signal, namely, fixed-length and adaptive segmentation. In the fixed-length segmentation, a signal is divided into constant parts. This method is very simple and fast to implement, however, the performance of this method is unacceptable [1]. In the adaptive segmentation, the boundaries of the signals are specified automatically, so the length of the signal may not be equal. There are many adaptive segmentation methods such as Varri, modified Varri, generalized likelihood ratio (GLR), nonlinear energy operator (NLEO), segmentation based on fractal dimension (FD) and short-time Fourier transform (STFT) [2-12].

Varri's method has not acceptable efficiency, so Krajca proposed a new method using Varri's method named modified Varri. In this method two windows move along the signal and calculate $\sum_{k=1}^{l}|x_k|$ and $\sum_{k=1}^{l}|x_k - x_{k-1}|$ then G function is defined as:

$$G_m = A_1\left|A_{dif_{m+1}} - A_{dif_m}\right| + F_1\left|F_{dif_{m+1}} - F_{dif_m}\right| \qquad (1)$$

where l and x_k show the window length and the k^{th} data point, respectively. Also m signifies the number of the window. Local maximums in the G function, above a threshold that is defined before, indicate boundaries of each segment. Modified Varri has an important problem: this method has three parameters (length of the windows, A_1 and F_1) which are chosen experimentally [1].

The other method that is based on the fractal dimension (FD) has slow speed, so it may not use in the online segmentation. Inasmuch as each change in the amplitude and/or frequency is detected in FD, this method is a good technique to segment a signal.

In GLR method two sliding windows move along whole the signal and then the data is modeled by using autoregressive model (AR) in each window. When both of the windows are placed in one segment of the signal, their AR are approximately similar. However, when second window enter in a new segment, difference between AR of windows starts to increase. When boundary of the two windows placed in the boundary of the signal, the same as Figure 1, between of the segments in case of AR will be maximum. Wavelet transform (WT) is used in a variety of signal processing applications such as selecting features from the time-frequency domain, signal segmentation and signal de-noising. To improve GLR, in [12] combining wavelet to GLR was proposed and it was named wavelet generalized likelihood ratio (WGLR). Although these methods has a good performance, the computational burden of WGLR or GLR is offensive.

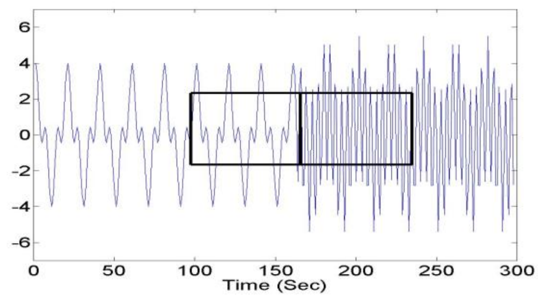

Fig. 1. Illustration of joint sliding windows and the time that boundary of the windows is placed in boundary of the segments.

In NLEO method, the same as GLR, two sliding windows moved along the signal are used. Each window computes the term that is defined as below [3]:

$$\psi_d[x(n)] = x^2(n) - x(n-1)\,x(n+1) \tag{2}$$

If the *x(*n*)* is a sinusoidal wave, then, $\psi_d[x(n)]$ will be defined as:

$$Q_d(n) = \psi_d[ACos(\omega_0\, n+\theta)] = A^2\, Sin^2\, \omega_0 \tag{3}$$

When ω_0 is much smaller than the sampling frequency, $Q_d(n)$ is equaled to $A^2\,{\omega_0}^2$. In other words, any change of amplitude (*A*) and/or frequency (ω_0) can be realized in $Q_d(n)$. When the signal is a multi-component wave, in [3] was indicated that the linear operation creates cross-terms which reduce performance of the NLEO method. In order to reduce the effects of cross-terms in NLEO method, in [3] it has been proposed to utilize WT. This new method is named improved nonlinear energy operator (INLEO). It should be mentioned that INLEO method can just slightly improve NLEO.

In this paper we use two sliding windows that move along the signal as shown in Figure 1. For each window, the standard deviation of the signal is computed. If the sliding windows fall within a segment, inasmuch as the both windows have similar statistical properties, the change between the two windows (modeling error) is low. But if both sliding windows aren't placed in the same segment, the modeling error become large.

Defining a suitable threshold level, if the local maximum of modeling error is above this level, a segment boundary point is detected. Also in this paper, using a variable length threshold is proposed. One problem that reduces the reliability of the proposed method is that the standard deviation can not distinguish shift effect in the consecutive segments. To overcome this problem we use integral as pre-processing step. The integral have tow advantages: beside the remove the shifting effects, it can smooth or filter the signal that it causes false boundaries decrease.

For real EEG signal, we propose to use discrete WT (DWT) as pre-processing step. The noises of the real EEG signal often happen in high frequencies (larger than 100 Hz) [13]. In other words, DWT can take away destructive noises such as electrooculogram (EOG), electrocardiogram (ECG), electromyogram (EMG) that often occurs in high frequency [13].

The rest of this paper is organized as followed. Next section briefly explains standard deviation and DWT. In the third section the proposed method is described. The performance of the suggested method is assessed by synthetic data and real EEG signal which is represented in Section 4. Finally, the conclusions are given in section 5.

2. BAKGROUND KNOWLEDGE FOR THE PROPOSED METHOD

2.1 Standard Deviation

In probability theory and statistics, the measure which indicates how far a set of numbers are spread out from each other is named the variance. In fact the variance is a parameter that describes the actual probability distribution of numbers or the theoretical probability distribution of a not fully observed population of numbers. Unlike expected absolute deviation, the variance of a signal has elements and each element is the square of its corresponding element. For instance a signal measured in meters has a variance measured in square meters. In order to conquer this problem, in this paper the standard deviation is used.

The standard deviation of the *f(x)*, denoted by ∂_x , is the positive square root of variance of the *f(x)* the same as follows:

$$std(f(x)) = \left[\mathrm{var}(f(x))\right]^{0.5} = \left[\int_{-\infty}^{+\infty} x^2 f(x)dx - \left[\int_{-\infty}^{+\infty} xf(x)dx\right]^2\right]^{0.5}$$

(4)

It should be noted that the properties of the standard deviation is close to the variance, i.e., these statistical definitions can both be utilized as an indicator of the spread of a distribution.

2.2 Discrete Wavelet Transform

DWT is a useful tool to analyze non-stationary and/or multi-component signals. DWT can be used to decompose different frequency bands of a time-domain signal with different time and frequency resolutions. In a general manner, the decomposed signal indicates the slowly and rapidly changing features of the signal in the lower frequency and higher frequency bands, respectively.

As expressed before, DWT can remove destructive noises like EOG, ECG, and EMG which these often occurred in higher frequencies of EEG signal [14, 15].

As described before, DWT decomposes a signal into different scales with different level of resolution. Majority of signal information is in low frequency which is the most important part of the signal and the information in high frequency indicates details of the signal. Because of this, DWT use low-pass and high-pass filters that are shown in Figure 2 [16].

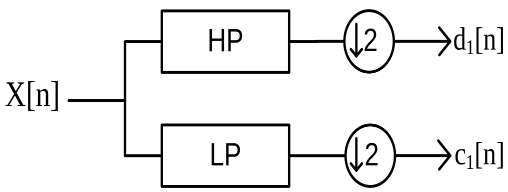

Fig 2: Decomposition of X[n] into one-level.

where X[n] is the input data and $d_1[n]$ and $d_2[n]$ depict the sub-bands signal components. After filtering, since each of the two outputs has the same length with the input signal, these outputs are downsampled by a factor of two [17].

3. PROPOSED METHOD

In this method two similar scrolling windows move along the signal. For each window, the standard deviation is computed. Inasmuch as the aim of the signal segmentation is that a signal is divided into pieces that have the same statistical characterizations such as amplitude and frequency, we prove that standard deviation is depending on amplitude and frequency. We assume $f(x) = a\cos w(x)$ in equation (4). Therefore standard deviation of $f(x)$ is equaled to:

$$std(a\cos w(x)) = \left[\mathrm{var}(a\cos w(x))\right]^{0.5} = \left[a^2\,\mathrm{var}(\cos w(x))\right]^{0.5}$$
$$= a\left[\mathrm{var}(\cos w(x))\right]^{0.5} \qquad (5)$$

and variance of the $\cos(wx)$ is attained:

$$\mathrm{var}(\cos(wx)) = \int_0^l x^2\cos(wx)dx - \left[\int_0^l x\cos(wx)dx\right]^2 =$$
$$-\frac{3}{2w^4} - \frac{l^2}{2w^2} + \left[\frac{l^2}{w} + \frac{2l-2}{w^3}\right]\sin(wl)$$
$$+\left[\frac{2l}{w^2} + \frac{2}{w^4}\right]\cos(wl) + \frac{-l}{w^3}\sin(2wl)$$
$$+\left[\frac{l^2}{2w^2} - \frac{1}{2w^4}\right]\cos(2wl) \qquad (6)$$

because the length of the window (l) is selected much more than $\frac{1}{w}$, it can be concluded:

$$\mathrm{var}(\cos(wx)) = -\frac{l^2}{2w^2} + \left[\frac{l^2}{w} + \frac{2l}{w^3}\right]\sin(wl)$$
$$+\left[\frac{2l}{w^2} + \frac{2}{w^4}\right]\cos(wl) + \frac{-l}{w^3}\sin(2wl) + \frac{l^2}{2w^2}\cos(2wl)$$
$$= -\frac{l^2}{2w^2} + \left[\left(\frac{l^2}{w} + \frac{2l}{w^3}\right)^2 + \left(\frac{2l}{w^2} + \frac{2}{w^4}\right)^2\right]^{0.5}\cos(wl + \theta_1)$$
$$+\left[\frac{l^2}{w^6} + \frac{l^4}{4w^4}\right]^{0.5}\cos(2wl + \theta_2) \qquad (7)$$

and θ_1 and θ_2 are defined as follows:

$$\theta_1 = \arcsin\frac{\frac{2l}{w^2} + \frac{2}{w^4}}{\left[\left(\frac{l^2}{w} + \frac{2l}{w^3}\right)^2 + \left(\frac{2l}{w^2} + \frac{2}{w^4}\right)^2\right]^{0.5}} \qquad (8)$$

$$\theta_2 = \arcsin\frac{\frac{l^2}{2w^2}}{\left[\frac{l^2}{w^6} + \frac{l^4}{4w^4}\right]^{0.5}} \qquad (9)$$

after shorting the equation (7):

$$\mathrm{var}(\cos(wx)) = -\frac{l^2}{2w^2} +$$
$$\frac{2l}{w^2}\left[l + \frac{1}{2w^2}\right]^{0.5}\cos(wl + \arcsin(0.5l^{0.5}))$$
$$+\frac{l^2}{2w^2}\cos(2wl + \frac{\pi}{4}) \qquad (10)$$

Finally, it can be concluded:

$$std(a\cos(wx)) = \frac{a}{w}[-\frac{l^2}{2} +$$
$$2l(l + \frac{1}{2w^2})^{0.5}\cos(wl + \arcsin(0.5l^{0.5}))$$
$$+l^2\cos(2wl + \frac{\pi}{4})]^{0.5} \qquad (11)$$

Thus, when the amplitude (a) and/or frequency (w) of the signal change, its standard deviation will change. It should be mentioned that l is a constant number that is meant the length of the window defined by us.

Standard deviation of the signal can not indicate a shifting effect as follows:

$$std(\cos(wx) + \mathrm{constant}) = std(\cos(wx)) \qquad (12)$$

To overcome this problem we use the integral as pre-processing step as follows:

$$\int \cos(wx) + k = \frac{1}{w}\sin(wx) + kx \qquad (13)$$

where k is a constant number. As can be seen in equation (13) the term kx causes standard deviation that will calculate in the next step changes considerably. It should be mentioned that the integral has advantages such as smoothing or filtering a signal and also it causes the frequency is shown in amplitude (term $\frac{1}{w}$).

H function is used to detect boundaries of signal's segments as follows:

$$H_a = \left| std_{a+1} - std_a \right| \qquad a=1, 2, \ldots, m-1 \qquad (14)$$

where *a* and *m* are the number of analyzed windows and the total number of analyzed windows, respectively. If the local maximum is bigger than the threshold, the current time is selected as a boundary of the segment.

Determining a threshold is one of the important problems in segmentation of the signal. In many researches, the mean value or the mean value added to standard deviation and so on are proposed as threshold. If the defined threshold is large, several boundaries of segments may not be indicated. Also, when the threshold is slow, several boundaries of segments may be selected inaccurately. To conquer this subject, we propose a local threshold obtained by a window that moves along the obtained signal (*H*) as follows:

$$T_H = mean[H_a : 1 : H_{a+l-1}] \quad a = 1, 2, \ldots, n\text{-}l + 1 \qquad (15)$$

where *n* and *l* are the length of the *H* and window, respectively. Depending on the application, the length of the window must be experimentally selected. For example for when we have less than three boundaries (four segments) for a signal, it is better that this length (*l*) is chosen as the length of *H*. Otherwise, we suggest to have *l* equal to the number of the segments.

4. PERFORMANCE EVALUATION

GLR and suggested method are implemented using MATLAB R2009a from Math Works. The performance and efficiency of this method is evaluated using synthetic multi-component data, and real EEG data.

4.1 Synthetic Signal

In order to assess the performance of the suggested method, this algorithm and GLR method are applied on a synthetic multi-component signal which defined as follows:
Epoch 1: $3.5\cos(9\pi t) + 4.5\cos(6\pi t)$,
Epoch 2: $3.5\cos(9\pi t) + 4.5\cos(6\pi t) + 1$,
Epoch 3: $8.5\cos(6\pi t) + 5.5\cos(10\pi t)$,
Epoch 4: $4.5\cos(2\pi t) + 5.5\cos(8\pi t)$,
Epoch 5: $5\cos(2\pi t) + 5.5\cos(5\pi t) + 2\cos(8\pi t)$,
Epoch 6: $7\cos(5\pi t) + 6\cos(7\pi t)$,
Epoch 7: $9\cos(3\pi t) + 7.5\cos(9\pi t)$,
Epoch 8: $9\cos(4\pi t) + 8\cos(4\pi t) + 3\cos(3\pi t)$.

Figure 3.a, 3.b and 3.c show 500 seconds of the original signal, integral of the original signal, and the result of applying the proposed method, respectively. As can be seen the boundaries for all eight segments can be accurately detected. Figure 4.a and 4.b show 500 seconds of original signal and result of *H* function without using integral as pre-processing step, respectively. The boundary between first and second segment is not detected due to shifting effect. To demonstrate emphasis of this algorithm, in Figure 5, the output of the GLR method is shown. The results show the proposed method has high performance for signal segmentation.
In order to make the signals more similar to real signals, we added Gaussian noise to original signals and then evaluated the performance of the proposed method. Three different parameters that is the true positive (TP) miss or false negative (FN) and false positive (FP) ratios are used to evaluate the performance and effectiveness of the proposed method. These parameters defined as $TP = \left(N_t / N \right)$, $FN = \left(N_m / N \right)$, and $FP = \left(N_f / N \right)$.
where N_t, N_m and N_f represent the number of true, missed and falsely detected and *N* shows the actual number of segment boundaries.
In table 1 the results of segmentation for 50 synthetic data using the proposed method is shown next to the results of GLR method. It should be noted that TP, FN and FP ratios for all these proposed method is very better than GLR method. By using the proposed method we can achieve 100% accuracy on a set of 50 synthetic signals.
This table reveals that this method has a better accuracy compared with the GLR method. Although GLR method has acceptable true positive ratio, the false positive ratio of this method has high false positive ratio. Thus, GLR method has low reliability and may not be suitable in segmentation of signals.

Table 1: Effect of applying proposed method and GLR method on set of synthetic data.

The proposed method				
SNR	5 dB	10 dB	15 dB	Without Noise
TP	100%	100%	100%	100%
FN	0%	0%	0%	0%
FP	35.3%	27.9%	20.5%	17.6%
GLR method				
SNR	5 dB	10 dB	15 dB	Without Noise
TP	95.5%	95.5%	97%	97%
FN	4.5%	4.5%	3%	3%
FP	350%	220%	180%	140%

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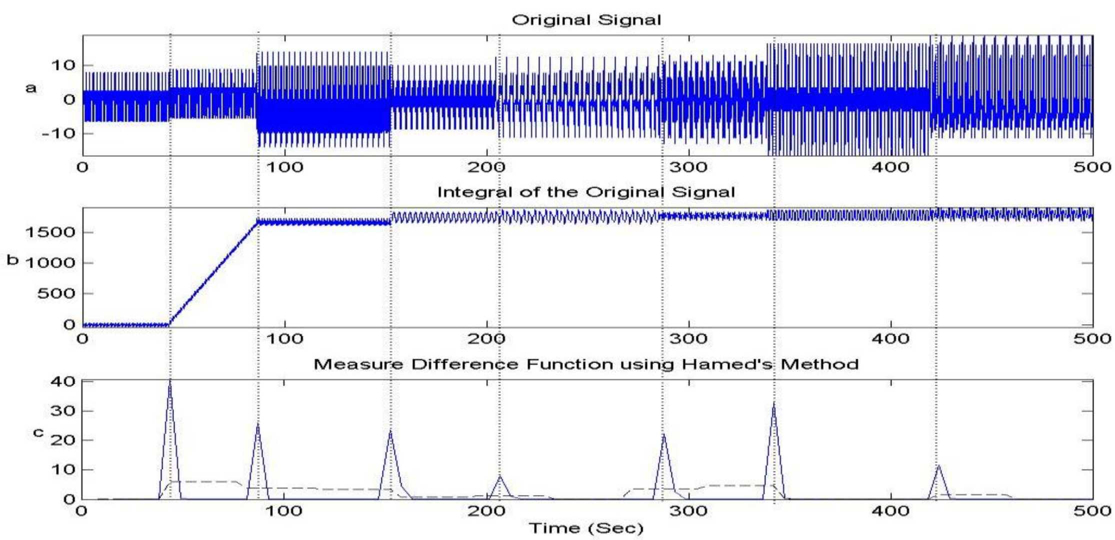

Fig 3: Signal segmentation in synthetic signal with using the proposed method; (a) original signal, (b) integral of the original signal, and (c) the variation of the *H* function.

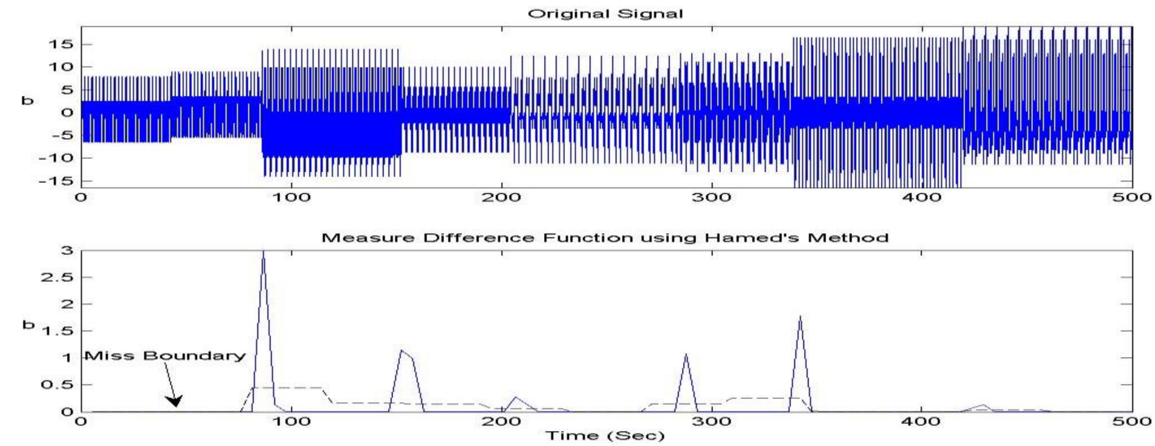

Fig 4: Signal segmentation in synthetic signal with using the proposed method without using integral; (a) original signal, and (b) the variation of the *H* function.

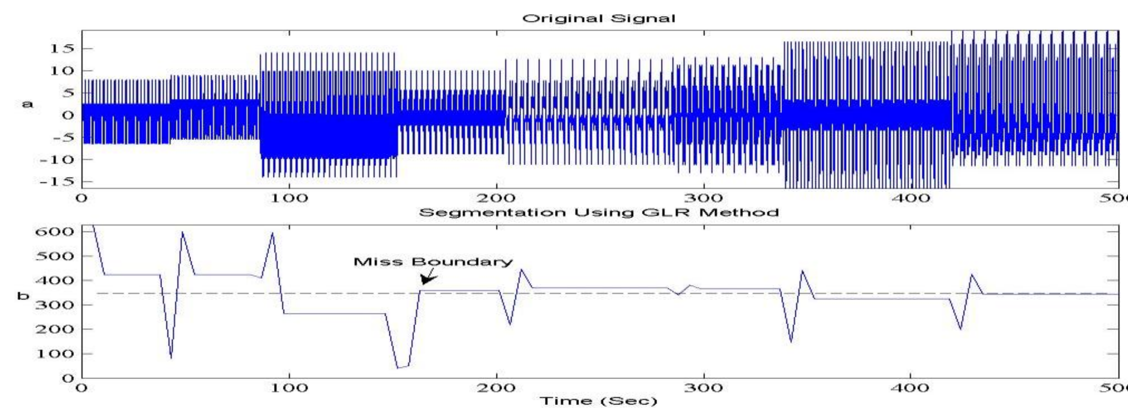

Fig 5: Signal segmentation in synthetic signal using GLR method; (a) original signal, and (b) Output of the GLR method.

4.2. Real EEG

As described before, segmenting a signal is a pre-processing step for EEG signal. In this part one epoch of a real newborn EEG signal that shown in Figure 6.a has been used. The result of applying the proposed method is shown in Figure 6.c. It can be seen that all three segments can be accurately segmented. Also, in this paper suggested using DWT as pre-processing step. Figure 7 show the result of this method. Also, this method can indicate three segments accurately.

In order to represent efficiency of the proposed method, in Figure 7.a, GLR is used for real newborn EEG signal the same as Figure 8.a. Output of GLR method is shown in Figure 8.b. We can see the influence of this method on the achieved outputs. It should be mentioned that due to three attained segmentation, we determine *L* equal to length of the obtained signal.

5. CONCLUSION

Previous methods for segmenting a signal had problems and difficulties such as slow speed, several parameters which must define experimentally, and low performance. In this paper a novel method to segment a signal in general and real EEG signal in particular using standard deviation, integral, discrete wavelet transform, and variable threshold has been proposed. In this paper we proved the standard deviation can indicate changes in the amplitude and/or frequency. To remove the effect of the shifting effect and smooth the signal, the integral is used as pre-processing step. Also, for enhancing the performance of the proposed method we use the variable threshold. In order to evaluate the performance of these methods, we use synthetic and real EEG signals. Discrete wavelet transform is a wide-spreading tool to remove destructive noises like EMG and EOG in EEG signals. Although the proposed method is easy and fast for segmenting a signal, the results have illustrated that the performance of this method is better than that of GLR method known to be an acceptable method for segmenting a signal. Also, this method has fewer variables comparing with several existing methods such as Modified Varri.

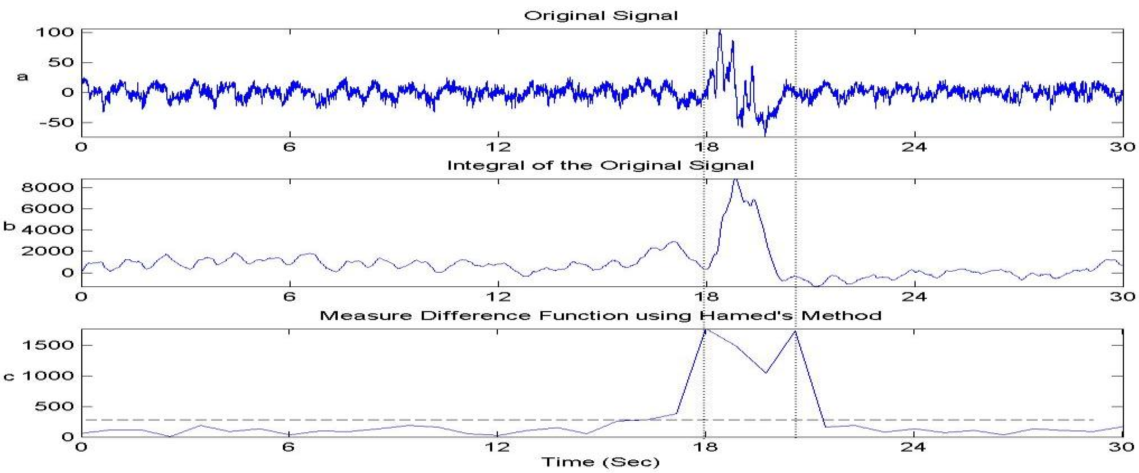

Fig 6: Signal segmentation in real EEG signal with using the proposed method; (a) original signal, (b) integral of the original signal, and (c) the variation of the *H* function.

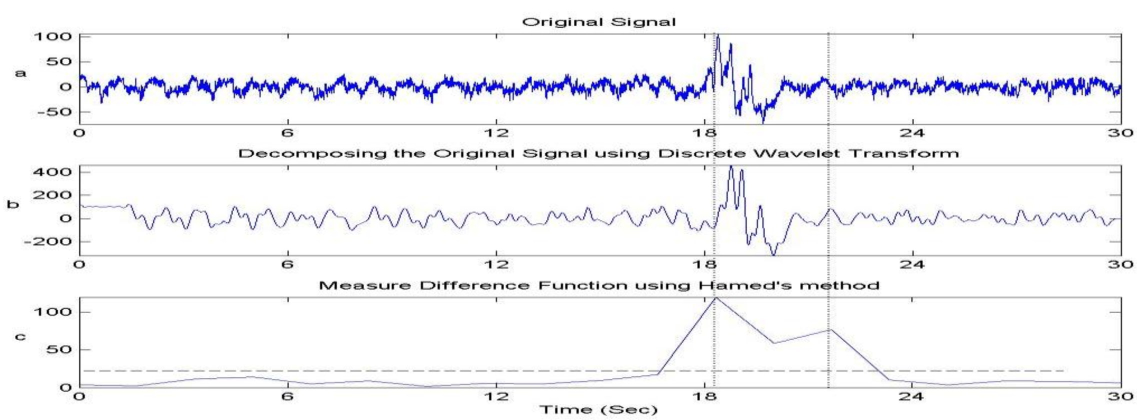

Fig 7: Signal segmentation in real EEG signal with using the proposed method and DWT; (a) original signal, (b) the decomposed signal using DWT, and (c) the variation of the *H* function.

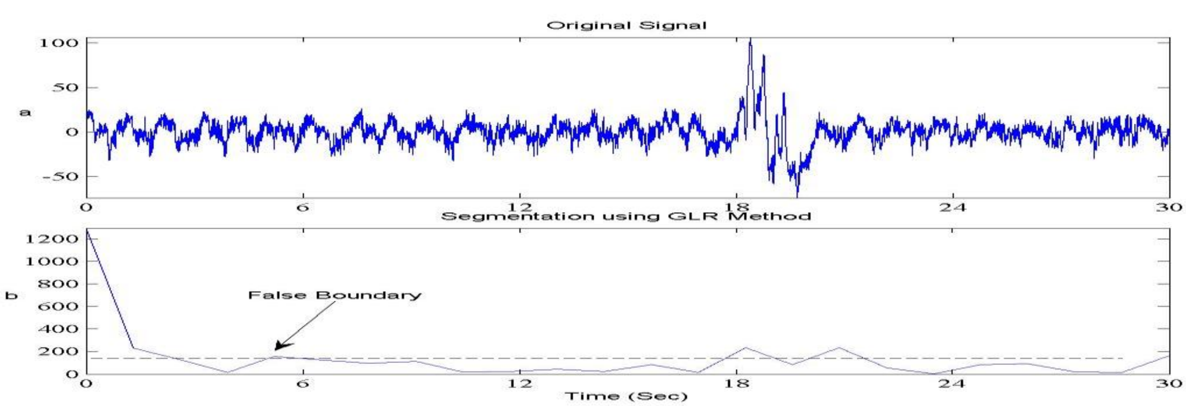

Fig 8: Signal segmentation in real EEG signal; (a) original signal, and (b) output of the GLR.

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