Fuzzy Focal Elements in Dempster-Shafer Theory of Evidence: Case study in Risk Analysis

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ABSTRACT:

Evidence Theory is an important tool of uncertainty modelling when both epistemic and aleatory uncertainties are present in the problem under consideration. In the absence of empirical data, experts in related fields provide necessary information. The fundamental objects of this theory of evidence are called focal elements, and the primitive function associated with it is called basic probability assignment (bpa). Focal elements are usually crisp subsets of some universal set. However in certain situations focal elements may also be represented by fuzzy numbers. In this paper we discuss Dempster-Shafer theory of evidence with fuzzy focal elements. We have considered two hypothetical case studies in risk analysis in this setting.

Keywords: Fuzzy Focal elements, Dempster-Shafer theory of evidence, Generalized fuzzy number with height, Risk analysis.

1. INTRODUCTION:

Probability theory is a very strong and well established mathematical tool to deal with objective uncertainty (i.e., uncertainty arises from heterogeneity or the random character of natural processes). However, all uncertainties arising in different situations are not of objective type. Such problems cannot be handled by traditional probability theory. Uncertainty may arise due to scarce or incomplete information or data, measurement error or data obtain from expert judgment or subjective interpretation of available data or information. These are of subjective nature. Traditional probability theory is inappropriate to represent subjective uncertainty (i.e., uncertainty arises from the partial character of our knowledge of the natural world). To overcome the limitation of probabilistic method, Dempster put forward a theory and now it is known as evidence theory or Dempster-Shafer theory (1976). This theory is nowadays widely used for the objective and subjective uncertainty analysis. The use of Dempster-Shafer theory in risk analysis has many advantages over the conventional probabilistic approach.

Experts opinion are sought when encountering subjective uncertainty. This is usually done in situations like cost of technical difficulties involved; it is difficult/impossible to make enough observations to quantify the models with real data etc. Sometimes these are also use to refine the estimate obtained from real data as well. Generally in Dempster-Shafer theory of evidence, experts provide basic probability assignments (bpa) for interval focal elements. Presence of uncertainty data can be treated as triangular fuzzy number (TFN) because TFN encodes only most likely value (mode) and the spread (confidence interval). Thus we get an extended version of Dempster-Shafer theory of evidence. In this paper we consider modelling focal elements as fuzzy number. The use of fuzzy focal elements is found in ([8], [9], [10]). In these papers the basic framework of DST is used in medical diagnosis. Every disease is associated with a set of symptoms. The symptoms are usually of fuzzy nature (e.g., low blood pressure, high body temperature etc.) and they represent the fuzzy focal elements in DST. Membership functions for these symptoms can be defined in consultation with an expert (a physician) or during training data investigation. Then bpas are assigned to the focal elements. In the calculation of belief and plausibility for the disease only those focal elements (symptoms) will take part for which the membership value corresponding to the observed value (laboratory test), exceeds some given threshold value. [Bel(D), Pl(D)] determines the credibility of the diagnosis. In this paper we also consider DST with fuzzy focal elements but the calculation of Bel and Pl is fundamentally different from that in ([8], [9], [10]).

2. BASIC CONCEPT OF FUZZY SET THEORY:

To estimate the effects of environmental pollution on human, risk assessment is performed. However, environmental data tends to be vague and imprecise, so uncertainty is associated with any study related with risk assessment. Fuzzy set theory is a tool which is used to characterize imprecisely defined variables, as well as to define relationships between variables based on expert knowledge and use them to compute results. In this section, some necessary backgrounds and notions of fuzzy set theory that will be required in the sequel are reviewed ([1], [2]).

Definition 2.1: Let X be a universal set. Then the fuzzy subset A of X is defined by its membership function

$$\mu_A : X \rightarrow [0,1]$$

Which assign a real number $\mu_A(x)$ in the interval [0, 1], to each element $x \in A$, where the value of $\mu_A(x)$ at x shows the grade of membership of x in A.

Definition 2.2: Given a fuzzy set A in X and any real number $\alpha \in [0, 1]$. Then the α -cut or α -level or cut worthy set of A, denoted by ${}^{\alpha}A$ is the crisp set

$${}^{\alpha}A = \{x \in X : \mu_A(x) \geq \alpha\}$$

The strong a cut, denoted by ${}^{\alpha+}A$ is the crisp set

$${}^{\alpha}A = \{x \in X : \mu_A(x) > \alpha\}$$

Definition 2.3: The support of a fuzzy set A defined on X is a crisp set defined as

Supp (A) = $\{x \in X : \mu_A(x) > 0\}$

Definition 2.4: The height of a fuzzy set A, denoted by $h(A)$ is the largest membership grade obtain by any element in the set and it is denoted as $h(A) = \sup_{x \in X} \mu_A(x)$

Definition 2.5: A fuzzy number is a convex normalized fuzzy set of the real line R whose membership function is piecewise continuous.

Definition 2.6: A triangular fuzzy number A can be defined as a triplet [a, b, c]. Its membership function is defined as:

$$\mu_A(x) = \begin{cases} \frac{x-a}{b-a}, a \le x \le b \\ \frac{c-x}{c-b}, b \le x \le c \end{cases}$$

Definition 2.7: A trapezoidal fuzzy number A can be expressed as [a, b, c, d] and its membership fuzzy number is defined as:

$$\mu_A(x) = \begin{cases} \frac{x-a}{b-a}, a \le x \le b \\ 1, b \le x \le c \\ \frac{d-x}{d-c}, c \le x \le d \end{cases}$$

2.8 Intersection of fuzzy number:

The operation *intersection* of two fuzzy numbers A and B whose membership function are μ_A and μ_B respectively is defined as

$$\mu_{A \cap B}(x) = \min(\mu_A(x), \mu_B(x)) \qquad \ldots(1)$$

2.9 Generalized Fuzzy Numbers (GFN):

The membership function of GFN ([5], [6]) $A = [a, b, c, d; w]$ where $a \le b \le c \le d, 0 < w < 1$ is defined as

$$\mu_A(x) = \begin{cases} 0, x < a \\ \frac{x-a}{b-a}, a \le x \le b \\ w, b \le x \le c \\ \frac{x-c}{d-c}, c \le x \le d \\ 0, x > d \end{cases} \qquad \ldots(2)$$

If $w = 1$, then GFN A is a normal trapezoidal fuzzy number $A = [a, b, c, d]$. If $a = b$ and $c = d$, then A is a crisp interval .If $b = c$ then A is a generalized triangular fuzzy number. If $a = b = c = d$ and $w = 1$ then A is a real number. Compared to normal fuzzy number the GFN can deal with uncertain information in a more flexible manner because of the parameter w that represent the degree of confidence of opinions of decision maker's.

2.10 Generalized Fuzzy Numbers (GFN) with left height (w_l) and right height (w_r):

Intersection two normal fuzzy numbers is in general not a fuzzy number. We have named such fuzzy sets as generalized fuzzy number with height. We defined generalized fuzzy number with left height (w_l) and right height (w_r) as

$$\mu_A(x) = \begin{cases} 0; x < a \\ \frac{x-a}{b-a}; a \le x \le b \\ \frac{x-b}{c-b}(w_r - w_l) + w_l; b \le x \le c \\ \frac{x-c}{d-c}; c \le x \le d \\ 0, x > d \end{cases} \quad \ldots(3)$$

It is denoted by $A = [a, b, c, d; w_l, w_r]$, where $a \le b \le c \le d, 0 < w_l, w_r < 1$.

If $w_l = w_r$, then A is a generalised trapezoidal fuzzy number $A = [a, b, c, d; w_l \ (or\ w_r)]$. If $w_l = w_r$ and $a = b$ and $c = d$, then A is a crisp interval .If $w_l = w_r$ and $b = c$ then A is a generalized triangular fuzzy number $[a, b, c, d; w_l \ (or\ w_r)]$. If $a = b = c = d$ and $w_l = w_r = 1$ then A is a real number.

For example, intersection of the triangular fuzzy numbers [10, 16, 20] and [8, 20, 32] produce the generalized fuzzy number [10, 12, 17, 20; 0.33, 0.75] with left height 0.33 and right height 0.75 and which is given in figure 1.

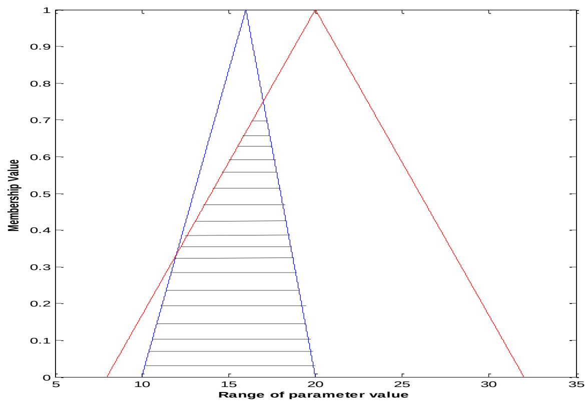

Figure 1: Intersection of fuzzy number (Shaded area)

2.11 Normalization of generalized fuzzy number:

To normalize a generalized fuzzy number we divide the membership function of the fuzzy number by its maximum height (highest membership grade). Then the generalized fuzzy number expressed by (2) becomes

$$\mu_A(x)=\begin{cases}\frac{x-a}{b-a}, a\le x\le b\\ 1, b\le x\le c\\ \frac{d-x}{d-c}, c\le x\le d\end{cases} \quad ...(4)$$

Here *A* indicates normal trapezoidal fuzzy number. If *b=c* then *A* is a normal triangular fuzzy number.

For generalized fuzzy number with left height (w_l) and right height (w_r), we divide the membership function by maximum of w_l and w_r. Then the fuzzy number (3) will be

$$\mu_A(x)=\begin{cases}0; x<a\\ \frac{x-a}{b-a}; a\le x\le b\\ \frac{\left\{\frac{x-b}{c-b}(w_r-w_l)+w_l\right\}}{\max(w_l,w_r)}; b\le x\le c\\ \frac{x-c}{d-c}; c\le x\le d\\ 0, x>d\end{cases} \quad ... (5)$$

3. BASIC CONCEPTS OF DEMPSTER - SHAFER THEORY OF EVIDENCE:

Dempster Shafer theory of evidence is widely used for modelling both epistemic and aleatory uncertainty. The basic underlying set considered in this theory is called a frame of discernment Θ which is a set of mutually exclusive and exhaustive propositional hypotheses, one and only one of them is true [7].

Evidence theory is based on two dual non-additive measure, i.e. belief measure and plausible measure. The primitive function in Dempster-Shafer theory used to define belief measure and plausible measure is known as basic probability assignment (bpa or mass function) and it is usually denoted by m.

Mathematically, bpa is a function $m: 2^{\Theta}\to[0,1]$ satisfying the following two conditions:

$$\left.\begin{array}{l} m(\phi)=0\\ \sum_{A\subseteq\Theta} m(A)=1\end{array}\right\} \quad ...(6)$$

Where ϕ is an empty set and A is any subset of Θ .

Given a frame, Θ , for each source of evidence, a mass function assigns a mass to every subset of Θ , which represents the degree of belief that one of the hypotheses in the subset is true, given the source of evidence.

A subset *A* of frame Θ is called the focal element of m , if $m(A)>1$.

Using the basic probability assignment (bpa), belief measure and plausibility measure are respectively determined as

$$Bel(A)=\sum_{B\subseteq A} m(B), A\subseteq\Theta \text{ and } Pl(A)=\sum_{B\cap A=\phi} m(B)...(7)$$

Here $m(B)$ is the degree of evidence in the set *B* alone, whereas *Bel(A)* is the total evidence in set *A* and all subset *B* of *A* and the plausibility of an event A is the total evidence in set *A*, plus the evidence in all sets of the universe that intersect with *A*.

Where *Bel*(*A*) and *Pl*(*A*) represent the lower bound and upper bound of belief in *A*. Hence, interval [*Bel*(*A*) , *Pl*(*A*)] is the range of belief in *A* .

4. DEMPSTER'S RULE OF OMBINATION:

If two basic probability assignments (mass functions) m_1 and m_2 are given by two different evidence sources Dempster's combination rule for can be use to combine them as:

$$m(C)=\frac{\sum_{A\cap B=C} m_1(A)m_2(B)}{1-\sum_{A\cap B=\phi} m_1(A)m_2(B)} \quad ... (8)$$

5. FOCAL ELEMENTS AS FUZZY NUMBERS:

In this section we show how belief and plausibility measures can be constructed when focal elements are fuzzy sets. Suppose Θ is a universe of discourse and information regarding some parameter, say, X, expert provide basic probability assignments (bpa) of focal elements as triangular fuzzy number.

i.e., say, $m([a_i,b_i,c_i])=p_i, i=1,2,3,...n$ satisfying

$$\sum_{i=1}^{n} p_i=1$$

We can calculate α -cut of each fuzzy number. α -cut of $[a_i,b_i,c_i]$ are $[a_i+(b_i-a_i)\alpha, c_i-(c_i-b_i)\alpha]$ for $\alpha\in[0,1]$.

As α -cut gives closed interval for continuous fuzzy number, therefore we can calculate cumulative belief and plausibility measure for each α -cut using classical Dempster-Shafer theory of evidence. Then we will have a collection of cumulative belief and plausibility for each α -cut. That is, for $\alpha=0:0.1:1,$ we get 21cumulative distribution functions (cdfs); 10 cumulative belief measures, 10 cumulative plausibility measures, while there is one cdf where both belief and plausibility measure coincide and which corresponds to 1-cut $(\alpha=1)$. From these cumulative belief and plausibility measures, membership functions (fuzzy numbers) of risk at different fractiles can be generated ([3], [4]).

For example suppose for a variable *X*, the Dempster-Shafer structure is given as in table I. We need to calculate their cumulative belief and plausibility.

Focal Elements	Basic Probability assignment
[15, 22.5, 30]	0.05
[30, 37.5, 45]	0.1
[30, 45, 60]	0.2
[45, 60, 70]	0.3
[75, 82.5, 90]	0.1
[60, 75, 90]	0.2
[90, 97.5, 105]	0.05

Table I: basic probability assignment of the fuzzy focal elements

The representations of the focal elements are available in the form of triangular fuzzy number whose general form of alpha-cut together with the basic probability assignment are given in table II.

Focal Elements	Basic probability Assignment
[15+7.5α, 30-7.5α]	0.05
[30+7.5α, 45-7.5α]	0.1
[30+15α, 60-15α]	0.2
[45+15α, 75-15α]	0.3
[75+7.5α, 90-7.5α]	0.1
[60+15α, 90-15α]	0.2
[90+7.5α, 105-7.5α]	0.05

Table II: Alpha-cuts of fuzzy focal elements

The graphical representation of cumulative belief and plausibility of the focal elements when α = 0, 0.5 and 1 (for simple and clear representation of Belief and Plausibility we have consider α = 0, 0.5 and 1) are also depicted in figure 2.

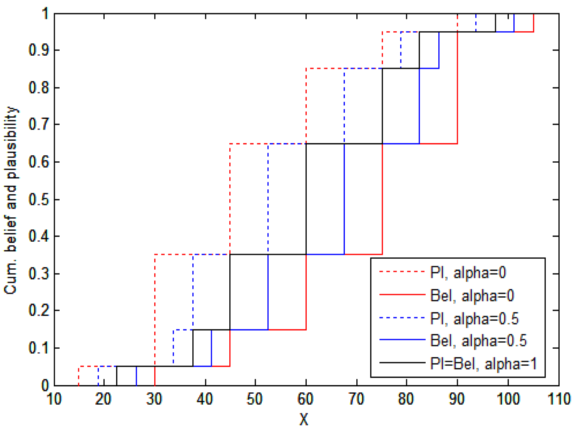

Figure 2: Bel and Pl of fuzzy focal elements when α=0, 0.5 and 1

We consider membership function of *X* at 90th fractile. For α = 0, 75 and 90 are values of *X*. Similarly, 78.75 and 86.25 are value of the variable *X* for α=0.5 and 82.5 is the value of the variable *X* for α=0**.** These are depicted in figure 3 and 4.

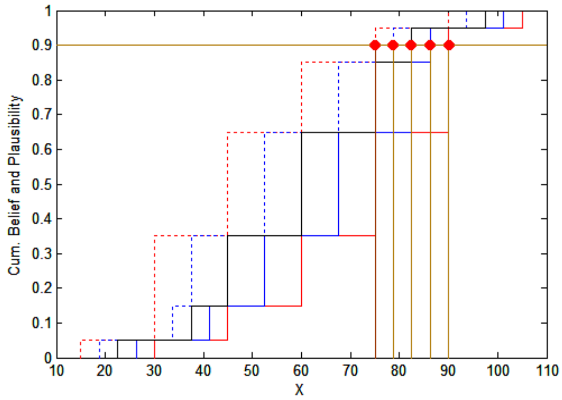

Figure 3: Bel & Pl of *X* for α=0.0, 0.5 & 1.0

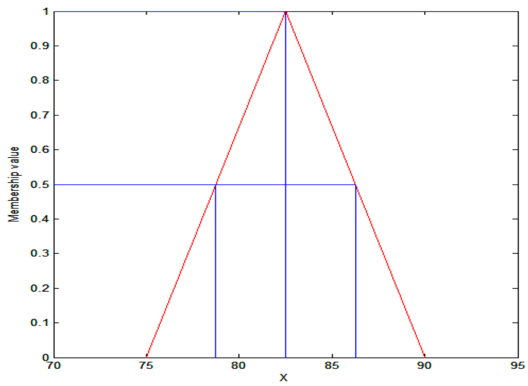

Figure 4: The membership function of *X* at 90th fractile

6. DEMPSTER'S COMBINATION RULE FOR FUZZY FOCAL ELEMENTS:

When information regarding the same parameter is given by two sources of evidence, it can be combined to a single source by Dempster's rule of combination. We extend the combination rule for fuzzy focal elements. Let us assume that two experts provide basic probability assignments (bpa) for focal elements which are taken as triangular fuzzy numbers, say, $m_1([a_i,b_i,c_i]) = p_i, i = 1,2,3,...n$ where $\sum_{i=1}^{n} p_i = 1$ and $m_2([a_j,b_j,c_j]) = p_j, j = 1,2,3,...m$ where $\sum_{j=1}^{m} p_j = 1$ respectively. To combine these two experts' mass function (bpa) we use Dempster's rule of combination. Here intersection of focal elements will be fuzzy set (generalised fuzzy number) as initial focal elements are considered as fuzzy number.

For example let's assume experts1 and expert 2 provide basic probability assignment (bpa) for fuzzy focal elements and their Dempster-Shafer structure is given in table VI and table VII. We need to combine their opinions.

Fuzzy Focal Elements	Basic Probability Assignment
[6, 8, 10]	0.3
[10, 15, 20]	0.6
[20, 25, 30]	0.1

Table VI: BPA fuzzy focal elements assigned by expert 1

Fuzzy Focal Elements	Basic Probability Assignment
[6, 18, 30]	0.6
[10, 15, 20]	0.4

Table VII: BPA fuzzy focal elements assigned by expert 2

Replacing crisp focal element by fuzzy focal element we can obtain the combined Dempster's structure as ([6, 9.432, 10; 0.286], 0.2143), ([10, 12.86, 15.94, 20; 0.572, 0.828], 0.4286), ([10, 15, 20; 1], 0.2857) and ([20, 22.92, 30; 0.59], 0.07143).

7. SAMPLING TECHNIQUE FOR POSSIBILITY THEORY:

Sampling technique to generate random numbers generally used in probabilistic method can also be used for possibility theory. Here, uniformly distributed random numbers between 0 and 1 are generated. Random variables are generated by equating these numbers to belief function and plausibility function. Two numbers are generated in this process, one corresponding to belief function (x_b) and the other corresponding to the plausibility function (x_p). This process is repeated for all the uncertainty variables present in the model.

For a uniformly distributed random number u the uncertain variable x_n and uncertainty variable x_p are obtained as

$$x_n = Bel^{-1}(u) \text{ and } x_p = Pl^{-1}(u).$$

For example, if figure 5 is the graphical representation of cumulative belief and plausibility whose Dempster-Shafer structure is given by an expert. For the uniformly distributed random number 0.75, using possibility sampling we have 25 and 35 are the values of the uncertain variable corresponding to plausibility measure and belief measure respectively.

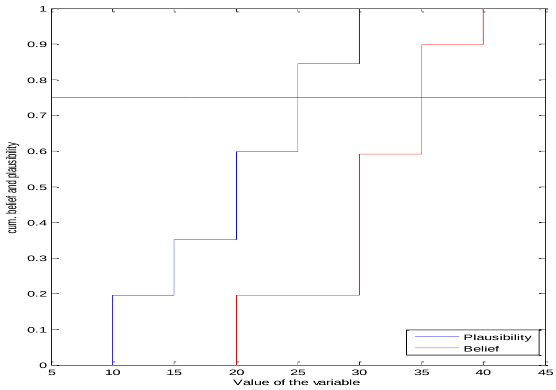

Figure 5: Cumulative belief and plausibility

8. CASE STUDY:

In this section, we perform non-cancer human health risk assessment with hypothetical data. Two cases are considered and in both cases two parameters are expressed in terms of Dempster-Shafer structure. Possibility sampling technique is used to calculate the output parameter.

Human being is always exposed to radiation either from natural or anthropogenic sources in the environment. Besides natural nuclides being present in the environment since the beginning of the earth's existence, manmade nuclides are being released from nuclear installations and fallouts from the nuclear test and nuclear accident. Further produced water is a significant source of waste generated in the production phase of oil and gas operations. When produce water is discharged into the ocean, a number of heavy metals and poly aromatic hydrocarbon present in it may introduce toxicity and bioaccumulation in aquatic organisms. These compounds are harmful to fish and therefore human can be affected through intake of such fishes. Consequently human health is indirectly (or directly) affected through different pathways such as inhalation, ingestion, submersion and dermal contact. When hazardous substances are released into the environment, an evaluation is necessary to determine the possible impact such substances may have on human health and ecology. For this purpose, risk assessment is performed to quantify the potential detriment to human and evaluate the effectiveness of proposed remediation measures.

A lot of organic and inorganic pollutants exist in produced water. However, here we consider only the heavy metal arsenic (As) because of its toxicity and high concentration in produced water.

The general form of a comprehensive food chain risk assessment model [11] as provided by EPA, 2001 is follows:

$$CDI = \frac{C_f \times FIR \times FR \times EF \times ED \times CF}{BW \times AT} \quad \dots (9)$$

Where CID = Chronic daily intake (mg/kg-day), FIR = fish ingestion rate (g/day), FR = fraction of fish from contaminated source, EF = exposure frequency (day/year), ED = exposure duration (years), CF = conversion factor (= 10^{-9}), BW = body weight (kg), AT = averaging time (days) and C_f = chemical concentration of fish tissue (mg/kg). The chemical concentration in fish tissue (C_f) can be computed as

$$C_f = PEC \times BCF \quad \dots (10)$$

Where PEC = predicted environmental concentration (mg/l) and BCF is the chemical bioaccumulation factor in fish (l/kg).

The non-cancer risk model for fish ingestion is expressed as:

$$Risk_{non-cancer} = \frac{CDI}{Rfd} \quad \dots (11)$$

Where, *Rfd* is the reference dose.

Case I: In this case, representation of the parameters predicted environmental concentration (PEC) and chemical bioaccumulation factor (BCF) are considered to be epistemic nature i.e., bpa are assigned for fuzzy focal elements. Other parameters are taken to be constant. Values of the parameters for the calculation of non-cancer human health risk assessment are given in the table VIII.

Parameter	Units	Type of variable	Value/distribution
Average Time (AT)	Days	constant	25550
Body Weight (BW)	Kg	constant	70
Exposure Duration (ED)	Years	constant	30
Exposure frequency (EF)	Days/year	constant	350
Fraction of contaminated Fish (FR)	-	constant	0.5
Fish Ingestion Rate (FIR)	g/day	constant	170
Conversion Factor (CF)	-	constant	1E-09
PEC for As	ug/l	epistemic	Given in table(IX)
BCF for As	l/kg	epistemic	Given in table (X)
Oral Rfd for As	mg/(kg.day)	constant	3.0E-04

Table VIII: Parameters used in the risk assessment

Fuzzy focal elements	Basic probability assignment
[1.5, 2.25, 3.0]	0.20
[2.0, 3.0, 4.0]	0.25
[4.0, 5.5, 7.0]	0.35
[7.0, 8.0, 9.0]	0.20

Table IX: Dempster-Shafer structure for PEC

Fuzzy focal elements	Basic probability assignment
[30, 35, 40]	0.15
[40, 45, 50]	0.30
[45,50,55]	0.35
[55, 57.5, 60]	0.20

Table X: Dempster-Shafer structure for BCF

The result of the non-cancer human health risk assessment using equation (11) is depicted in figure (6).

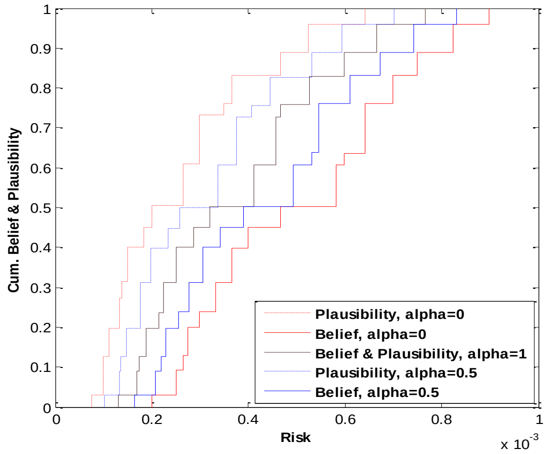

Figure 6: Non-cancer human health risk

The cumulative belief and plausibility corresponding to lower and upper value of each alpha-cut are used to generate membership function of the resulting risk at 80^{th}, 85^{th} and 95^{th} fractiles of risk. The membership functions of the resulting risk at these fractiles are depicted in the following figure 7.

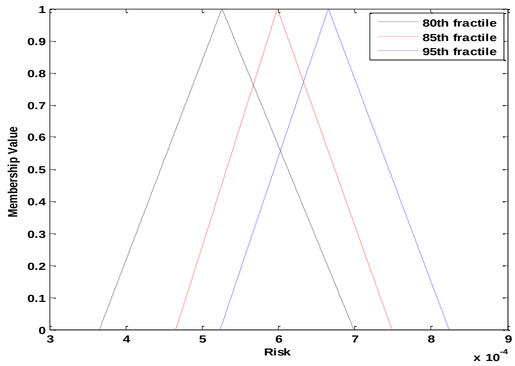

Figure 7: The membership function of risk at 80^{th}, 85^{th} and 95^{th} fractiles.

Case II: In this case, representation of the parameters chemical bioaccumulation factor (BCF) and oral reference dose (Rfd) are considered to be epistemic nature i.e., bpa are assigned for fuzzy focal elements. Other parameters are keeping constant. Values of the parameters for the calculation of non-cancer human health risk assessment are given in the table XI.

Parameter	Units	Type of variable	Value/distribution
Average Time (AT)	Days	constant	25550
Body Weight (BW)	Kg	constant	70
Exposure Duration (ED)	Years	constant	30
Exposure frequency	Days/year	constant	350

(EF)			
Fraction of contaminated Fish (FR)	-	constant	0.5
Fish Ingestion Rate (FIR)	g/day	constant	170
Conversion Factor (CF)	-	constant	1E-09
PEC for As	ug/l	constant	5.25
BCF for As	l/kg	epistemic	Given in table (X)
Oral Rfd for As	mg/(kg.day)	epistemic	Given in table (XII)

Table XI: Parameters used in the risk assessment

Fuzzy focal elements	Basic probability assignment
[2.0e-04, 2.3e-04, 2.6e-04]	0.05
[2.3e-04, 2.65e-04, 3.0e-04]	0.35
[3.0e-04, 3.3e-04, 3.6e-04]	0.40
[3.4e-04, 3.7e-04, 4.0e-04]	0.20

Table XII: Dempster-Shafer structure for Rfd

The non cancer risk assessment has been performed using the model (11) and result is depicted in the following figure 8.

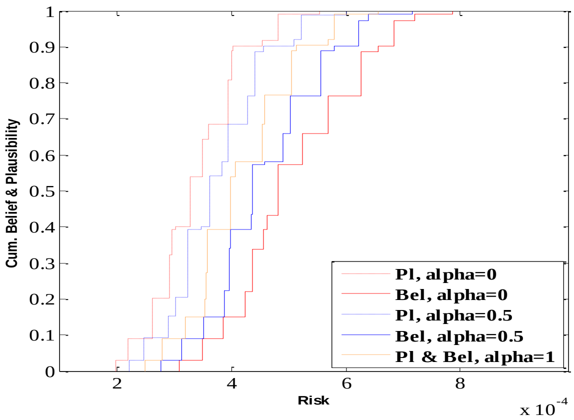

Figure 8: Non-cancer human health risk

In a similar fashion as above from cumulative belief and plausibility corresponding to lower and upper value of each alpha-cut level are used to generate membership function of the resulting risk at 80th, 85th and 95th fractiles of risk and which are given in figure 9.

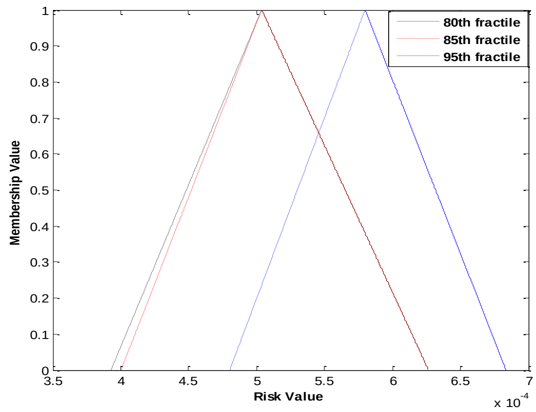

Figure 9: The membership function of risk at 80th, 85th and 95th fractiles.

In this study, case 1 uses parameters predicted environmental concentration (PEC) and chemical bioaccumulation factor (BCF) as Dempster-Shafer structure while case 2 uses parameters chemical bioaccumulation factor (BCF) and oral reference dose (Rfd) as Dempster-Shafer structure. In which focal elements of Dempster-Shafer structure are taken as fuzzy number for both the cases so the resulting risk is also a fuzzy number. The membership function of risk at 80th, 85th and 95th fractiles of risk case 1 and case 2 are depicted in figure 7 and figure 9 respectively. The membership functions of risk at different fractiles have fuzzy number which can be interpreted as risk at a certain fractile of risk being around the most likely value. For instant, at 95th fractile risk is around 6.654e-04 in which the possible range (i.e., support of the membership function) of the risk at the corresponding fractile is 5.926e-04 to 8.234e-04 for case 1. For case 2, 5.794e-04 is the most likely value i.e., risk is around 5.794e-04 while range is from 4.803e-04 to 6.835e-04 for the same fractile of risk. Similarly at 85th and 80th fractiles of risk, for case 1, 5.988e-04 and 5.261e-04 are the most likely value while ranges are [4.658e-04, 7.485-04] and [3.659e-04, 6.986e-04] respectively and for case 2, 5.038e-04 is the most likely value for both the fractiles while [4.003e-04, 6.265e-04] and [3.93e-04, 6.265e-04] are the ranges for 85th and 80th fractiles respectively.

9. CONCLUSION:

Dempster-Shafer theory of evidence is one of the important tool for decision making under uncertainty. Dempster-Shafer theory is more fruitful in situation when cost of technical difficulties involved or uniqueness of the situation under study makes it difficult/impossible to make enough observations to quantify the models with real data. Then experts provide opinion in terms of basic probability assignment for focal elements. Usually, it is seen that experts provide basic probability assignment for interval (or crisp) focal elements. However due to presence of uncertainty focal elements can sometimes be treated as triangular fuzzy number (TFN) instead of intervals or crisp set. TFN encodes only most likely value (mode) and the spread.

In this paper, we study Dempster-Shafer theory of evidence by considering focal elements as triangular fuzzy number. We have devised a method for obtaining belief and plausibility measure from bpa's assigned to fuzzy foal elements.

For that we calculate alpha-cut of the fuzzy for focal elements. Each alpha-cut produces a collection of crisp focal elements from which we calculate cumulative belief and plausibility measure. The process is repeated for sufficient number of each alpha-cut. From this we get a family of cumulative belief and plausibility measures and using these cumulative belief and plausibility measure membership function at different fractiles are generated.

We have also defined Dempster's rule of combination for fuzzy focal elements when two sources of evidences are provided with. For this purpose, we have defined generalized fuzzy number with left height and right height as intersection of fuzzy numbers does not always produce generalized fuzzy number but also produce generalized fuzzy number with left height and right height. We have seen that Dempster's combination rule for fuzzy focal elements produce generalized fuzzy numbers with left and right height.

Finally to demonstrate use of fuzzy focal elements in Dempster-Shafer theory we performed non-cancer human health risk assessment with hypothetical data by considering two cases. In case 1 parameters predicted environmental concentration (PEC) and chemical bioaccumulation factor (BCF) are provided as body of evidence and in case 2 uses parameters chemical bioaccumulation factor (BCF) and oral reference dose (Rfd) are represented by body evidence. In both the cases focal elements are taken as triangular fuzzy number. After calculation of belief and plausibility, membership functions of the risk are generated at different fractiles. The membership functions of risk at different fractiles are fuzzy number since the focal elements are taken as fuzzy numbers. We can interpret the Membership function of risk at some given fractile as risk being around the most likely value. The membership function of risk at a certain fractile provides important information to the analyst. From the membership function it is obvious that the possibility of occurrence of risk values having zero membership values for a given fractile are zero, whereas for membership value one risk is most likely. The possible ranges of the risk at the corresponding fractile are provided by the support of the membership function. The shape also provides extra information about the resulting uncertainty which is the effect of fuzzy focal elements.

ACKNOWLEDGEMENT

The work done in this paper is under a research project funded by Board of Research in Nuclear Sciences, Department of Atomic Energy, Govt. of India.

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