λ - Closed Sets in Intuitionistic Fuzzy Topological Spaces

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ABSTRACT
In this paper we introduce a new class of intuitionistic fuzzy closed set namely intuitionistic fuzzy λ-closed set and intuitionistic fuzzy λ-open set and studied some of their basic properties.

Keywords:
Intuitionistic fuzzy topology, Intuitionistic fuzzy λ –closed sets and intuitionistic fuzzy λ –open sets.

AMS subject classification: 54A40, 03F55

1. INTRODUCTION
After the introduction of fuzzy sets by Zadeh [10] in 1965 and fuzzy topology by Chang [4] in 1968, several researches were conducted on the generalizations of the notions of fuzzy sets and fuzzy topology. The idea of "intuitionistic fuzzy set" was first introduced by Atanassov [3] in 1986 and various concept of fuzzy mathematics have extended for intuitionistic fuzzy sets. In this present paper the concept of intuitionistic fuzzy λ–closed sets and intuitionistic fuzzy λ–open sets are introduced and studied some of their basic properties.

2. PRELIMINARIES
Definition 2.1 [1]
Let X be a nonempty fixed set. An intutionistic fuzzy set A in X is an object having the form A = {< x, $\mu_A(x)$, $\upsilon_B(x)$ > :x∈X}, where the function $\mu_A : X \to [0,1]$ and $\upsilon_A : X \to [0,1]$ denotes the degree of membership $\mu_A(x)$ and the degree of non membership $\gamma_A(x)$ of each element x∈X to the set A respectively and $0 \le \mu_A(x) + \upsilon_A(x) \le 1$ for each x∈ X.

Definition 2.2 [1]
Let X be a non empty fixed set. Let A and B be the intuitionistic fuzzy sets in the form A = {<x, $\mu_A(x)$, $\upsilon_A(x)$ > : x∈ X} and B = {<x, $\mu_B(x)$, $\upsilon_B(x)$ > : x∈ X} Then

(a)A ⊆ B if and only if $\mu_A(x) \le \mu_B(x)$ and $\upsilon_A(x) \ge \upsilon_B(x)$ for all x∈ X

(b)A = B if and only if A ⊆ B and B ⊆ A

(c)A^C = {<x, $\upsilon_A(x)$, $\mu_A(x)$> / x∈X}

(d)A∩B = {<x, $\mu_A(x) \wedge \mu_B(x)$, $\upsilon_A(x) \vee \upsilon_B(x)$> / x∈ X}

(e)A∪B = {x, $\mu_A(x) \vee \mu_B(x)$, $\upsilon_A(x) \wedge \upsilon_B(x)$> / x∈X}

(f) $0_\sim$ = {<x, 0, 1> / x∈X} and $1_\sim$ = { <x, 1, 0> / x∈X} are respectively the empty set and the whole set of X.

Definition 2.3 [5]
An intuitionistic fuzzy topology on X is a family τ of intuitionistic fuzzy topology sets in X satisfying the following axioms.

(i)0, 1 ∈ τ

(ii)$G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$

(iii)$\cup G_i \in \tau$ for any family $\{G_i / i \in I\} \subseteq \tau$.

In this case the pair (X, τ) is called an intuitionistic fuzzy topological space and each intuitionistic fuzzy set in τ is known as an intuitionistic fuzzy open set in X.

Definition 2.4 [5]
The complement A^C of an intuitionistic fuzzy open set A in an intuitionistic fuzzy topological space (X, τ) is called intuitionistic fuzzy closed set in X.

Definition 2.5 [5]
Let (X, τ) be in intuitionistic fuzzy topological space and A = {<x, μ_A (x), $\upsilon_A(x)$> / x∈X} be an intuitionistic fuzzy set in X. Then the intuitionistic fuzzy interior and the intuitionistic fuzzy closure are defined by

Cl(A) = ∩ {K / K is an intuitionistic fuzzy closed sets in X and A ⊆ K}

Int(A) = ∪ {G / G is an intuitionistic fuzzy open sets in X and G ⊆ A}.

Remark 2.6 [5]
For any intuitionistic fuzzy set A in (X, τ) we have

(i) $cl(A^C) = [int(A)]^C$, (ii) $int(A^C) = [cl(A)]^C$,

(iii)A is an intuitionistic fuzzy closed in X ⇔ Cl(A) = A

(iv)A is an intuitionistic fuzzy open set in X ⇔ int(A) = A

Definition 2.7
An intuitionistic fuzzy set A of an intuitionistic fuzzy topological space (X,τ) is called (i).intuitionistic fuzzy generalized closed set [9] (intuitionistic fuzzy g – closed) if cl(A)⊆U whenever A ⊆ U and U is intuitionistic fuzzy open (ii) intuitionistic fuzzy g – open set[9] if the complement of an intuitionistic fuzzy g – closed set is called intuitionistic fuzzy g - open set. (iii) intuitionistic fuzzy semi open (resp. intuitionistic fuzzy semi closed) if there exists an intuitionistic fuzzy open set ∪ (resp. intuitionistic fuzzy closed) such that ∪ ⊆ A ⊆ cl (∪) (resp. int (U) ⊆ A ⊆ U).

Remark 2.8 [9]
Every intuitionistic fuzzy closed set [9] (intuitionistic fuzzy open set) is intuitionistic fuzzy g-closed (intuitionistic fuzzy g- open set) but the converse may not be true.

3. INTUITIONISTIC FUZZY λ – CLOSED SET
In this section we introduce intuitionistic fuzzy λ-closed set and studied some of its properties.

Definition 3.1
An intuitionistic fuzzy set A of an intuitionistic fuzzy topological space (X, τ) is called intuitionistic fuzzy λ-closed if A ⊇ cl(U) whenever A ⊇ U and U is intuitionistic fuzzy open in X.

Theorem 3.2
Every intuitionistic fuzzy closed set is intuitionistic fuzzy λ-closed.

Proof: Let A be an intuitionistic fuzzy closed set in the intuitionistic topological space (X, τ). Let U be the intuitionistic fuzzy open set in X such that A ⊇ U, then cl(A) ⊇ cl(U) since A is intuitionistic fuzzy closed we have cl(A) = A. Therefore A ⊇ cl(U). Hence A is intuitionistic fuzzy λ-closed set.

Remark 3.3
The converse of the above theorem may not be true as seen in the following example.

Example 3.4
Let X = {a, b} and τ = { $\underset{\sim}{0}$, $\underset{\sim}{1}$, ∪} be an intuitionistic fuzzy topology on X where ∪ = {<a, 0.5, 0.5>, <b, 0.3, 0.6}. Then the intuitionistic fuzzy set A = {a, 0.5, 0.5>, <b, 0.8, 0.2>} is intuitionistic fuzzy λ-closed set but it is not intuitionistic fuzzy closed.

Theorem 3.5
If A and B are two intuitionistic fuzzy λ-closed sets in an intuitionistic fuzzy topological space (X, τ), then A∩B is an intuitionistic fuzzy λ-closed set.

Proof:
Let U be an intuitionistic fuzzy open set in X such that A∩B ⊇ U. Since A and B and B are intuitionistic fuzzy λ-closed sets we have A ⊇ cl(U) and B ⊇ cl(U) . Therefore A∩B ⊇ cl(U) .Hence A∩B is intuitionistic fuzzy λ-closed set.

Remark: 3.6
The union of two intuitionistic fuzzy λ-closed sets in an intuitionistic fuzzy topological space (X, τ) may not be intuitionistic fuzzy λ-closed as seen from the following example.

Example 3.7
Let X = {a, b} and ∪, A and B be the intuitionistic fuzzy sets of X defined as follows:
∪ = {< a, 0.5. 0.5 >, < b, 0.6, 0.3 >}, A = {< a, 0.5. 0.5 >, < b, 0.5, 0.3 >} and B = { < a, 0.5. 0.5 >, < b, 0.6, 0.4 >}
Let τ = { $\underset{\sim}{0}$ $\underset{\sim}{1}$, ∪} be intuitionistic fuzzy topology on X. Then A and B are intuitionistic fuzzy λ-closed in (X, τ). But A∪B is not intuitionistic fuzzy λ-closed.

Remark 3.8
The concept of intuitionistic fuzzy λ-closed sets and intuitionistic fuzzy g-closed sets are independent as seen from the following examples.

Example 3.9
Let X = {a, b} and τ = { $\underset{\sim}{0}$, $\underset{\sim}{1}$, ∪ } be an intuitionistic fuzzy topology on X ,where ∪ = { <a, 0.5, 0.5>, <b, 0.5,0.2>}. Then the intuitionistic fuzzy set A = {<a, 0.5, 0.5>, <b, 0.5, 0.4>} is not intuitionistic fuzzy g-closed but intuitionistic fuzzy λ-closed.

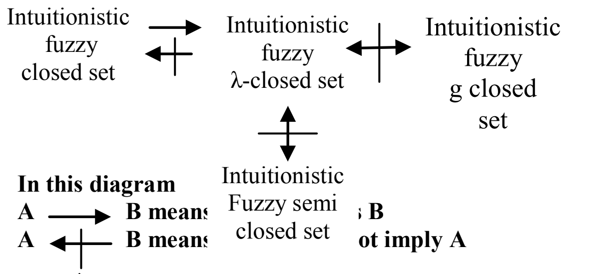
Example 3.10
Let X = {a, b} and τ = { $\underset{\sim}{0}$ $\underset{\sim}{1}$, ∪ } be an intuitionistic fuzzy topology on X where ∪ = {<a, 0.5, 0.5>, <b, 0.6, 0.3>} Then the intuitionistic fuzzy set A = { <a, 0.5, 0.5>, <b, 0.6,0.2>} is intuitionistic not fuzzy λ-closed set but intuitionistic fuzzy g-closed set.

Remark 3.11
The concept of intuitionistic fuzzy λ-closed sets and intuitionistic fuzzy semi closed sets are independent as seen from the following examples.

Example 3.12
Let X = {a, b} and τ = { $\underset{\sim}{0}$ $\underset{\sim}{1}$, ∪} be an intuitionistic fuzzy topology on X where U = {<a, 0.5, 0.5>, <b, 0.5, 0.2>}. Then the intuitionistic fuzzy set A = {<a, 0.5, 0.5>, <b, 0.5, 0.4>} is Intuitionistic fuzzy λ-closed but not intuitionistic fuzzy semi closed.

Example 3. 13
Let X = {a, b} and τ = { $\underset{\sim}{0}$, $\underset{\sim}{1}$, {<a,0.5, 0.5>, <b,0.3, 0.5>}, { <a,0.5, 0.5>, <b,0.4, 0.5>} be an intuitionistic fuzzy topology on X . Then intuitionistic fuzzy set A= {<a, 0.5, 0.5>, <b, 0.3, 0.5>} is not an intuitionistic fuzzy λ-closed but intuitionistic fuzzy semi closed set.

Remark 3.14
Theorem 3.2, Remark 3.3, 3.8, and 3.11 reveals the following diagram of implication.



In this diagram
A ⟶ B means ; B
A ↚ B means ot imply A

A ↮ B means that A and B are independent to each other

Definition 3.15

An intuitionistic fuzzy set A of an intuitionistic fuzzy topological space (X, τ) is called intuitionistic fuzzy λ – open set if and only if its complement A^C is an intuitionistic fuzzy λ-closed.

Remark 3.16

Every intuitionistic fuzzy open set is an intuitionistic fuzzy λ- open set in (X,τ) but the converse may not be true as seen from the following example.

Example 3.15

Let X = {a, b} and τ = {$\underset{\sim}{0}$ $\underset{\sim}{1}$, ∪} be an intuitionistic fuzzy topology on X. Where ∪ = {<a, 0.5, 0.5>, <b, 0.3, 0.6>} then the intuitionistic fuzzy set B = {<a, 0.5, 0.5>, <b, 0.2, 0.8>} is intuitionistic fuzzy λ- open set but it is not intuitionistic fuzzy open in (X, τ)

Remark 3.16

The intersection of two intuitionistic fuzzy λ- open sets is an intuitionistic fuzzy topological space (X, τ) may not be intuitionistic fuzzy λ- open for example.

Example 3.17

Let X = {a, b} and ∪, C and D be the intuitionistic fuzzy sets of X defined by follows.∪ = {<a, 0.5, 0.5>, <b, 0.6, 0.3>}, C = { <a, 0.5, 0.5>, <b, 0.3, 0.5>} and D = { <a, 0.5, 0.5>, <b, 0.4, 0.6>}.Let τ = {$\underset{\sim}{0}$, $\underset{\sim}{1}$, ∪ } be intuitionistic fuzzy topology on X. Then C and D are intuitionistic fuzzy λ-open in (X, τ). But A∩B is not intuitionistic fuzzy λ- open.

Theorem 3.18

Proof: Assume that α is a intuitionistic fuzzy λ- open set in X. Then α^C is a intuitionistic fuzzy
A fuzzy set α in a fuzzy topological space X is intuitionistic fuzzy λ-open if α ⊇ int(σ) whenever α ⊇ σ and σ is intuitionistic fuzzy closed set in X.

λ – closed set in X. Let σ be a intuitionistic fuzzy closed in X such that α⊇σ which implies $\alpha^C \supseteq \sigma^C$. Since α^C intuitionistic fuzzy λ – closed and σ^C is intuitionistic fuzzy open we have $\alpha^C \supseteq$ cl (σ^C). But cl (σ^C) = $[\text{int }(\sigma)]^C$ And so $\alpha^C \supseteq$ $[\text{int }(\sigma)]^C \Rightarrow \alpha \subseteq \text{int }(\sigma)$.

Conversely assume that α ⊆ int (σ) where α ⊆ (σ) and σ is intuitionistic fuzzy closed in X. We want to prove that α is intuitionistic fuzzy λ- open. We have to prove that α^C is intuitionistic fuzzy λ – closed. Let β be a intuitionistic fuzzy open set such that then $\alpha^C \supseteq \beta$ which implies $\alpha \subseteq \beta^c$ then by our hypothesis we have $\alpha \subseteq \text{int}(\beta^c)$. But $\text{int}(\beta^c) = [\text{Cl}(\beta)]^c$. We have $\alpha \subseteq \text{Cl}(\beta)^c$. Then $\alpha^c \supseteq \text{Cl}(\beta)$. Therefore α^c is intuitionistic fuzzy λ-closed set . Therefore α is intuitionistic fuzzy λ- open set.

4. CONCLUSION

In this paper we have introduced intuitionistic fuzzy λ- closed sets and studied some of its basic properties. Also we study the relationship between the newly introduced sets namely intuitionistic fuzzy λ- closed sets and some of the intuitionistic fuzzy closed sets that already exists. In this paper also we introduce intuitionistic fuzzy λ- open sets and studied some of its basic properties.

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