

An Application of Fuzzy Soft Sets in Medical Diagnosis using Fuzzy Soft Complement

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ABSTRACT

For the complement of a fuzzy soft set as initiated by Maji, the set theoretic axioms of contradiction and exclusion are not valid. In this context, Neog and Sut have reintroduced the notion of complement of a fuzzy soft set and showed that all the properties of complement of a set in classical sense are satisfied by fuzzy soft sets also according to the proposed definition of complement. In this paper we introduce a matrix representation of fuzzy soft set and extend Sanchez's approach for medical diagnosis using our notion of fuzzy soft complement.

Keywords

Fuzzy soft set, fuzzy membership function, fuzzy reference function, membership value matrix.

1. INTRODUCTION

In the fuzzy set theory initiated by Zadeh [13] in 1965, it has been accepted that the classical set theoretic axioms of exclusion and contradiction are not satisfied. In this regard, H. K. Baruah [2, 3] proposed that two functions, namely fuzzy membership function and fuzzy reference function are necessary to represent a fuzzy set. Accordingly, Baruah [2, 3] reintroduced the notion of complement of a fuzzy set in a different way and proved that indeed the set theoretic axioms of exclusion and contradiction are valid for fuzzy sets also.

In 1999, Molodstov [8] introduced the theory of soft sets, which is a new mathematical approach to vagueness. In recent years the researchers have contributed a lot towards fuzzification of soft set theory. Maji et al. [6] initiated the concept of fuzzy soft sets with some properties regarding fuzzy soft union, intersection, complement of a fuzzy soft set, De Morgan's Law etc. These results were further revised and improved by Ahmad and Kharal [1]. They defined arbitrary fuzzy soft union and intersection and proved De Morgan's Inclusions and De Morgan's Laws in Fuzzy Soft Set Theory. In 2011, Neog and Sut [9] have reintroduced the notion of fuzzy soft sets and redefined the complement of a fuzzy soft set accordingly. They have shown that the modified definition of complement of a fuzzy soft set meets all the requirements that complement of a set in classical sense really does.

Now a days, applications of Fuzzy Soft Set Theory in many disciplines and real life situations have been studied by many researchers. De et.al.[5] have studied Sanchez's [11, 12] method of medical diagnosis using intuitionistic fuzzy set. Saikia et.al.[10] have extended the method in [5] using intuitionistic fuzzy soft set theory. In [4], Chetia and Das have studied Sanchez's approach of medical diagnosis through IVFSS obtaining an improvement of the same presented in De et.al. [5]. Using the representation of interval valued fuzzy matrix,

Meenakshi et.al.[7] have provided the techniques to study Sanchez's approach of medical diagnosis of Interval valued fuzzy matrices. They have compared their technique with the One found in [4], for the same hypothetical case study presented in [4]. They have introduced the arithmetic mean (*am*) matrix of an IVFM A and directly applied Sanchez's method of medical diagnosis for the $am(A)$, which is a fuzzy matrix. In this paper, by using the definition of complement of a fuzzy soft set proposed by Neog and Sut [9] and the notion of fuzzy sets reintroduced by H.K. Baruah [2, 3], we put forward a matrix representation of fuzzy soft set and extend Sanchez's approach for medical diagnosis. We have considered the hypothetical case study taken by Chetia et.al. [4] and later by Meenakshi et.al. [7] and arrived at the same conclusion in our way as was obtained in [4] and [7].

2. PRELIMINARIES

Definition 2.1[8] A pair (F, E) is called a soft set (over U) if and only if F is a mapping of E into the set of all subsets of the set U .

In other words, the soft set is a parameterized family of subsets of the set U . Every set $F(\varepsilon), \varepsilon \in E$, from this family may be considered as the set of ε - elements of the soft set (F, E) , or as the set of ε - approximate elements of the soft set.

Definition 2.2[6] A pair (F, A) is called a fuzzy soft set over U where $F : A \rightarrow \tilde{P}(U)$ is a mapping from A into $\tilde{P}(U)$.

Definition 2.3[1] Let U be a universe and E a set of attributes. Then the pair (U, E) denotes the collection of all fuzzy soft sets on U with attributes from E and is called a fuzzy soft class.

Definition 2.4[9] A fuzzy soft set (F, A) over U is said to be null fuzzy soft set (with respect to the parameter set A), denoted by $\tilde{\varphi}$ if $\forall \varepsilon \in A, F(\varepsilon)$ is the null fuzzy set φ .

Definition 2.5[9] A fuzzy soft set (F, A) over U is said to be absolute fuzzy soft set (with respect to the parameter set A), denoted by \tilde{A} if $\forall \varepsilon \in A, F(\varepsilon)$ is the absolute fuzzy set U .

Definition 2.6[9] The complement of a fuzzy soft set (F, A) is denoted by $(F, A)^c$ and is defined by $(F, A)^c = (F^c, A)$ where $F^c : A \rightarrow \tilde{P}(U)$ is a mapping given by $F^c(\alpha) = [F(\alpha)]^c$, $\forall \alpha \in A$.

3. MATRIX REPRESENTATION OF A FUZZY SOFT SET

Let $U = \{c_1, c_2, c_3, \dots, c_m\}$ be the universal set and E be the set of parameters given by $E = \{e_1, e_2, e_3, \dots, e_n\}$. Then the fuzzy soft set (F, E) can be expressed in matrix form as $A = [a_{ij}]_{m \times n}$ or simply by $[a_{ij}]$, $i = 1, 2, 3, \dots, m$; $j = 1, 2, 3, \dots, n$ and $a_{ij} = (\mu_{j1}(c_i), \mu_{j2}(c_i))$; where $\mu_{j1}(c_i)$ and $\mu_{j2}(c_i)$ represent the fuzzy membership function and fuzzy reference function respectively of c_i in the fuzzy set $F(e_j)$ so that $\delta_{ij}(c_i) = \mu_{j1}(c_i) - \mu_{j2}(c_i)$ gives the fuzzy membership value of c_i . We shall identify a fuzzy soft set with its fuzzy soft matrix and use these two concepts interchangeable. The set of all $m \times n$ fuzzy soft matrices over U will be denoted by $FSM_{m \times n}$.

Example 3.1 Let $U = \{c_1, c_2, c_3, c_4\}$ be the universal set and E be the set of parameters given by $E = \{e_1, e_2, e_3\}$. We consider a fuzzy soft set

$$\begin{aligned} (F, E) &= \{F(e_1) = \{(c_1, 0.7, 0), (c_2, 0.1, 0), (c_3, 0.2, 0), (c_4, 0.6, 0)\}, \\ &F(e_2) = \{(c_1, 0.8, 0), (c_2, 0.6, 0), (c_3, 0.1, 0), (c_4, 0.5, 0)\}, \\ &F(e_3) = \{(c_1, 0.1, 0), (c_2, 0.2, 0), (c_3, 0.7, 0), (c_4, 0.3, 0)\}\} \end{aligned}$$

We would represent this fuzzy soft set in matrix form as

$$[a_{ij}]_{4 \times 3} = \begin{bmatrix} (0.7, 0) & (0.8, 0) & (0.1, 0) \\ (0.1, 0) & (0.6, 0) & (0.2, 0) \\ (0.2, 0) & (0.1, 0) & (0.7, 0) \\ (0.6, 0) & (0.5, 0) & (0.3, 0) \end{bmatrix}_{4 \times 3}$$

Definition 3.1 We define the membership value matrix corresponding to the matrix A as $MV(A) = [\delta(A)_{ij}]_{m \times n}$, where $\delta(A)_{ij} = \mu_{j1}(c_i) - \mu_{j2}(c_i) \quad \forall i = 1, 2, 3, \dots, m$ and $j = 1, 2, 3, \dots, n$, where $\mu_{j1}(c_i)$ and $\mu_{j2}(c_i)$ represent the fuzzy membership function and fuzzy reference function respectively of c_i in the fuzzy set $F(e_j)$.

Definition 3.2 Let $A = [a_{ij}]_{m \times n}$, $a_{ij} = (\mu_{j1}(c_i), \mu_{j2}(c_i))$; where $\mu_{j1}(c_i)$ and $\mu_{j2}(c_i)$ represent the fuzzy membership function and fuzzy reference function respectively of c_i , so that $\delta_{ij}(c_i) = \mu_{j1}(c_i) - \mu_{j2}(c_i)$ gives the fuzzy membership value of c_i . Also let $B = [b_{ij}]_{n \times p}$, $b_{ij} = (\chi_{j1}(c_i), \chi_{j2}(c_i))$; where $\chi_{j1}(c_i)$ and $\chi_{j2}(c_i)$ represent the fuzzy membership function and fuzzy reference function respectively of c_i , so that $\delta_{ij}(c_i) = \chi_{j1}(c_i) - \chi_{j2}(c_i)$ gives the fuzzy membership value of c_i . We now define $A.B$, the product of A and B as,

$$\begin{aligned} A.B &= [c_{ij}]_{m \times p} \\ &= [\max \min(\mu_{j1}(c_i), \chi_{j1}(c_i)), \min \max(\mu_{j2}(c_i), \chi_{j2}(c_i))]_{m \times p} \end{aligned}$$

If $\mu_{j2}(c_i) = \chi_{j2}(c_i) = 0 \quad \forall i, j$ then

$$A.B = [c_{ij}]_{m \times p}$$

$$\begin{aligned} &= [\max \min(\mu_{j1}(c_i), \chi_{j1}(c_i)), \min \max(0, 0)]_{m \times p} \\ &= [\max \min(\mu_{j1}(c_i), \chi_{j1}(c_i)), 0]_{m \times p} \\ &= [\max \min(\mu_{j1}(c_i), \chi_{j1}(c_i))]_{m \times p}, \end{aligned}$$

which is the definition of product of two fuzzy matrices in the usual sense where fuzzy reference function is 0.

4. FUZZY SOFT SETS IN MEDICAL DIAGNOSIS

Let us assume that S is the set of symptoms of some diseases, D is the set of diseases related to these symptoms and P is the set of patients showing the symptoms present in the set S . We construct a fuzzy soft set (F, D) over S . A relation matrix R_1 is obtained from the fuzzy soft set (F, D) . We would call this matrix the *symptom - disease matrix*. Similarly its complement $(F, D)^c$ gives another relation matrix R_2 , called *non symptom diseases matrix*. Analogous to Sanchez's notion of medical knowledge, we call the matrices R_1 and R_2 as medical knowledge of a fuzzy soft set. Again we construct another fuzzy soft set (F_1, S) over P . This fuzzy soft set gives the relation matrix Q_1 called *patient-symptom matrix* and its complement $(F_1, S)^c$ gives the relation matrix Q_2 , called *patient - non symptom matrix*. Then using Definition 3.2 above, we obtain two new relation matrices $T_1 = Q_1.R_1$ and $T_2 = Q_1.R_2$ called *patient symptom disease matrix* and *patient symptom non disease matrix* respectively. In a similar way, we obtain the relation matrices $T_3 = Q_2.R_1$ and $T_4 = Q_2.R_2$ called *patient non symptom disease matrix* and *patient non symptom non disease matrix* respectively.

Now,

$$T_1 = Q_1.R_1, T_2 = Q_1.R_2, T_3 = Q_2.R_1, T_4 = Q_2.R_2 \quad (4.1)$$

Using Definition 3.1, we then obtain the corresponding membership value matrices $MV(T_1)$, $MV(T_2)$, $MV(T_3)$ and

$MV(T_4)$.

We calculate the diagnosis score S_{T_1} and S_{T_2} for and against the disease respectively as

$$S_{T_1} = [\gamma(T_1)_{ij}]_{m \times n}, \text{ where } \gamma(T_1)_{ij} = \delta(T_1)_{ij} - \delta(T_3)_{ij} \quad (4.2)$$

and

$$S_{T_2} = [\gamma(T_2)_{ij}]_{m \times n}, \text{ where } \gamma(T_2)_{ij} = \delta(T_2)_{ij} - \delta(T_4)_{ij} \quad (4.3)$$

Now if $\max_j (S_{T_1}(p_i, d_j) - S_{T_2}(p_i, d_j))$ occurs for

exactly (p_i, d_k) only, then we would be in a position to accept that diagnostic hypothesis for patient p_i is the disease d_k . In case there is a tie, the process is repeated for patient p_i by reassessing the symptoms.

5. ALGORITHM

1. Input the fuzzy soft set (F, D) and compute $(F, D)^c$. Compute the corresponding matrices R_1 and R_2 .
2. Input the fuzzy soft set (F_1, S) and compute $(F_1, S)^c$. Compute the corresponding matrices Q_1 and Q_2 .
3. Compute $T_1 = Q_1.R_1$, $T_2 = Q_1.R_2$, $T_3 = Q_2.R_1$, $T_4 = Q_2.R_2$
4. Compute the corresponding membership value matrices $MV(T_1)$, $MV(T_2)$, $MV(T_3)$ and $MV(T_4)$.
5. Compute the diagnosis scores S_{T_1} and S_{T_2}

6. Find $S_k = \max_j (S_{T_1}(p_i, d_j) - S_{T_2}(p_i, d_j))$. We conclude that

the patient p_i is suffering from the disease d_k .

7. If S_k has more than one value, then go to step (1) and repeat the process by reassessing the symptoms for the patient.

6. CASE STUDY

Suppose that there are three patients p_1, p_2, p_3 in a hospital with symptoms temperature, headache, cough and stomach problem. Let the possible diseases related to the above symptoms be viral fever and malaria. We consider the set $S = \{e_1, e_2, e_3, e_4\}$ as universal set where e_1, e_2, e_3 and e_4 represent the symptoms temperature, headache, cough and stomach problem respectively and the set $D = \{d_1, d_2\}$, where d_1 and d_2 represent the parameters viral fever and malaria respectively.

Suppose that fuzzy soft set (F, D) over S , where F is a mapping $F: D \rightarrow \tilde{F}(S)$, gives an approximate description of fuzzy soft medical knowledge of the two diseases and their symptoms.

Let

$$(F, D) = \{F(d_1) = \{(e_1, 0.85, 0), (e_2, 0.25, 0), (e_3, 0.55, 0), (e_4, 0.30, 0)\}, \\ F(d_2) = \{(e_1, 0.75, 0), (e_2, 0.50, 0), (e_3, 0.45, 0), (e_4, 0.45, 0)\}\}$$

Complement of (F, D) i.e. $(F, D)^c$ is given by

$$(F, D)^c = \{F^c(d_1) = \{(e_1, 1, 0.85), (e_2, 1, 0.25), (e_3, 1, 0.55), (e_4, 1, 0.30)\}, \\ F^c(d_2) = \{(e_1, 1, 0.75), (e_2, 1, 0.50), (e_3, 1, 0.45), (e_4, 1, 0.45)\}\}$$

We represent the fuzzy soft sets (F, D) and $(F, D)^c$ by the following matrices R_1 and R_2 respectively.

$$R_1 = \begin{matrix} & d_1 & d_2 \\ e_1 & (0.85, 0) & (0.75, 0) \\ e_2 & (0.25, 0) & (0.50, 0) \\ e_3 & (0.55, 0) & (0.45, 0) \\ e_4 & (0.30, 0) & (0.45, 0) \end{matrix} \text{ and } R_2 = \begin{matrix} & d_1 & d_2 \\ e_1 & (1, 0.85) & (1, 0.75) \\ e_2 & (1, 0.25) & (1, 0.50) \\ e_3 & (1, 0.55) & (1, 0.45) \\ e_4 & (1, 0.30) & (1, 0.45) \end{matrix}$$

Again we take $P = \{p_1, p_2, p_3\}$ as the universal set where p_1, p_2 and p_3 represent three patients respectively and $S = \{e_1, e_2, e_3, e_4\}$ as the set of parameters, where e_1, e_2, e_3 and e_4 represent the symptoms temperature, headache, cough and stomach problem respectively.

Let (F_1, S) fuzzy soft set, where F_1 is a mapping $F_1: S \rightarrow \tilde{F}(P)$, gives a collection of an approximate description of the patient symptoms in the hospital.

Let

$$(F_1, S) = \{F_1(e_1) = \{(p_1, 0.75, 0), (p_2, 0.40, 0), (p_3, 0.70, 0)\}, \\ F_1(e_2) = \{(p_1, 0.40, 0), (p_2, 0.50, 0), (p_3, 0.40, 0)\}, \\ F_1(e_3) = \{(p_1, 0.90, 0), (p_2, 0.30, 0), (p_3, 0.60, 0)\}, \\ F_1(e_4) = \{(p_1, 0.75, 0), (p_2, 0.40, 0), (p_3, 0.30, 0)\}\}$$

We represent this fuzzy soft set (F_1, S) by the following matrix Q_1 , called *patient-symptom matrix*.

$$Q_1 = \begin{matrix} & e_1 & e_2 & e_3 & e_4 \\ p_1 & (0.75, 0) & (0.40, 0) & (0.90, 0) & (0.75, 0) \\ p_2 & (0.40, 0) & (0.50, 0) & (0.30, 0) & (0.40, 0) \\ p_3 & (0.70, 0) & (0.40, 0) & (0.60, 0) & (0.30, 0) \end{matrix}$$

Complement of (F_1, S) i.e. $(F_1, S)^c$ is given by

$$(F_1, S)^c = \{F_1^c(e_1) = \{(p_1, 1, 0.75), (p_2, 1, 0.40), (p_3, 1, 0.70)\}, \\ F_1^c(e_2) = \{(p_1, 1, 0.40), (p_2, 1, 0.50), (p_3, 1, 0.40)\}, \\ F_1^c(e_3) = \{(p_1, 1, 0.90), (p_2, 1, 0.30), (p_3, 1, 0.60)\}, \\ F_1^c(e_4) = \{(p_1, 1, 0.75), (p_2, 1, 0.40), (p_3, 1, 0.30)\}\}$$

We represent this fuzzy soft set $(F_1, S)^c$ by the following matrix Q_2 , called *patient-non symptom matrix*.

$$Q_2 = \begin{matrix} & e_1 & e_2 & e_3 & e_4 \\ p_1 & (1, 0.75) & (1, 0.40) & (1, 0.90) & (1, 0.75) \\ p_2 & (1, 0.40) & (1, 0.50) & (1, 0.30) & (1, 0.40) \\ p_3 & (1, 0.70) & (1, 0.40) & (1, 0.60) & (1, 0.30) \end{matrix}$$

Thus we have

$$T_1 = Q_1.R_1 = \begin{matrix} & d_1 & d_2 \\ p_1 & (0.75, 0) & (0.75, 0) \\ p_2 & (0.40, 0) & (0.50, 0) \\ p_3 & (0.70, 0) & (0.70, 0) \end{matrix}$$

and

$$T_2 = Q_1.R_2 = \begin{matrix} & d_1 & d_2 \\ p_1 & (0.90, 0.25) & (0.90, 0.45) \\ p_2 & (0.50, 0.25) & (0.50, 0.45) \\ p_3 & (0.70, 0) & (0.70, 0) \end{matrix}$$

We have now the following membership value matrices $MV(T_1)$ and $MV(T_2)$.

$$MV(T_1) = \begin{matrix} & d_1 & d_2 \\ p_1 & 0.75 & 0.75 \\ p_2 & 0.40 & 0.50 \\ p_3 & 0.70 & 0.70 \end{matrix}$$

and

$$MV(T_2) = \begin{matrix} & d_1 & d_2 \\ p_1 & 0.65 & 0.45 \\ p_2 & 0.25 & 0.05 \\ p_3 & 0.70 & 0.70 \end{matrix}$$

Again

$$T_3 = Q_2.R_1 = \begin{matrix} & d_1 & d_2 \\ p_1 & (0.85, 0.4) & (0.75, 0.4) \\ p_2 & (0.85, 0.3) & (0.75, 0.3) \\ p_3 & (0.85, 0.3) & (0.75, 0.3) \end{matrix}$$

and

$$d_1 \quad d_2$$

$$T_4 = Q_2.R_2 = \begin{matrix} p_1 & \begin{bmatrix} (1,0.40) & (1,0.50) \\ (1,0.40) & (1,0.45) \\ (1,0.30) & (1,0.45) \end{bmatrix} \\ p_2 \\ p_3 \end{matrix}$$

We have the following membership value matrices $MV(T_3)$ and $MV(T_4)$.

$$MV(T_3) = \begin{matrix} d_1 & d_2 \\ p_1 & \begin{bmatrix} 0.45 & 0.35 \\ 0.55 & 0.45 \\ 0.55 & 0.45 \end{bmatrix} \\ p_2 \\ p_3 \end{matrix}$$

and

$$MV(T_4) = \begin{matrix} d_1 & d_2 \\ p_1 & \begin{bmatrix} 0.60 & 0.50 \\ 0.60 & 0.55 \\ 0.70 & 0.55 \end{bmatrix} \\ p_2 \\ p_3 \end{matrix}$$

Finally we calculate the diagnosis score S_{T_1} and S_{T_2} for and against the disease as below

$$S_{T_1} = \begin{matrix} d_1 & d_2 \\ p_1 & \begin{bmatrix} 0.30 & 0.40 \\ -0.15 & 0.05 \\ 0.15 & 0.15 \end{bmatrix} \\ p_2 \\ p_3 \end{matrix} \quad \text{and} \quad S_{T_2} = \begin{matrix} d_1 & d_2 \\ p_1 & \begin{bmatrix} 0.05 & -0.05 \\ -0.35 & -0.50 \\ 0 & 0.15 \end{bmatrix} \\ p_2 \\ p_3 \end{matrix}$$

Now, we have the difference for and against the diseases as

$S_{T_1} - S_{T_2}$	d_1	d_2
p_1	0.25	0.45
p_2	0.20	0.55
p_3	0.15	0

We conclude that the patient p_3 is suffering from the disease d_1 and patients p_1 and p_2 both are suffering from disease d_2 .

7. CONCLUSION

In this work, we have considered the same hypothetical case study taken by Chetia et.al. [4] and later by Meenakshi et.al.[7] and applying Sanchez’s approach, we have arrived at the same conclusion as was obtained in [4] and [7]. Our approach is more rational in the sense that we are using our new notion of fuzzy soft complement initiated in [9] and the fuzzy sets in our work have been replaced with extended fuzzy sets initiated by Baruah in [2, 3]. We have put forward a matrix representation of fuzzy soft set and the operation “product” for fuzzy matrices. Future work in this regard would be required to study whether the notions put forward in this paper yield a fruitful result.

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9. REFERENCES

- [1] Ahmad , B. and Kharal, A. , “On Fuzzy Soft Sets”, Advances in Fuzzy Systems, Volume 2009, pp. 1-6, 2009.
- [2] Baruah, H. K. “Towards Forming A Field Of Fuzzy Sets”, International Journal of Energy, Information and Communications, Vol. 2, Issue 1, pp. 16-20, February 2011.
- [3] Baruah, H. K. “The Theory of Fuzzy Sets: Beliefs and Realities”, International Journal of Energy, Information and Communications, Vol. 2, Issue 2, pp. 1-22, May 2011.
- [4] Chetia, B., and Das, P. K. (2010). “An Application of Interval valued fuzzy soft set in medical diagnosis”, Int.J.contempt. math., sciences, vol. 5, 38, 1887 - 1894.
- [5] De, S. K., Biswas, R., and Roy, A.R. (2001). “An Application of Intuitionistic fuzzy sets in medical diagnosis”, Fuzzy Sets and Systems, 117, 209 213.
- [6] Maji, P. K., Biswas, R. and Roy, A. R. “Fuzzy Soft Sets”, Journal of Fuzzy Mathematics, Vol 9, No. 3, pp. 589-602, 2001
- [7] Meenakshi , A. R. and Kaliraja , M. “An Application of Interval Valued Fuzzy Matrices in Medical Diagnosis”, Int. Journal of Math. Analysis, Vol. 5, 2011, no. 36, 1791 - 1802
- [8] Molodstov, D.A., “Soft Set Theory - First Result”, Computers and Mathematics with Applications, Vol. 37, pp. 19-31, 1999
- [9] Neog, T. J. and Sut, D. K. , “Theory of Fuzzy Soft Sets From a New Perspective”, International Journal of Latest Trends in Computing, Vol. 2, No. 3 September 2011, pp. 439 - 450
- [10] Saikia, B.K., Das, P. K., and Borkakati, A.K.(2003). “An Application of Intuitionistic fuzzy soft sets in medical diagnosis”, Bio Science Research Bulletin, 19(2), 121-127.
- [11] Sanchez, E. (1976). “Resolution of composite fuzzy relation equations, Information and control”, 30, 38 - 48.
- [12] Sanchez, E. (1979). “Inverse of fuzzy relations, application to possibility distributions and medical diagnosis”, Fuzzy sets and Systems, 2 (1), 7586.
- [13] Zadeh, L. A. “Fuzzy Sets”, Information and Control, 8, pp. 338-353, 1965.