International Journal of Computer Applications (0975 – 8887)
Volume 33– No.9, November 2011

An Economic Order Quantity Model for Items Having Linear Demand under Inflation and Permissible Delay

Sarbjit Singh
Institute of Management Technology, Nagpur

ABSTRACT
This study considers deteriorating items having linear demand pattern, although this demand pattern is not new and lot of work has been done on this demand pattern. But this study is unique in itself, as in this study constant part of linear demand changes with each cycle, thus it gives better picture of demand then earlier models. The earlier models used to consider the constant factor of linear demand pattern as constant for the whole year which is an absurd. In this paper, the effect of permissible delay in payments is also considered, which is usual practice in most of the business i.e. purchasers are allowed a period to pay back for the goods brought without paying any interest. To make it more suitable to the present environment the effect of inflation and time value of money is also considered. As, the product considered in this paper are perishable product, hence shortages are allowed and are fully backlogged. The effect of inflation and time value of money are also taken into account. Numerical illustrations are also incorporated to show, the effect of changing constant part of linear demand after each cycle.

Keywords
Deterioration, Linear demand pattern, Permissible delay, Inflation, Time value of money, Allowable shortages

1. INTRODUCTION
Many mathematical models have been developed for controlling inventory. The majority of the earlier inventory models consider that demand rate is constant. Even today most of researchers consider that demand for items is constant. This is a feature of static environment, while in today's dynamic environment nothing is fixed or constant. In present scenario, in most of the cases the demand for items increases with time. Most of the companies are working towards increasing demand of their items with time. Normally, it is considered that items stored for future use, will remain fresh and fit for use, which is also an absurd. But most of the items stored for future use, always loose part of their value with passage of time. In language of inventory, this phenomenon is known as deterioration of items. This paper has incorporated both the features in proposed model.

Inventory problems involving time variable demand patterns have received attention from several researchers. Silver and Meal [33] established an approximate solution technique of a deterministic inventory model with time-dependent demand. Donaldson [13] developed an optimal algorithm for solving classical no-shortage inventory model analytically with linear trend in demand over fixed time horizon. He used simple calculus to develop a computational procedure for finding the optimal reorder times over a finite planning horizon. Silver [34] also developed a simple inventory replenishment policy for linear trend in demand. Ritchie [30] gave a simple optimal solution for the economic order quantity with linear increasing demand. Suresh Goyal, Martin Kusy & Rajan Soni [16] has written a note on the economic replenishment interval of an item with a linear trend in demand. S.K. Goyal [17] worked on improving replenishment policies for linear trend in demand. Dutta and Pal [12] considered the both deterministic and probabilistic version of power demand pattern with variable rate of deterioration. U. Dave [11] proposed a deterministic lot-size inventory model with shortages and a linear trend in demand. Goswami and Chaudhuri [14] discussed different types of inventory models with linear trend in demand. Moncer Hariga discussed the inventory replenishment problem with a linear trend in demand. Bose et-al [5] proposed an economic order quantity model for deteriorating with linear time dependent demand rate and shortages under inflation and time discounting. Hariga and Goyal [19] also studied the effects of inflation and time value of money on an inventory model with time-dependent demand rate and shortages. Jinn-Tsair Teng [37] developed a deterministic inventory replenishment model with a linear trend in demand. Bhunia et al [3] developed an inventory model of deteriorating items with lot-size dependent replenishment cost and linear trend in demand. Balki and Benkherouf [2] worked on the optimal replenishment schedule for an inventory system with deteriorating items and time-varying demand and production rates. Bhunia et al [22] extended their study to a deterministic inventory model with two levels of storage, a linear demand trend for a fixed time horizon. Chung and Tsai [10] developed an algorithm to determine the EOQ for deteriorating items with shortage and a linear trend in demand. Wen Yang Lo, Chi-Hung Tsai & Rong-Kwei Li [25] developed an exact solution of inventory replenishment policy for a linear trend in demand-two equation model. Khanra and Chaudhuri [23] discussed an order level decaying inventory model with such time dependent quadratic demand. Zhou et al [40] gave note on an inventory model for deteriorating items with stock dependent and time varying demand rate. Jinn-Tsair Teng, Liang Yuh Ouyang & Chun-Tao Chang [38] discussed a deterministic economic production quantity models with time varying demand and cost. Hsin Rau & Bing-Chang Ouyang [28] developed a general and optimal approach for three inventory models with a linear trend in demand.

Most of the physical goods undergo decay or deterioration overtime. Researches in decaying or deteriorating items are important because in real life, milk, blood, drugs, fruits, vegetables, foodstuffs etc suffer from depletion by direct spoilage while kept in store. Highly volatile liquids like gasoline, alcohol, turpentine etc undergo physical depletion over time through the process of evaporation. Electronic goods, radioactive substances, photographic film, grain etc deteriorate through gradual loss of potential or utility with the passage of time. This decay or deterioration of physical goods in stock is a very realistic feature and inventory modelers' felt the need to consider this factor. In real life situation there is loss of inventory by deterioration and the inventory value at the time of evaluation. Hariga [21] also worked on the inventory replenishment problem with a linear trend in demand.

In recent years the study of deteriorating items has gained great importance, Ghare and Schrader [15] were the first to use the concept of deterioration; they developed an economic order quantity model for an item with exponential decay and constant demand. Shah[32] who proposed an order level lot size inventory model for deteriorating items.. In nineties, Wee H.A. [39] formulated a deterministic lot size inventory model for deteriorating items with shortage and declining market followed by T. Chakrabrati and K.S. Chaudari [7] who developed an Economic order quantity model for deteriorating items with a linear trend in demand and shortages in all cycles. Moncer Hariga [20] has obtained an optimal inventory policy for perishable items with time dependent demand. Kun-Jen Chung and Su-Fu Tsai [10] framed an algorithm to determine the economic order quantity for deteriorating items with shortage and a linear trend in demand. Yong-WU Zhou & Hon-Shiang Lau [40] considered an economic lot size model for deteriorating items with lot size dependent replenishment cost and time varying demand. IIkyeong Moon, Bibhas Chandra Giri, Byungsung [27] discussed EOQ models for ameliorating/deteriorating items under inflation and time discounting.

Most of the conventional inventory models did not take into account the effect of inflation and time value of money. Perhaps it was believed that inflation would not influence the cost and price components to any significant degree. But during last two decades, the economic conditions of most of the countries has changed to such an extent due to large-scale inflation and consequent sharp decline in the purchasing power of money, that it is not possible now, to ignore the effects of inflation and time value of money. Buzacott [6] was the first to consider effect of inflation in economic order quantity model, Bierman and Thomas [4], investigated the inventory models with inflation followed by Mishra [26]. Mishra also developed a discount model in which the effect of both inflation and time value of money was considered. In nineties, Datta and Pal [12] investigated a finite time horizon inventory model following Mishra [26] with a linear time-dependent demand rate, allowing shortages and considering the effects of inflation and time value of money. J. Ray and K.S. Chaudhari [29] formulated an economic order quantity model with stock-dependent demand, shortage, inflation and time discounting.

Typically, inventory planning takes into account only data from operations concerns. Therefore, the interdependencies among the operations, financing and marketing concerns are not considered. In most business transactions, the supplier would allow a specified credit period (say, 30 days) to the retailer for payment without penalty to stimulate demand of his/her items. This credit term in financial management is known as “net 30”. J.T. Teng [35] indicated that the permissible delay produces two benefits to the supplier: (I) it attracts new customers who consider it as a type of price reduction; and (II) it can cause a reduction in sales outstanding, since some established customer will pay more promptly in order to take advantage of trade credit more frequently.

Goyal [18] was the first to introduce the concept of permissible delay. Aggarwal and Jaggi [1] considered the inventory model with an exponential deterioration rate under the condition of permissible delay in payments followed by Khouja and Mehrez [24] who formulated an optimal inventory policy under different supplier credit. Kun-Jen Chung [8] developed a theorem on the determination of economic order quantity under conditions of permissible delay in payments. Sarkar, Jamal and Wang [31] developed a supply chain model for perishable products under inflation and permissible delay in payment**.** J.T. Teng [35] considered the EOQ under condition of permissible delay in payment which is further extended by Ken Chung Kun and Yun-Fu-Huang [9] for limited storage capacity. Teng et-al [36] obtained an optimal pricing and ordering policy under permissible delay in payment.

In the present paper, an attempt has been made to develop a deterministic inventory model for an item having time dependent demand pattern i.e., linear demand pattern. In this study, the constant part of demand changes with each cycle. The items considered in the paper are deteriorating items having constant rate of deterioration.

To make this study more realistic and present environment suitable the effect of inflation and time value is also considered. The items considered here are perishable items with constant rate of deterioration. In earlier study on linear demand pattern, the constant part of linear demand is constant for entire year, which is an absurd, as its value changes with each cycle. In this study, we have tried to remove that error and effect of that error has also be analyzed with numerical illustrations

In most of the businesses the retailer is given some grace period to make his payment, which in terms of inventory is known as permissible delay. In this paper fixed grace period is given to the buyer and he can plan his inventory, according to this grace period. The paper has been divided into parts depending upon whether depletion time of inventory is more than permissible delay or less than permissible delay. The total cost equation for both the cases has been obtained.

The product consider in this paper is perishable product, having constant rate of deterioration. That’s why planned shortages have been incorporated in this model. The shortages are fully backlogged. The purpose of this study is to aid retailers in efficiently stocking the inventory under the influence of different decision criteria such as trade credit, time value of money, inflation, deterioration and time dependent demand. In addition, we prove that changing constant part of linear demand is necessary, if we have more than one cycle in a year, as, the total variable cost of inventory and shortages increases with each cycle.

2. NOTATIONS

f	Inflation rate.
i	Inventory carrying rate.
A	Ordering cost of inventory, Rs./order.
S	Period with shortage
T	Length of inventory cycle, time units
T_1	Length of period with positive stock of the items period with no shortage
θ	Rate of deterioration per unit time
i_p	$I_p - r$
r	Discount rate representing the time value of money
I_p	Nominal interest paid per Rupees per unit time at time

i_e	$I_e - r$
I_e	Nominal interest at time t=0
$I_e(t)$	Rate of interest earned at time t Rupees per Rupee per unit time.
$I_p(t)$	Interest rate paid at time t Rupees per Rupee per unit time.
I_T^i	Total interest earned per cycle with inflation.
M	Delay Period in settling the account.
P_T^i	Interest payable per cycle with inflation.
c	Unit cost of per item at time $t = 0$, Rs./unit
C_0^b	Present value of the inflated backorder cost c_b, Rs. /unit
C_D^i	Total cost of deterioration per cycle with inflation
C_H^i	Total holding cost per cycle with inflation
D_T	Amount of materials deterioration during a cycle time, T
S	Period with shortage

3. ASSUMPTIONS

1) The items considered in this model are deteriorating items with constant rate of deterioration.
2) Replenishment are instantaneous.
3) The supplier proposes a certain credit period, and sales revenue generated during the credit period is deposited in an interest bearing account. At the end of the period, the credit is settled and the retailer starts paying the capital opportunity cost for the items in stock.
4) Shortages are allowed and fully backlogged.
5) Cycle time is T, which includes inventory depletion time and shortage period.
6) The demand rate considered is time dependent, increasing demand rate.
7) The constant part of linear demand pattern changes with each cycle
8) N cycles have been considered in a year.
9) Effect of inflation and time value of money both are considered.
10) The lead time is zero for sake of simplicity.
11) There is no repair or replacement of deteriorated inventory during a given cycle.
12) Interest earned is less than the interest paid.

4. MATHEMATICAL MODEL AND ANALYSIS

In this mathematical model variable rate of demand is considered in which constant part changes after each cycle with constant rate of deterioration. Depletion of the inventory occurs due to demand (supply) as well as due to deterioration which occurs only when there is inventory i.e., during the period [0, T_1]. The deterioration is possible when the products are available in the stock so there is no deterioration during [T_1, T] and hence there is no loss of products during shortage period.

For First Cycle

We consider constant $a_1 = a$, so linear differential equation for first cycle is given by

$$\frac{dI(t)}{dt} + \theta I(t) = -(a+bt)$$

$$0 \le t \le T_1 \qquad (1)$$

$$I(t) = \frac{-a}{\theta} - \frac{bt}{\theta} + \frac{b}{\theta^2} + C_1 e^{-\theta t}$$

Where at t = 0, I (t) = I_0 (Initial Inventory Level) Putting this value in the above equation, we get

$$I_0 = \frac{-a}{\theta} + \frac{b}{\theta^2} + C_1$$

Which gives

$$I(t) = I_0 e^{-\theta t} + \frac{a(e^{-\theta t} - 1)}{\theta} + \frac{b(e^{-\theta t} + 1)}{\theta^2} - \frac{b}{\theta} \qquad 0 \le t \le T \qquad (2)$$

It is obvious that at t = T_1, I (T_1) = 0. So equation (2) yields

$$I_0 = \frac{bT_1 e^{\theta T_1}}{\theta} + \frac{a(e^{\theta T_1} - 1)}{\theta} - \frac{b(e^{\theta T_1} - 1)}{\theta^2} = Q - b^1 \qquad (3)$$

where b^1 being the maximum backorder (shortage) level permitted. Substituting the value of I_0 in equation (2) we get

$$I(t) = \frac{b}{\theta}\left(T_1 e^{(\theta T_1 - \theta t)} - t\right) + \frac{a\left(e^{\theta(T_1 - t)} - 1\right)}{\theta} - \frac{b\left(e^{\theta(T_1 - t)} - 1\right)}{\theta^2}$$

$$0 \le t \le T_1 \qquad (4)$$

and I(t) = 0 when, $T_1 \le t \le T$. The total demand during T_1 is $\int_0^{T_1} d(a+bt)dt$. Thus it can be easily seen that the amount of items deteriorates during one cycle is given by

$$D_T = I_o - \int_0^{T_1}(a+bt)dt = \frac{bT_1 e^{\theta T_1}}{\theta} + \frac{a(e^{\theta T_1} - 1)}{\theta} - \frac{b(e^{\theta T_1} - 1)}{\theta^2} - aT_1 - \frac{bT_1^2}{2} \qquad (5)$$

For inflation rate f, the continuous time inflation factor for the time period is e^{ft} which means that item costing Rs c at time t=0 will cost ce^{ft} at time t. for a discount rate, r, representing the time value of money, the present value of an amount at time t, is ce^{-rt}. Hence, the present value of the inflated price of an item at time t=0, $ce^{ft}\,e^{-rt}$ is given by

$c_0 = ce^{(f-r)t} = ce^{Rt}$, where R = f – r (6)

in which c is inflated through time t to ce^{ft}, ce^{-rt} is the factor deflating the future worth to its present value and R is the present value of the inflation rate similarly, the present value of the inflated backorder cost c_b C_O^b is given by

$$C_0^b = c_b e^{(f-r)t} = c_b e^{Rt} \quad (7)$$

The Inflation Model

There are two distinct cases in this type of inventory system based on permissible delay in payments

I. Payment at or before the total depletion of inventory($M \le T_1 < T$)

II. After depletion payment ($T_1 < M$)

Case I. (i.e., payment at or before the total depletion of inventory)

(a) Since the ordering is made at time t=0, the inflation does not affect the ordering cost. Thus the ordering cost for items is fixed at A Rs/order.

(b) Since $I_0 = \frac{bT_1 e^{\vartheta T_1}}{\theta} + \frac{a(e^{\theta T_1}-1)}{\theta} - \frac{b(e^{\theta T_1}-1)}{\theta^2}$, the value of this inventory at time t=0, is c I_0. The present value of the items sold is $\int_0^{T_1} c_0 \left(a+bt\right) dt$. Hence the cost of deterioration per cycle time T under inflation, C_D^i is given by

$$C_D^i = cI_0 - \int_0^{T_1} c_0 (a+bt)dt$$

$$= \frac{cbT_1 e^{\vartheta T_1}}{\theta} + \frac{ca(e^{\theta T_1}-1)}{\theta} - \frac{cb(e^{\theta T_1}-1)}{\theta^2} - \frac{ca(e^{RT_1}-1)}{R} - \frac{cb(T_1 e^{RT_1})}{R} + \frac{cb(e^{RT_1}-1)}{R^2} \quad (8)$$

(c) The holding cost under inflation is given by

$$C_H^i = i\int_0^{T_1} c_0 I(t)dt$$

$$= \frac{icbT_1}{\theta}\left(\frac{e^{RT_1}-e^{\vartheta T_1}}{R-\theta}\right) - \frac{icbT_1 e^{RT_1}}{\theta R} + \frac{icb}{\theta}\left(\frac{e^{RT_1}-1}{R^2}\right) + \frac{icae^{\theta T_1}}{\theta(R-\theta)}\left(e^{(R-\theta)T_1}-1\right) - \frac{ica}{\theta}\left(\frac{e^{RT_1}-1}{R}\right) - \frac{icb}{\theta^2}\left(\frac{e^{RT_1}-e^{\theta T_1}}{R-\theta}\right) + \frac{icb}{\theta^2}\left(\frac{e^{RT_1}-1}{R}\right) \quad (9)$$

The interest payable rate at time t is $e^{I_p t} - 1$ rupees per rupees, so the present value (at t=0) of interest payable rate at time t is $I_p(t) = (e^{I_p t} - 1)e^{-rt}$ rupees per rupees. Therefore the interest payable per cycle for the inventory not sold after the due date M is given by

$$P_T^i = \int_M^T cI_p(t)I(t)dt = \int_M^{T_1} cI_p(t)I(t)dt$$

Since I (t) = 0 for $T_1 \le t \le T$

$$= \frac{bcT_1 e^{\vartheta T_1}}{\theta}\left(\frac{e^{(i_p-\theta)T_1}-e^{(i_p-\theta)M}}{i_p-\theta}\right) - \frac{bc}{\theta}\left(\frac{T_1 e^{i_p T_1}-Me^{i_p M}}{i_p}\right) + \frac{bc}{\theta}\left(\frac{e^{i_p T_1}-e^{i_p M}}{i_p^2}\right) + \frac{bcT_1 e^{\vartheta T_1}}{\theta}\left(\frac{e^{-(r+\theta)T_1}-e^{-(r+\theta)M}}{r+\theta}\right)$$
$$- \frac{bc}{\theta}\left(\frac{T_1 e^{-rT_1}-Me^{-rM}}{r}\right) - \frac{bc}{\theta}\left(\frac{e^{-rT_1}-e^{-rM}}{r^2}\right) + \frac{ace^{\theta T_1}}{\theta}\left(\frac{e^{(i_p-\theta)T_1}-e^{(i_p-\theta)M}}{i_p-\theta}\right) + \frac{dace^{\theta T_1}}{\theta}\left(\frac{e^{-(r+\theta)T_1}-e^{-(r+\theta)M}}{r+\theta}\right)$$
$$- \frac{ac}{\theta}\left(\frac{e^{i_p T_1}-e^{i_p M}}{i_p}\right) - \frac{ac}{\theta}\left(\frac{e^{-rT_1}-e^{-rM}}{r}\right) - \frac{bc}{\theta^2}e^{\theta T_1}\left(\frac{e^{(i_p-\theta)T_1}-e^{(i_p-\theta)M}}{i_p-\theta}\right) + \frac{bc}{\theta^2}\left(\frac{e^{i_p T_1}-e^{i_p M}}{i_p}\right)$$
$$- \frac{bc}{\theta^2}e^{\theta T_1}\left(\frac{e^{-(r+\theta)T_1}-e^{-(r+\theta)M}}{r+\theta}\right) + \frac{bc}{\theta^2}\left(\frac{e^{-rT_1}-e^{-rM}}{r}\right) \quad (10)$$

where $i_p = I_p - r$

Similarly, the present value of the interest earned at time t, $I_e(t)$ $I_e(t)$ is $(e^{I_e t} - 1)e^{-rt}$. Considering inflated unit cost at time t as $c_t = ce^{Rt}$, the interest earned per cycle, I_T^i, is the interest earned up to time T_1 and it is given by

$$I_T^i = \int_0^T c_o I_e(t) t(a+bt)dt$$

$$= \frac{caT_1 e^{(R+i_e)T_1}}{(R+i_e)} - ca\left(\frac{e^{(R+i_e)T_1}-1}{(R+i_e)^2}\right) + cdbT_1^2 \frac{e^{(R+i_e)T_1}}{R+i_e} - cb\left(\frac{2T_1 e^{(R+i_e)T_1}}{(R+i_e)^2}\right) + 2cb\left(\frac{e^{(R+i_e)T_1}-1}{(R+i_e)^3}\right)$$
$$- \frac{caT_1 e^{(R-r)T_1}}{(R-r)} + ca\left(\frac{e^{(R-r)T_1}-1}{(R-r)^2}\right) - cbT_1^2 \frac{e^{(R-r)T_1}}{R-r} + cb\left(\frac{2T_1 e^{(R-r)T_1}}{(R-r)^2}\right) - 2cb\left(\frac{e^{(R-r)T_1}-1}{(R-r)^3}\right)$$

(11)

The backorder cost per cycle under inflation, C_B^i is given by

$$C_B^i = \int_0^{T-T_1} c_b e^{R(T_1+t)} t(a+bt)dt$$

Since the backorder starts at $t=T_1$

$$= c_b e^{RT_1} \frac{a}{R^2}\left[\left(RT - RT_1 - 1\right)e^{R(T-T_1)} + 1\right]$$

$$+ \frac{c_b e^{RT_1} b}{R^3}\left[\left(R^2T^2 + R^2T_1^2 + 2R^2TT_1 - 2RT + 2RT_1 + 2\right)e^{R(T-T_1)} - 2\right]$$

(12)

The Variable Cost Function

The total variable cost per cycle C_{VT}, is defined as sum of ordering cost, deterioration cost, holding cost, interest paid, backorder cost minus interest earned

$$C_{VT}(T_1, T) = A + C_D^i + C_H^i + P_T^i - I_T^i + C_B^i$$

(13)

Substituting the values from equation (8)-(12) in equation (13), we have C_{VT} in terms of T_1 and T

$$
\begin{aligned}
C_{VT} &= A + \frac{cbT_1e^{\vartheta T_1}}{\theta} + \frac{ca(e^{\theta T_1}-1)}{\theta} \\
&- \frac{cb(e^{\theta T_1}-1)}{\theta^2} - \frac{ca(e^{RT_1}-1)}{R} \\
&- \frac{cb(T_1e^{RT_1})}{R} + \frac{cb(e^{RT_1}-1)}{R^2} \\
&\frac{icbT_1}{\theta}\left(\frac{e^{RT_1}-e^{\vartheta T_1}}{R-\theta}\right) - \frac{icbT_1e^{RT_1}}{\theta R} \\
&+ \frac{icb}{\theta}\left(\frac{e^{RT_1}-1}{R^2}\right) \\
&+ \frac{icae^{\theta T_1}}{\theta(R-\theta)}\left(e^{(R-\theta)T_1}-1\right) \\
&- \frac{ica}{\theta}\left(\frac{e^{RT_1}-1}{R}\right) - \frac{icb}{\theta^2}\left(\frac{e^{RT_1}-e^{\theta T_1}}{R-\theta}\right) \\
&+ \frac{icb}{\theta^2}\left(\frac{e^{RT_1}-1}{R}\right) \\
&+ \frac{bcT_1e^{\vartheta T_1}}{\theta}\left(\frac{e^{(i_p-\theta)T_1}-e^{(i_p-\theta)M}}{i_p-\theta}\right) \\
&- \frac{dbc}{\theta}\left(\frac{T_1e^{i_pT_1}-Me^{i_pM}}{i_p}\right) \\
&+ \frac{bc}{\theta}\left(\frac{e^{i_pT_1}-e^{i_pM}}{i_p^2}\right) \\
&+ \frac{bcT_1e^{\vartheta T_1}}{\theta}\left(\frac{e^{-(r+\theta)T_1}-e^{-(r+\theta)M}}{r+\theta}\right) \\
&- \frac{bc}{\theta}\left(\frac{T_1e^{-rT_1}-Me^{-rM}}{r}\right) - \frac{bc}{\theta}\left(\frac{e^{-rT_1}-e^{-rM}}{r^2}\right) \\
&+ \frac{ace^{\theta T_1}}{\theta}\left(\frac{e^{(i_p-\theta)T_1}-e^{(i_p-\theta)M}}{i_p-\theta}\right) \\
&+ \frac{ace^{\theta T_1}}{\theta}\left(\frac{e^{-(r+\theta)T_1}-e^{-(r+\theta)M}}{r+\theta}\right) \\
&- \frac{ac}{\theta}\left(\frac{e^{i_pT_1}-e^{i_pM}}{i_p}\right) \\
&- \frac{ac}{\theta}\left(\frac{e^{-rT_1}-e^{-rM}}{r}\right) \\
&- \frac{bc}{\theta^2}e^{\theta T_1}\left(\frac{e^{(i_p-\theta)T_1}-e^{(i_p-\theta)M}}{i_p-\theta}\right) \\
&+ \frac{bc}{\theta^2}\left(\frac{e^{i_pT_1}-e^{i_pM}}{i_p}\right) \\
&- \frac{bc}{\theta^2}e^{\theta T_1}\left(\frac{e^{-(r+\theta)T_1}-e^{-(r+\theta)M}}{r+\theta}\right) \\
&+ \frac{bc}{\theta^2}\left(\frac{e^{-rT_1}-e^{-rM}}{r}\right) \\
&- \frac{caT_1e^{(R+i_e)T_1}}{(R+i_e)} + ca\left(\frac{e^{(R+i_e)T_1}-1}{(R+i_e)^2}\right) - cbT_1^2\frac{e^{(R+i_e)T_1}}{R+i_e} \\
&+ cb\left(\frac{2T_1e^{(R+i_e)T_1}}{(R+i_e)^2}\right) + 2cb\left(\frac{e^{(R+i_e)T_1}-1}{(R+i_e)^3}\right) \\
&+ c_be^{RT_1}\frac{a}{R^2}\left[\left(RT-RT_1-1\right)e^{R(T-T_1)}+1\right] \\
&+ \frac{c_be^{RT_1}b}{R^3}\left[\begin{pmatrix} R^2T^2+R^2T_1^2+2R^2TT_1 \\ -2RT+2RT_1+2 \end{pmatrix}e^{R(T-T_1)}-2\right]
\end{aligned}
$$

(14)

Multiple Inventories Cycle per Year

For Second cycle we have $a_2 = a + bT$ i.e. $a_2 = a_1 + bT$

For Third Cycle we get $a_2 = a + 2bT$ i.e. $a_3 = a_2 + bT$

For Nth Cycle $a_N = a + (N-1)bT$

Now Using Arithmetic Progression we get

Average Value of Constant part of linear demand pattern as

$$a_{av} = \frac{2a + (N-1)bT}{2}$$

Replacing a by a_{av}, in equation (14) will give us average variable cost per cycle.

The inflation and time value of money exist for each cycle of replenishment, so we need to consider the effect over the time horizon NT. Let there be N complete cycle during a year. Hence, NT=1. The total cost during total time is given by

$$C_T = C_{VT} \times \left[1 + e^{2RT} + e^{3RT} + \dots\dots\dots + e^{(N-1)RT}\right]$$

$$C_{VT} \times \left[\frac{1-e^{NRT}}{1-e^{RT}}\right] = C_{VT} \times \left[\frac{1-e^{R}}{1-e^{RT}}\right] \quad (15)$$

The value of T and T_1 which minimize C_T may be obtained by simultaneously solving

$$\frac{\partial C_T}{\partial T}(T_1, T) = 0 \quad \text{and} \quad \frac{\partial C_T}{\partial T_1}(T_1, T) = 0$$

Now $\frac{\partial C_T}{\partial T} = \frac{\partial C_{VT}}{\partial T} \times \left(\frac{1-e^R}{1-e^{RT}}\right) + C_{VT} \times \frac{\partial}{\partial T}\left(\frac{1-e^R}{1-e^{RT}}\right)$ (16)

in which $\frac{\partial C_{VT}}{\partial T} = c_ba(T-T_1)e^{RT} + 4\frac{c_b}{R}bT_1e^{RT} + c_bb\left(T^2+T_1^2+2T_1\right)e^{RT}$

and $$\frac{\partial}{\partial T}\left(\frac{1-e^R}{1-e^{RT}}\right) = \left(\frac{1-e^R}{(1-e^{RT})^2}\right)R$$

Therefore

$$\frac{\partial C_T}{\partial T} = S \times \left(\frac{1-e^R}{1-e^{RT}}\right) + C_{VT} \times \frac{\partial}{\partial T}\left(\frac{1-e^R}{1-e^{RT}}\right) \quad (17)$$

in which is expressed in known quantities from equation (14), similarly

$$\frac{\partial C_T}{\partial T_1} = \frac{\partial C_{VT}}{\partial T_1} \times \left(\frac{1-e^R}{1-e^{RT}}\right)$$

(18) Solution of equation (17) and (18) will yield optimal T and T_1.

Optimal Solution

By direct search approach it can be shown that given by (15) is convex in feasible domain of T and T_1. Therefore the optimum value of T and T_1 minimizing C_T can be obtained by simultaneously solving equations $\frac{\partial C_T}{\partial T_1} = 0$ and $\frac{\partial C_T}{\partial T} = 0$ .The expression for the total cost involves higher order exponential terms, it is not easy to evaluate the Hessians in closed form, to conclude about its positive definiteness directly, and thus it is not trivial to see whether the total cost function is convex.

Case II T<M (i.e., after depletion payment)

The deterioration cost C_D^i , carrying cost, C_H^i and the backorder cost C_B^i per cycle are the same as in the equation (8), (9) and (12) respectively. The interest paid P_T^i per cycle is equal to zero when T_1 <M because the supplier can pay in full at the end of permissible delay, M. The interest earned per cycle is the interest earned during the positive inventory period plus the interest earned from the cash invested during the time period (T_1, M) after the inventory is exhausted at time I_T^i , it is given by

$$
\begin{aligned}
I_T^i &= \int_0^T c_o I_e(t) t(a+bt)dt + \\
&(e^{i_e(M-T_1)} - 1)\int_0^{T_1} c_0 t(a+bt)dt \\
&= \frac{caT_1 e^{(R+i_e)T_1}}{(R+i_e)} - ca\left(\frac{e^{(R+i_e)T_1}-1}{(R+i_e)^2}\right) + cbT_1^2 \frac{e^{(R+i_e)T_1}}{R+i_e} \\
&-cb\left(\frac{2T_1 e^{(R+i_e)T_1}}{(R+i_e)^2}\right) + 2cb\left(\frac{e^{(R+i_e)T_1}-1}{(R+i_e)^3}\right) \\
&-\frac{caT_1 e^{(R-r)T_1}}{(R-r)} + ca\left(\frac{e^{(R-r)T_1}-1}{(R-r)^2}\right) \\
&-cdbT_1^2 \frac{e^{(R-r)T_1}}{R-r} + cb\left(\frac{2T_1 e^{(R-r)T_1}}{(R-r)^2}\right) \\
&-2cb\left(\frac{e^{(R-r)T_1}-1}{(R-r)^3}\right)(e^{i_e(M-T_1)} - 1) \\
&\times cb\left(\frac{T_1^2 e^{RT_1}}{R} - \frac{2T_1 e^{RT_1}}{R^2} + \frac{2(e^{RT_1}-1)}{R^3}\right)
\end{aligned}
\qquad (19)
$$

Incorporating the modification of I_T^i in equation (19) and P_T^i =0 into equation (13) because of the changes in assumption for case II, value in equation has changed from that in equation (14). The solution structure for the total annual cost remains the same as in equation (15). So a similar solution structure may be applied for the optimal solution of T_1 and T as it was done earlier from multiple cycles per year in equations (15)-(18)

6. NUMERICAL ILLUSTRATIONS

Taking a =1000 and b=50 considering six cycles in a year i.e. N=6, we have

Table1.

Cycle	**Demand each cycle with constant a**	**Demand each cycle with changing a**
1	1100	1100
2	1100	1200
3	1100	1300
4	1100	1400
5	1100	1500
6	1100	1600

Which shows that model with constant value of ***a*** is absurd, and it is not viable to use that model if we have more than one cycle in a year. As shortages will start increasing with each cycle and will also affect our total cost. Thus, it is necessary to change the value of ***a*** after every cycle or we must take the average value of ***a*** for the complete year.

7. CONCLUSION

In this paper, some realistic features are incorporated. These features are likely to be associated with an inventory of consumer goods.

First, characteristic is its demand pattern. The demand pattern considered in this study is linear demand pattern, i.e., demand is linear function of time and with the passing of time demand increases linearly. The other important factor with this study is that the constant part of linear demand changes with each cycle. In earlier models, it has been considered to be constant, which means that after every cycle demand for items is equal to constant part (here it is denoted by a)which means that demand after every cycle will be equal to the constant part of linear demand again and it will start increasing with time and will reach the maximum value which it has reached in first cycle , which is an absurd if N cycles are considered in a year. On the contrary, shortages increases with each cycle and hence total cost also increases with each cycle. Second, the occurrence of shortages in inventory is a very natural phenomenon in real situations. Shortages in inventory are allowed in the present model. Third, the effects of inflation and time value of money can no longer be ignored as in the case of the other classical models. Today the economy of many countries (especially developing countries) is in the grip of a large-scale inflation and a consequent sharp decline in the purchasing power of money. Fourth, the effect of permissible delay is considered in this paper, which is a usual practice in all businesses, the whole study has been divided into two cases, whether permissible delay is greater than inventory depletion time or less than inventory depletion time (i.e., T_1).

The mathematical model and its solution have been incorporated. The present value of total cost incurred in this inventory is obtained first, and then an optimal order quantity and maximum allowable shortage are obtained by using a search procedure. This model will aid both retailers and suppliers in stocking their goods under the above mentioned conditions. This study can further be extended for other demand rates like ramp type demand, stock dependent demand, quadratic demand etc. Also, concept of fixed trade credit can be incorporated in this model. Thus this study can act as a catalyst for many future models.

8. REFERENCES

[1] Aggarwal, S.P., & Jaggi, C.K. 1995. Ordering policies of deteriorating items under permissible delay in payments. Journal of the Operational Research Society. 49, 658-662.

[2] Balki, Z.T., & Benkherouf, L. 1996. On the optimal replenishment schedule for an inventory system with deteriorating items with time varying demand and production rates. Computers & Industrial Engineering, 30, 823-29.

[3] Bhunia, A.K., Maiti. M. 1998. A two-warehouse inventory model for deteriorating items with linear trend in demand and shortages. Journal of the Operational Research Society. 49(3), 287-292.

[4] Bierman., & Thomas. 1977. Inventory decisions under inflationary condition. Decision Sciences. 8(7), 151-155.

[5] Bose, S., Goswami, A., Chaudhari, A., & Chaudhari, K.S. 1995. An EOQ model for deteriorating items with linear time-dependent demand rate and shortages under inflation and time discounting. Journal of Operational Research Society. 46, 771-782.

[6] Buzacott.1975. EOQ with inflation for deteriorating items. Operational Research Quarterly.26 (3), 553-558

[7] Chakrabarti, T., & Chaudhari , K.S. 1997. An EOQ model for deteriorating items with a linear demand and shortages in all cycles. International Journal of production Economics. 49(3), 205-213.

[8] Chung K.J. 1998. A theorem on the determination of economic order quantity under conditions of permissible delay in payments. Computers & Operations Research. 25(1), 49-52.

[9] Chung K.J. & Huang Y.F. (2004). Optimal replenishment policies for EOQ inventory model with limited storage capacity under permissible delay in payment Journal of Operational Research Society. 14, 17-22.

[10] Chung, K.S., & Tsai, S.F. 1997. An algorithm to determine the EOQ for deteriorating items with shortage and a linear trend in demand, International Journal of Production Economics, 51(3), 215-221.

[11] Dave, U. 1989. On a heuristic inventory-replenishment rule for items with a linearly increasing demand incorporating shortages. Journal of the Operational Research Society. 38(5), 459-463.

[12] Datta, T.K., & Pal, A.K. 1991. Effects of inflation time value of money on an inventory model with linear time dependent demand rate and shortages. European Journal of Operations Research. 52, 1-8

[13] Donaldson W.A. 1977. Inventory replenishment policy for a linear trend in demand-an analytical solution. Operational Research Quarterly, 28,663-670

[14] Goswami, A. and Chaudhri, K.S. 1991 An EOQ model for deteriorating items with a linear trend in demand. Journal of Operational Research Society. 42(12), 1105-1110

[15] Ghare & Schrader. 1963. A model for exponential decaying inventory. Journal of Industrial Engineering .14(3), 238-43

[16] Goyal, S., Kusy, M., & Soni, R. 1986. A note on the economic replenishment interval of an item with a linear trend in demand. Engineering Costs & Production Economics. 10(3), 253-255.

[17] Goyal, S.K. 1986. On improving replenishment policies for linear trend in demand. Engineering Costs & Production Economics. 10(1), 73-76.

[18] Goyal S. K. 1985. Economic order quantity under conditions of permissible delay in payments. Journal of the Operational Research Society. 36 (3), 335-38.

[19] Hariga, M. and Goyal, S.K. 1995. An alternative procedure for determining the optimal policy for an inventory item having linear trend in demand, Journal of Operational Research Society. 46(4), 521-527.

[20] Hariga, M. 1997. Optimal inventory policies for perishable items with time dependent demand. International Journal of Production Economics. 50(1), 35-41

[21] Hariga, M. 1993. The inventory replenishment problem with a linear trend in demand. Computer & Industrial Engineering. 24(2), 143-150

[22] Kar, S., Bhunia, A.K, Maiti, M. 2001. Deterministic inventory model with two levels of storage, a linear trend in demand and a fixed time horizon. Computers and Operation Research. 28, 1315-1331

[23] Khanra, S. & Chaudhuri, K.S. 2003. A note on an order-level inventory model for a deteriorating item with time dependent quadratic demand. Computers and Operations Research,2003, 30, 1901-1916

[24] Khouja, M., & Mehrez, A. 1996. Optimal inventory policy under different supplier credits. Journal of Manufacturing Systems. 15, 334-339

[25] Lo, W.Y., Tsai, C.H., & Li, R. K., 2002. Exact solution of inventory replenishment policy for a linear trend in demand- two equation model. International journal of Production Economics, 76(2), 111-120.

[26] Misra R.B. 1979. A study of inflation effects on inventory system. Logistics Spectrum .9(3), 260-268

[27] Moon, I., Giri, B.C., & Ko, B. 2005. Economic order quantity models for ameliorating/deteriorating items under inflation and time discounting. European Journal of Operational Research.162 (3), 773-785.

[28] Rau, H., & Ouyang, B.C. 2007. A general and optimal approach for three inventory models with a linear trend in demand. Computers and Industrial Engineering, 52(4), 521-532.

[29] Ray, J. & Chaudhari, K.S. 1997. An EOQ model with stock-dependent demand, shortage, inflation and time discounting. International Journal of production economics.53, 171-180.

[30] Ritchie, E. 1980. Practical inventory replenishment policies for a linear trend in demand followed by a period of steady demand. Journal of Operational Research Society.31, 605-613.

[31] Sarkar, B.R., Jamal, A.M.M., & Wang S. 2000. Supply chain models for perishable products under inflation and permissible delay in payment. Computers and Operation Research. 27, 59-75.

[32] Shah, Y.K. 1977. An order level lot size inventory for deteriorating items. AIIE Transactions. 9(2),108-112

[33] Silver, E.A., & Meal, H.C. 1969. A simple modification of the EOQ for the case of varying demand rate. Production & Inventory Management.10 (4), 52-65.

[34] Silver, E.A. 1979. A simple inventory replenishment decision rule for a linear trend in demand. Journal of Operational Research Society. 30, 71-75.

[35] Teng, J. T. 2002. On the economic order quantity under conditions of permissible delay in payment. Journal of the Operational Research Society. 53, 915.918.

[36] Teng, Chang & Goyal, S.K. (2005), Optimal Pricing and ordering policy under permissible delay in payment International Journal of Production Economics. 97, 121-129

[37] Teng, J.T. 1996. A deterministic inventory replenishment model with a linear trend in demand. Operations Research Letters.19 (1), 33-41.

[38] Teng, J.T., Ouyang, Y.L., & Chang, C.T. 2005. Deterministic economic production models with time varying demand and cost. Applied Mathematical Modeling, 29(10), 987-1003.

[39] Wee, H.A. 1995. A deterministic lot-size inventory model for deteriorating items with shortage and a declining market. Computers & Operations Research. 22(3), 345-356.

[40] Zhou, W.Y., & Lau, H.S.2000. An economic lot size model for deteriorating items with lot size dependent replenishment cost and time varying demand. Applied Mathematical Modeling. 24(10), 121-139.