Unsteady MHD flow and Heat Transfer past a Porous Flat Plate in a Rotating System

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ABSTRACT
An analysis is made on the unsteady MHD flow and heat transfer of a viscous incompressible electrically conducting viscous fluid bounded by an infinite porous flat plate. The plate is oscillating in its own plane with a velocity $u_0 e^{\beta^* t}\cos\omega t$, ω being the frequency of the oscillations. A uniform magnetic field of strength B_0 is imposed perpendicular to the plate. The governing equations along with the boundary conditions are solved analytically. It is found that with an increase in either magnetic parameter or suction parameter the primary velocity and the magnitude of secondary velocity decrease. The primary velocity and the magnitude of the secondary velocity increase with an increase in either accelerated parameter or frequency parameter or time. It is found that the solution also exists for the blowing at the plate. The temperature distribution is obtained on taking viscous and joule dissipation into account. The mean wall temperature as well as the rate of heat transfer are also obtained. It is found that with an increase of magnetic field intensity, the mean temperature increases.

Keywords: MHD flow, heat transfer, magnetic parameter, rotation parameter, Prandtl number and frequency parameter.

1. INTRODUCTION
Investigations on MHD flow and heat transfer of an incompressible viscous fluid over a porous plate find many important applications in modern metallurgical, metal-working processes and manufacturing processes. The heat treatment of materials travelling between a feed roll and a wind-up roll or on conveyor belts, the lamination, hot rolling, wire drawing, crystal growing, purification of molten metals from non-metallic inclusions and melt-spinning processes in the extraction of polymers possess the characteristics of moving plates/surfaces. MHD also finds applications in ion propulsion, controlled fusion research, plasma jets and chemical synthesis, etc. The hydromagnetic viscous incompressible fluid flow due to harmonic oscillations of a plane studied by Kakutani [1, 2] and exact solution are obtained for the cases of perfectly conducting and non-conducting planes. Hide and Roberts [3] studied the hydromagnetic flow due to an oscillating plane. The unsteady hydromagnetic flow in a rotating fluid was investigated by Soundalgekar and Pop [4]. Debnath [5] made an analysis on unsteady magnetohydrodynamic boundary layers in a rotating flow. The hydromagnetic Ekman layer near an accelerated plate was described by Datta and Mazumdar [6]. The flow of an incompressible viscous fluid near a porous oscillating infinite plate subject to suction or blowing was studied by Bühler and Zierep [7]. Turbatu et al.[8] investigated the flow of an incompressible viscous fluid on an infinite porous plate oscillating with increasing or decreasing velocity amplitude of oscillation. Attia [9] analyzed the transient Hartmann flow with heat transfer considering the ion slip. Flow in the Ekman layer on an oscillating porous plate was investigated by Gupta et al. [10]. Gupta et al. [11] studied the effects of suction or blowing on the velocity and temperature distribution in the flow past a porous flat plate of a power-law fluid. Mohyuddin et al. [13] investigated the unsteady magneto-fluid-dynamics fluid and heat flow with section. The MHD flow and heat transfer driving by a power-law shear over a semi-infinite flat plate was described by Akl [14].

In this paper, we study the unsteady MHD flow and heat transfer of an incompressible electrically conducting viscous fluid past an infinite heated porous flat plate. The plate is oscillating in its own plane with the velocity $u_0 e^{\beta^* t}\cos\omega t$, ω being the frequency of the oscillations. A uniform magnetic field of strength B_0 is imposed perpendicular to the plate. It is found that with an increase in either magnetic parameter M^2 or suction parameter S the primary velocity u_1 and the magnitude of secondary velocity w_1 of fluid decrease at a particular point in flow filed. The primary velocity u_1 and the magnitude of secondary velocity w_1 increase with an increase in either accelerated parameter β or frequency parameter n or time τ. The mean wall temperature $\theta_0(\eta)$ as well as the rate of heat transfer $\theta'(0)$ is also obtained. It is found that with an increase of magnetic field intensity the mean temperature $\theta_0(\eta)$ increases at a particular point in flow filed. Further, it is found that the magnitude of tangent of the phase angle of the rate of heat transfer oscillations $\tan\psi$ decreases with an increase in either rotation parameter K^2 or Prandtl number Pr while it increases with an increase in frequency parameter n for fixed values of K^2.

2. MATHEMATICAL FORMULATION AND ITS SOLUTION
Consider the unsteady flow of a viscous incompressible electrically conducting fluid past an infinite porous flat plate with uniform suction or blowing at the plate. The plate oscillates in its own plane with the velocity $u_0 e^{\beta^* t}\cos\omega t$ in a given direction. The amplitude of the oscillations decreases for acceleration $(\beta^* > 0)$ and the amplitude of the oscillations increases for deceleration $(\beta^* < 0)$. We choose the x - axis along the plate, y - axis perpendicular to the plate and z - axis normal to the xy - plane. The plate and the fluid are in a state of rigid body rotation with uniform angular velocity Ω about the y -axis. An external

uniform magnetic field of strength B_0 is imposed perpendicular to the plate [See Figure 1] and the plate is taken electrically non-conducting. The velocity components are (u,v,w) relative to a frame of reference rotating with the fluid. Since the plate occupying the plane $y=0$ is of infinite extent, all the physical quantities will be the function of y and t only. The equation of continuity $\nabla\cdot\vec{q}=0$ gives $\frac{\partial v}{\partial y}=0$ which on integration yields $v=-v_0(\text{constant})$, where $\vec{q}\equiv(u,v,w)$. The constant v_0 which denotes the normal velocity at the plate is positive for suction and negative for blowing. We assume that the magnetic Reynolds number for the flow is small so that the induced magnetic field can be neglected. This assumption is justified since the magnetic Reynolds number is generally very small for partially ionized gases. The solenoidal relation $\nabla\cdot\vec{B}=0$ for the magnetic field gives $B_y=B_0=$ constant everywhere in the fluid where $\vec{B}\equiv\left(B_x,B_y,B_z\right)$.

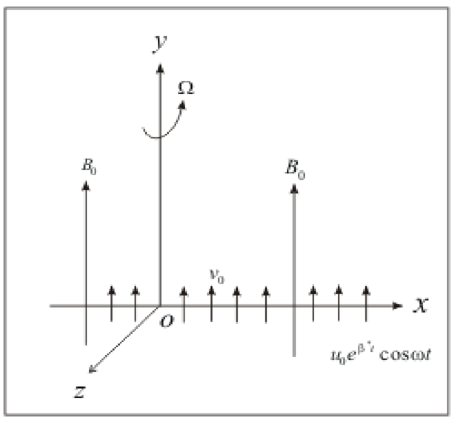

Figure 1 : Geometry of the problem

The momentum equations along x, y and z- directions are given by

$$\frac{\partial u}{\partial t}-v_0\frac{\partial u}{\partial y}-2\Omega\, w=\nu\frac{\partial^2 u}{\partial y^2}-\frac{\sigma B_0^2}{\rho}u, \tag{1}$$

$$0=-\frac{1}{\rho}\frac{\partial p}{\partial y}, \tag{2}$$

$$\frac{\partial w}{\partial t}-v_0\frac{\partial w}{\partial y}+2\Omega\, u=\nu\frac{\partial^2 w}{\partial y^2}-\frac{\sigma B_0^2}{\rho}w, \tag{3}$$

where μ_e, ρ, ν and p are respectively the magnetic permeability, the density of the fluid, the kinematic coefficient of viscosity, the fluid pressure.
The boundary conditions of the problem are

$$u=u_0e^{\beta^* t}\cos\omega t,\ w=0 \text{ at } y=0,$$
$$u\to 0, w\to 0 \text{ as } y\to\infty, \tag{4}$$

where u_0 is a constant and β^* is the accelerating index.
Introducing the non-dimensional variables

$$\eta=\frac{u_0\,y}{\nu},\ u_1=\frac{u}{u_0},\ w_1=\frac{w}{u_0},\ \tau=\frac{u_0^2\,t}{\nu},\ n=\frac{\nu\omega}{u_0^2}, \tag{5}$$

equations (1) and (3) become

$$\frac{\partial u_1}{\partial\tau}-S\frac{\partial u_1}{\partial\eta}-2K^2w_1=\frac{\partial^2u_1}{\partial\eta^2}-M^2u_1, \tag{6}$$

$$\frac{\partial w_1}{\partial\tau}-S\frac{\partial w_1}{\partial\eta}+2K^2u_1=\frac{\partial^2w_1}{\partial\eta^2}-M^2w_1, \tag{7}$$

where $K^2=\frac{2\omega\,\nu}{u_0^2}$ is the rotation parameter, $S=\frac{v_0}{u_0}$, the suction parameter and $M^2=\frac{\sigma B_0^2\nu}{\rho u_0^2}$, the magnetic parameter.
On the use of (5), the boundary conditions (4) become

$$u_1=e^{\beta\tau}\cos n\tau,\ w_1=0 \text{ at } \eta=0,$$
$$u_1\to 0, w_1\to 0 \text{ as } \eta\to\infty \tag{8}$$

where $\beta=\frac{\beta^*\nu}{u_0^2}$ is the accelerated parameter.
Combining (6) and (7), we get

$$\frac{\partial f}{\partial\tau}-S\frac{\partial f}{\partial\eta}+2i\,K^2f=\frac{\partial^2 f}{\partial\eta^2}-M^2f, \tag{9}$$

where

$$f(\eta,\tau)=u_1(\eta,\tau)+i\,w_1(\eta,\tau),\quad i=\sqrt{-1}. \tag{10}$$

The boundary conditions (8) now become

$$f(0)=\frac{1}{2}\left[e^{(\beta+in)\tau}+e^{(\beta-in)\tau}\right] \text{and } f(\infty)=0. \tag{11}$$

To solve equation (9) subject to the boundary conditions (11), we assume the solution in the following form

$$f(\eta,\tau)=f_1(\eta)e^{(\beta+in)\tau}+f_2(\eta)e^{(\beta-in)\tau}. \tag{12}$$

Substituting (12) in equation (9) we find that $f_1(\eta)$ and $f_2(\eta)$ satisfy the following equations

$$f_1''(\eta)+S\,f_1'(\eta)-\left(\beta+in+M^2+2iK^2\right)f_1(\eta)=0\ , \tag{13}$$

$$f_2''(\eta)+S\,f_2'(\eta)-\left(\beta-in+M^2+2i\,K^2\right)f_2(\eta)=0\ , \tag{14}$$

where prime denote the differentiation with respect to η. The corresponding boundary conditions for $f_1(\eta)$ and $f_2(\eta)$ are

$$f_1(0)=\frac{1}{2},\ f_2(0)=\frac{1}{2},\ \ f_1(\infty)=0,\ f_2(\infty)=0. \tag{15}$$

The solution of the equations (13) and (14) subject to the boundary conditions (15) are

$$f_1(\eta)=\frac{1}{2}e^{-\left(\frac{S}{2}+\alpha_1+i\,\beta_1\right)\eta},\ \ f_2(\eta)=\frac{1}{2}e^{-\left(\frac{S}{2}+\alpha_2\pm i\,\beta_2\right)\eta}, \tag{16}$$

where

$$\alpha_1,\beta_1=\frac{1}{\sqrt{2}}\left[\left\{\left(\frac{S^2}{4}+M^2+\beta\right)^2+\left(2K^2+n\right)^2\right\}^{\frac{1}{2}}\pm\left(\frac{S^2}{4}+M^2+\beta\right)\right]^{\frac{1}{2}}$$

$$\alpha_2,\beta_2=\frac{1}{\sqrt{2}}\left[\left\{\left(\frac{S^2}{4}+M^2+\beta\right)^2+\left(2K^2-n\right)^2\right\}^{\frac{1}{2}}\pm\left(\frac{S^2}{4}+M^2+\beta\right)\right]^{\frac{1}{2}}, \tag{17}$$

and the upper sign is for $n<2K^2$ and lower sign for $n>2K^2$.
Hence, using (16), equation (12) yields

$$f(\eta,\tau)=\begin{cases}\dfrac{1}{2}e^{\beta\tau}\begin{bmatrix}e^{-\left(\frac{S}{2}+\alpha_1+i\beta_1\right)\eta+in\tau}\\+e^{-\left(\frac{S}{2}+\alpha_2\pm i\beta_2\right)\eta-in\tau}\end{bmatrix} & \text{for } n\neq 2K^2\\ \dfrac{1}{2}\begin{bmatrix}e^{-\left(\frac{S}{2}+\alpha_1+i\beta_1\right)\eta+in\tau}\\+e^{-\left(\frac{S}{2}+\alpha_2^*\right)\eta-in\tau}\end{bmatrix} & \text{for } n=2K^2\end{cases} \quad (18)$$

where $\alpha_2^*=\left(\dfrac{S^2}{4}+M^2+\beta\right)^{\frac{1}{2}}$ and α_1 and β_1 are given by (17).

On separating into real and imaginary parts, we have

$$u_1(\eta,\tau)=\begin{cases}\dfrac{1}{2}e^{\beta\tau}\begin{bmatrix}e^{-\left(\frac{S}{2}+\alpha_1\right)\eta}\cos(n\tau-\beta_1\eta)\\+e^{-\left(\frac{S}{2}+\alpha_2\right)\eta}\cos(n\tau\pm\beta_2\eta)\end{bmatrix} & \text{for } n\neq 2K^2\\ \dfrac{1}{2}e^{\beta\tau}\begin{bmatrix}e^{-\left(\frac{S}{2}+\alpha_1\right)\eta}\cos(n\tau-\beta_1\eta)\\+e^{-\left(\frac{S}{2}+\alpha_2^*\right)\eta}\cos n\tau\end{bmatrix} & \text{for } n=2K^2\end{cases} \quad (19)$$

$$w_1(\eta,\tau)=\begin{cases}\dfrac{1}{2}e^{\beta\tau}\begin{bmatrix}e^{-\left(\frac{S}{2}+\alpha_1\right)\eta}\sin(n\tau-\beta_1\eta)\\-e^{-\left(\frac{S}{2}+\alpha_2\right)\eta}\sin(n\tau\pm\beta_2\eta)\end{bmatrix} & \text{for } n\neq 2K^2\\ \dfrac{1}{2}e^{\beta\tau}\begin{bmatrix}e^{-\left(\frac{S}{2}+\alpha_1\right)\eta}\sin(n\tau-\beta_1\eta)\\-e^{-\left(\frac{S}{2}+\alpha_2^*\right)\eta}\sin n\tau\end{bmatrix} & \text{for } n=2K^2\end{cases} \quad (20)$$

The above solutions are valid for both suction and blowing at the plate. If $K^2=0$, then the equation (19) is identical with the equations (12) of Mohyuddin et al. [13].

3. RESULTS AND DISCUSSION

It is seen from equations (19) and (20) that the velocity profile consists of two part, one parts oscillates with amplitude $\frac{1}{2}e^{-\left(\frac{S}{2}+\alpha_1\right)\eta}$ and the other one with $\frac{1}{2}e^{-\left(\frac{S}{2}+\alpha_2\right)\eta}$, where α_1 and α_2 are given by (17). It is seen from (17) that the wave length $\frac{2\pi}{\beta_2}$ is always greater than that of $\frac{2\pi}{\beta_1}$ because β_1 is always greater than β_2. The solution represents a flow in which the oscillations decay exponentially with the distance from the plate. The layer corresponding to the former part at distance η from the plate oscillates with phase lag of $\beta_1\eta$ while the layer corresponding to the latter part oscillates with phase advance of $\beta_2\eta$ when $n<2K^2$ and a phase lag when $n>2K^2$. It is interesting to note that the normal solution exists for $S=0$ and $n=2K^2$. This is due to the fact that $\beta_2>0$ when $S=0$ and $n=2K^2$. This result shows that the shear oscillations are also confined near the plate when $S=0$ and $n=2K^2$. Figure 2 shows that both the primary velocity u_1 and the magnitude of secondary velocity w_1 decrease with an increase in M^2. It clearly indicates that the transverse magnetic field opposes the transport phenomena. This is due to the fact that variation of the Hartmann number leads to the variation of the Lorentz force due to magnetic field and the Lorentz force produces more resistance to transport phenomena. It is observed from Figure 3 that the primary velocity u_1 decreases while the magnitude of secondary velocity w_1 first increases near the plate and it decreases away from the plate with an increase in K^2. Figures 4-6 show that both the primary velocity u_1 and the magnitude of secondary velocity w_1 increase with an increase in either accelerated parameter β or frequency parameter n or time τ. It is seen from Figures 7-8 that both the primary velocity u_1 and the magnitude of secondary velocity w_1 decrease with an increase in either suction parameter S or phase angle $n\tau$. This means that the suction at the plate or the phase angle have a retarding influence on the flow field.

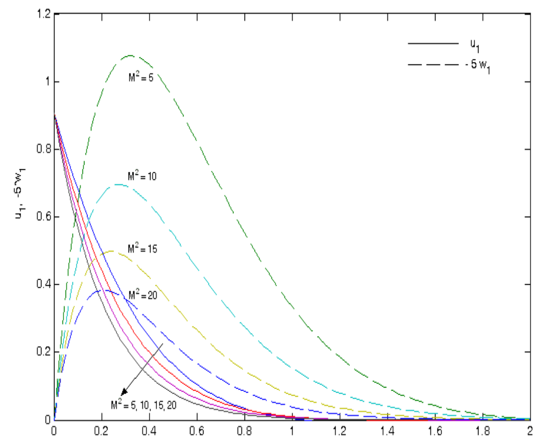

Figure 2: Variations of u_1 and w_1 for M^2 when $K^2=4$, $S=0.5$, $n=2$, $\beta=0.5$, $\tau=0.5$ and $n\tau=\frac{\pi}{4}$.

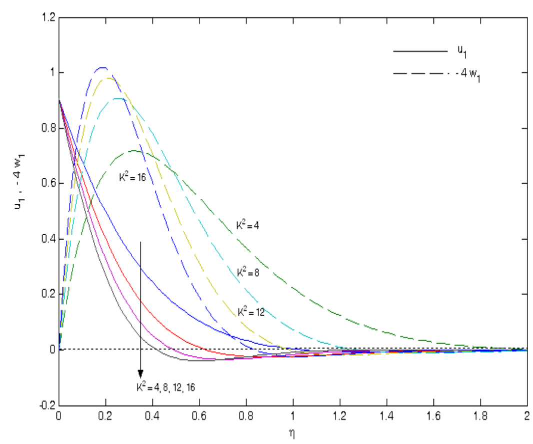

Figure 3: Variations of u_1 and w_1 for K^2 when $M^2=5$, $S=0.5$, $n=2$, $\beta=0.5$, $\tau=0.5$ and $n\tau=\frac{\pi}{4}$.

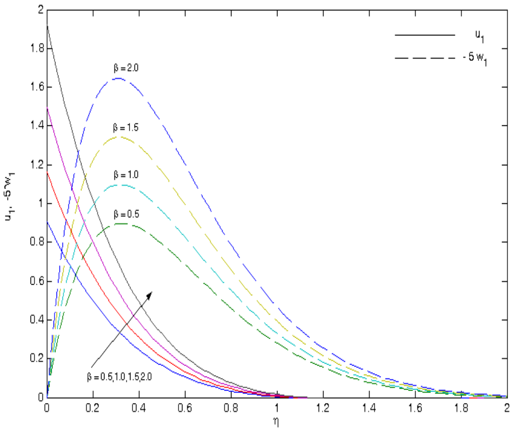

Figure 4: Velocity profiles for β when $M^2 = 5$, $K^2 = 4$, $S = 0.5$, $n = 2$, $\tau = 0.5$ and $n\tau = \frac{\pi}{4}$.

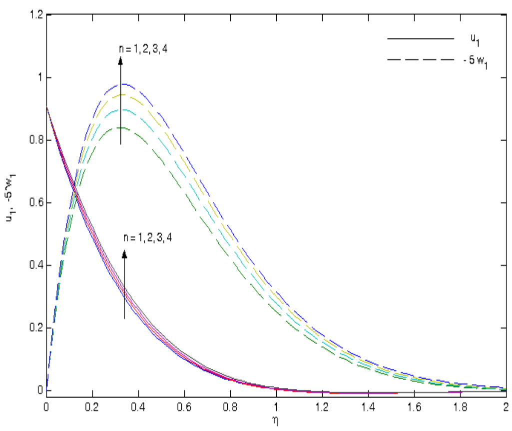

Figure 5: Variations of u_1 and w_1 for n when $M^2 = 5$, $K^2 = 4$, $S = 0.5$, $\beta = 0.5$, $\tau = 0.5$ and $n\tau = \frac{\pi}{4}$.

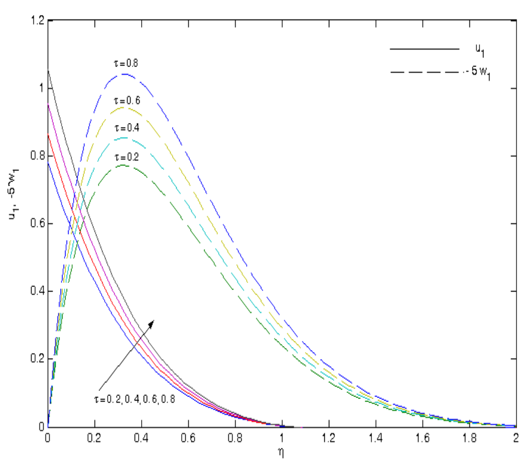

Figure 6: Variations of u_1 and w_1 for time τ when $M^2 = 5$, $K^2 = 4$, $S = 0.5$, $\beta = 0.5$, $n = 2$ and $n\tau = \frac{\pi}{4}$.

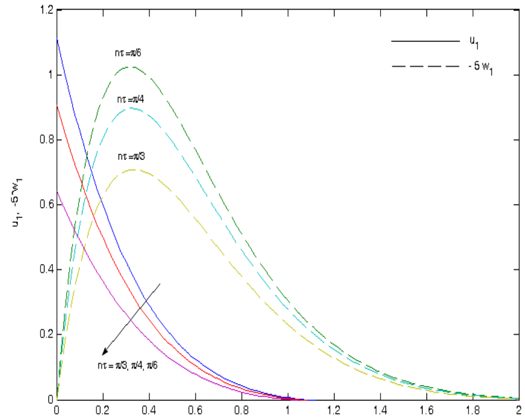

Figure 7: Variations of u_1 and w_1 for $n\tau$ when $M^2 = 5$, $K^2 = 4$, $S = 0.5$, $\beta = 0.5$, $\tau = 0.5$ and $n = 2$.

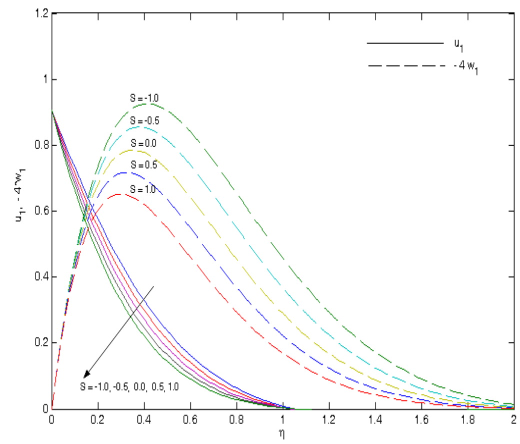

Figure 8: Variations of u_1 and w_1 for S when $M^2 = 5$, $K^2 = 4$, $n = 2$, $\beta = 0.5$, $\tau = 0.5$ and $n\tau = \frac{\pi}{4}$.

The non-dimensional shear stress at the plate $\eta = 0$ due to the primary flow is

$$\tau_x = \left(\frac{\partial u_1}{\partial \eta}\right)_{\eta=0} = -\frac{1}{2} R_1 \cos(n\tau + \theta_1), \tag{21}$$

where

$$R_1^2 = \left(S + \alpha_1 + \alpha_2\right)^2 + (\beta_1 \mp \beta_2)^2, \quad \tan\theta_1 = \frac{(\beta_1 \mp \beta_2)}{(S + \alpha_1 + \alpha_2)}, \tag{22}$$

Equation (22) shows that $0 < \tan\theta_1 < 1$. Hence from (21) and (17) we conclude that the shear stress due to the primary flow has a phase lead for $\theta_1 < \frac{\pi}{4}$ over the oscillations of the plate.

The non-dimensional shear stress at the plate $\eta = 0$ due to the secondary flow is

$$\tau_y = \left(\frac{\partial w_1}{\partial \eta}\right)_{\eta=0} = -\frac{1}{2} R_2 \cos(n\tau - \theta_1), \tag{23}$$

where

$$R_2^2 = (\beta_1 \pm \beta_2)^2 + (\alpha_1 - \alpha_2)^2, \ \tan\theta_2 = \frac{(\alpha_1 - \alpha_2)}{(\beta_1 \pm \beta_2)}. \qquad (24)$$

It can be seen from (24) that $0 < \tan\theta_2 < 1$. Hence it follows from (23) and (17) that the shear stress due to the secondary flow has a phase lag for $\theta_2 < \frac{\pi}{4}$ over the oscillations of the plate. The variations of amplitudes of shear stresses R_1, R_2 and the tangent of the phase angles of shear stresses $\tan\theta_1$ and $\tan\theta_2$ due to primary and the secondary flows respectively against K^2 for different values of M^2, β and n with $S = 1$ and $n\tau = \frac{\pi}{4}$ are shown in Figures 9-14. It is observed from Figures 9 and 10 that both the amplitudes R_1 and R_2 decrease with an increase in either magnetic parameter M^2 or frequency parameter n. Fig.11 reveals that the amplitude R_1 increases while the amplitude R_2 decreases with an increase in accelerated parameter β. It is seen from Figs.12 and 13 that both the tangent of the phase angles, $\tan\theta_1$ and $\tan\theta_2$ decrease with an increase in either magnetic parameter M^2 or accelerated parameter β. Figure 14 shows that both the tangent of the phase angles $\tan\theta_1$ and $\tan\theta_2$ increase with an increase in frequency parameter n. The kink in the curves of the Figures 12-14 indicates the tangent of the phase angles of the shear stresses in the critical case $n = 2K^2$.

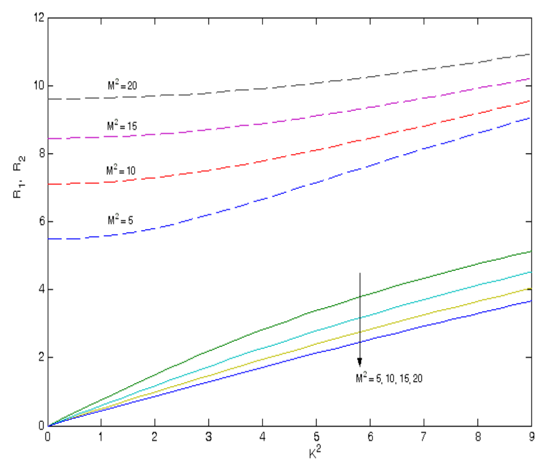

Figure 9: Variations of R_1 and R_2 for M^2 when $S = 0.5$, $\beta = 0.5$, $S = 0.5$ and $n\tau = \frac{\pi}{4}$.

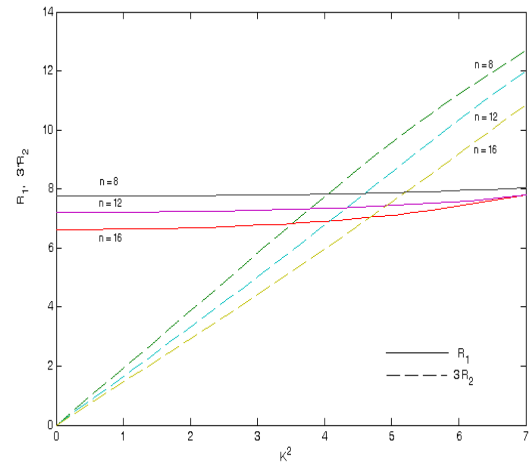

Figure 10: Variations of R_1 and R_2 fo n when $M^2 = 5$, $\beta = 0.5$, $S = 0.5$ and $n\tau = \frac{\pi}{4}$.

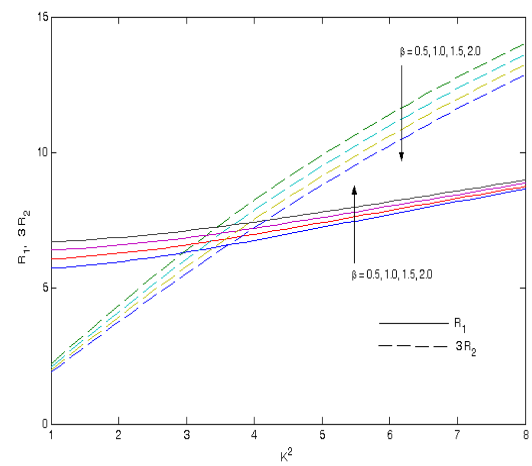

Figure 11: Variations of R_1 and R_2 for β when $M^2 = 5$, $S = 0.5$, $\beta = 0.5$, $n = 4$ and $n\tau = \frac{\pi}{4}$.

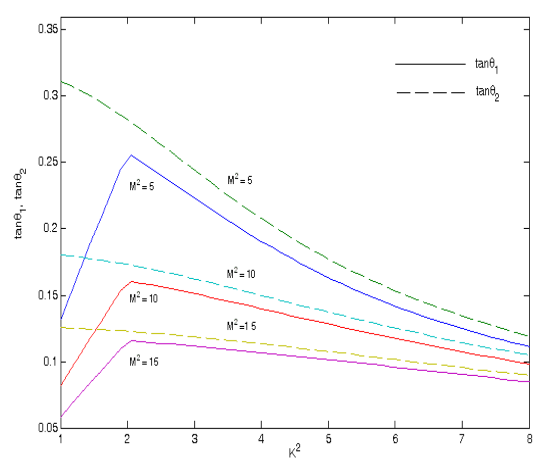

Figure 12: Variations of $\tan\theta_1$ and $\tan\theta_2$ for M^2 when $n = 4$, $\beta = 0.5$, $S = 0.5$ and $n\tau = \frac{\pi}{4}$.

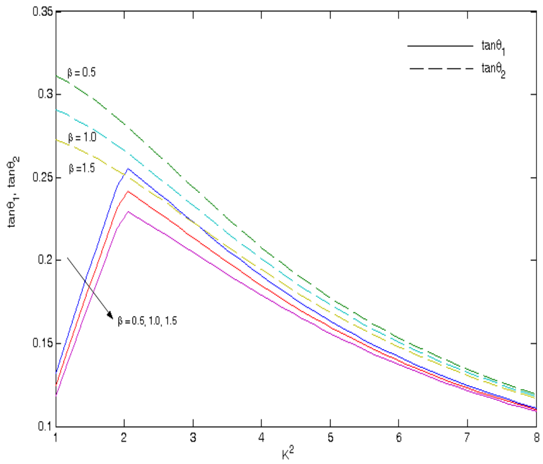

Figure 13: Variations of $\tan\theta_1$ and $\tan\theta_2$ for β when $M^2 = 5$, $n = 4$, $S = 0.5$ and $n\tau = \frac{\pi}{4}$.

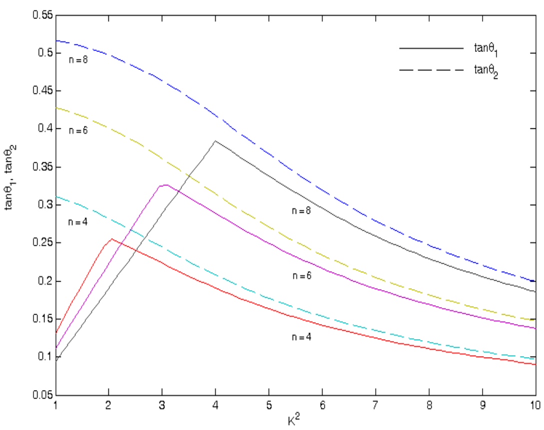

Figure 14: Variations of $\tan\theta_1$ and $\tan\theta_2$ for n when $M^2 = 5$, $\beta = 0.5$, $S = 0.5$ and $n\tau = \frac{\pi}{4}$.

4. HEAT TRANSFER

We now discuss the temperature distribution in oscillating flow past a porous flat plate subject to uniform suction at the plate in the presence of a uniform magnetic field perpendicular to the flow field. The equation of energy for the temperature distribution is

$$\rho C_p\left(\frac{\partial T}{\partial t} - v_0\frac{\partial T}{\partial y}\right) = k\frac{\partial^2 T}{\partial y^2} + \mu\left[\left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial y}\right)^2\right]$$
$$+\sigma B_0^2\left(u^2 + w^2\right), \quad (25)$$

where k, μ, C_p and σ are respectively the thermal conductivity, coefficient of viscosity, specific heat of the fluid and electrical conductivity. The last two terms within parenthesis are due to the viscous dissipation and Joule heating respectively.

The boundary conditions for temperature distribution are

$$T = T_w \text{ at } y = 0 \text{ and } T \to T_\infty \text{ as } y \to \infty, \quad (26)$$

where T_∞ is the constant ambient temperature of the surrounding fluid and $T_w > T_\infty$.

Introducing the non-dimensional variable

$$\theta = \frac{T - T_\infty}{T_w - T_\infty} \quad (27)$$

and using (5), equation (25) is reduced to

$$\frac{\partial\theta}{\partial t} - S\frac{\partial\theta}{\partial\eta} = \frac{1}{Pr}\frac{\partial^2\theta}{\partial\eta^2} + Ec\left[\left(\frac{\partial u_1}{\partial\eta}\right)^2 + \left(\frac{\partial w_1}{\partial\eta}\right)^2\right]$$
$$+M^2\left(u_1^2 + w_1^2\right), \quad (28)$$

where $Ec = \frac{u_0^2}{C_p(T_w - T_\infty)}$ is the Eckert number and $Pr = \frac{\rho C_p \nu}{k}$, the Prandtl number.

The boundary conditions (26) become

$$\theta = 1 \text{ at } \eta = 0 \text{ and } \theta \to 0 \text{ as } \eta \to \infty. \quad (29)$$

Since the velocity field given by (19) and (20) has zero mean,we assume the temperature distribution as

$$\theta(\eta,\tau) = \theta_0(\eta)e^{2\beta\tau} + \theta_1(\eta)e^{2(\beta+in)\tau} + \overline{\theta}_1(\eta)e^{2(\beta-in)\tau}, \quad (30)$$

where $\theta_0(\eta)$ represents the mean part and $\overline{\theta}_1(\eta)$ presents the complex conjugate of $\theta_1(\eta)$.

Substituting (30) in (28) and equating the harmonic coefficients to zero, we get

$$\frac{d^2\theta_0}{d\eta^2} + S\,Pr\frac{d\theta_0}{d\eta} = -EcPr\left[A_1e^{-(S+2\alpha_1)\eta} + A_2e^{-(S+2\alpha_2)\eta}\right], \quad (31)$$

$$\frac{d^2\theta_1}{d\eta^2} + S\,Pr\frac{d\theta_1}{d\eta} - 2(\beta+in)Pr\theta_1 = -EcPr(c_r + ic_i)e^{-(\alpha_3+i\beta_3)\eta}, \quad (32)$$

$$\frac{d^2\overline{\theta}_1}{d\eta^2} + S\,Pr\frac{d\overline{\theta}_1}{d\eta} - 2(\beta-in)Pr\overline{\theta}_1 = -Ec\,Pr(c_r - ic_i)e^{-(\alpha_3-i\beta_3)\eta}, \quad (33)$$

where

$$A_1 = \frac{1}{4}\left[\left(\frac{S}{2}+\alpha_1\right)^2 + \beta_1^2 + M^2\right],$$

$$A_2 = \frac{1}{4}\left[\left(\frac{S}{2}+\alpha_2\right)^2 + \beta_2^2 + M^2\right],$$

$$c_r = \frac{1}{4}\left[\left(\frac{S}{2}+\alpha_1\right)\left(\frac{S}{2}+\alpha_2\right) \pm \beta_1\beta_2 + M^2\right], \quad (34)$$

$$c_i = \frac{1}{4}\left[\beta_1\left(\frac{S}{2}+\alpha_2\right) \mp \beta_2\left(\frac{S}{2}+\alpha_1\right)\right],$$

$$\alpha_3 = S + \alpha_1 + \alpha_2,\ \beta_3 = \beta_1 \mp \beta_2.$$

where the upper sign is for $n < 2K^2$ and the lower sign is for $n > 2K^2$.

The boundary conditions for $\theta_0(\eta)$, $\theta_1(\eta)$ and $\overline{\theta}_1(\eta)$ become

$$\theta_0 = 1,\ \theta_1 = 0,\ \overline{\theta}_1 = 0 \text{ at } \eta = 0,$$
$$\theta_0 \to 0,\ \theta_1 \to 0,\ \overline{\theta}_1 \to 0 \text{ as } \eta \to \infty. \quad (35)$$

The solution of (31) subject to the conditions (35) is

$$\theta_0(\eta)=\begin{cases} e^{-SPr\eta}+Pr\,Ec\left[\dfrac{A_1\{e^{-(S+2\alpha_1)\eta}-e^{-SPr\eta}\}}{(S+2\alpha_1)(SPr-S-2\alpha_1)}\right. \\ \left.+\dfrac{A_2\{e^{-(S+2\alpha_2)\eta}-e^{-SPr\eta}\}}{(S+2\alpha_2)(SPr-S-2\alpha_2)}\right] \\ \text{for } S+2\alpha_1\neq SPr,\ S+2\alpha_2\neq SPr \\ e^{-SPr\eta}+Pr\,Ec\left[\dfrac{A_1}{SPr}\eta e^{-SPr\eta}\right. \\ \left.+\dfrac{A_2\{e^{-(S+2\alpha_2)\eta}-e^{-SPr\eta}\}}{(S+2\alpha_2)(SPr-S-2\alpha_2)}\right] \\ \text{for } S+2\alpha_1=SPr,\ S+2\alpha_2\neq SPr \\ e^{-SPr\eta}+Pr\,Ec\left[\dfrac{A_2}{SPr}\eta e^{-SPr\eta}\right. \\ \left.+\dfrac{A_1\{e^{-(S+2\alpha_1)\eta}-e^{-SPr\eta}\}}{(S+2\alpha_1)(SPr-S-2\alpha_1)}\right] \\ \text{for } S+2\alpha_1\neq SPr,\ S+2\alpha_2=SPr \end{cases} \quad (36)$$

where α_1 and α_2 are given by equation (17).

The solution of the equation (32) subject to boundary condition (35) is

$$\theta_1(\eta)=\frac{EcPr(c_r+ic_i)}{d_r+id_i}\left[e^{-(\alpha_4+i\beta_4)\eta}-e^{-(\alpha_3+i\beta_3)\eta}\right], \quad (37)$$

where

$$\alpha_4=\frac{1}{2\sqrt{2}}\left[\sqrt{2}SPr+\left\{\begin{array}{l}\left[\left(S^2Pr^2+8\beta Pr\right)^2+64Pr^2n^2\right]^{\frac{1}{2}}\\+\left(S^2Pr^2+8\beta Pr\right)\end{array}\right\}^{1/2}\right],$$

$$\beta_4=\frac{1}{2\sqrt{2}}\left\{\begin{array}{l}\left[\left(S^2Pr^2+8\beta Pr\right)^2+64Pr^2n^2\right]^{\frac{1}{2}}\\-\left(S^2Pr^2+8\beta Pr\right)\end{array}\right\}^{1/2}, \quad (38)$$

$$d_r=\alpha_3^2-SPr\alpha_3-\beta_3^2-2\beta Pr,.$$
$$d_i=2\alpha_3\beta_3-SPr\beta_3-2nPr\ .$$

Since the equation (33) is the complex conjugate of the equation (32) so the solution of the equation (33) is obtained on taking the complex conjugate of θ_1.

It is seen from Figures 15-17 that the mean temperature $\theta_0(\eta)$ increases with an increase in either magnetic parameter M^2 or rotation parameter K^2 or frequency parameter n. Further, Figures 18-19 show that the mean temperature $\theta_0(\eta)$ increases with an increase in either accelerated parameter β or suction parameter S. It is seen from Figure 20 that the mean temperature $\theta_0(\eta)$ increases near the plate and it decreases away from the plate with an increase in Prandtl number Pr. The increase of Prandtl number Pr means that the thermal diffusivity decreases. So the temperature decreases due to the decrease of thermal boundary layer. This characteristic indicates that the temperature dependent fluid viscosity plays a significant role in shifting the fluid away from the plate

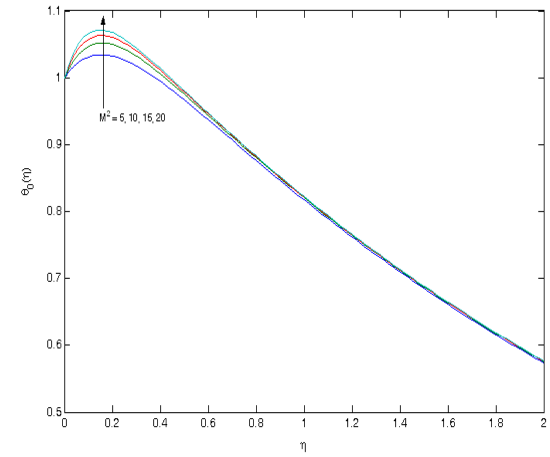

Figure 15: Variations of $\theta_0(\eta)$ for M^2 when $K^2=2$, $S=1$, $\beta=0.5$, $n=4$ and $Pr=0.71$.

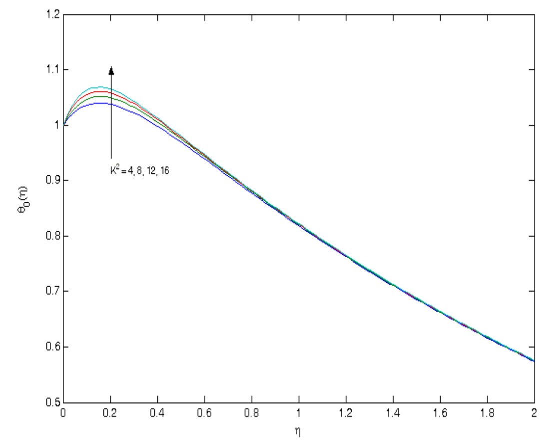

Figure 16: Variations of $\theta_0(\eta)$ for K^2 when $M^2=5$, $S=1$, $\beta=0.5$, $n=4$ and $Pr=0.71$.

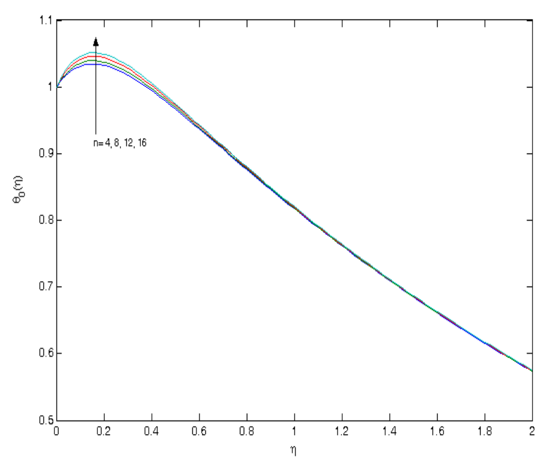

Figure 17: Variations of $\theta_0(\eta)$ for n when $M^2=5$, $K^2=2$, $S=1$, $\beta=0.5$ and $Pr=0.71$.

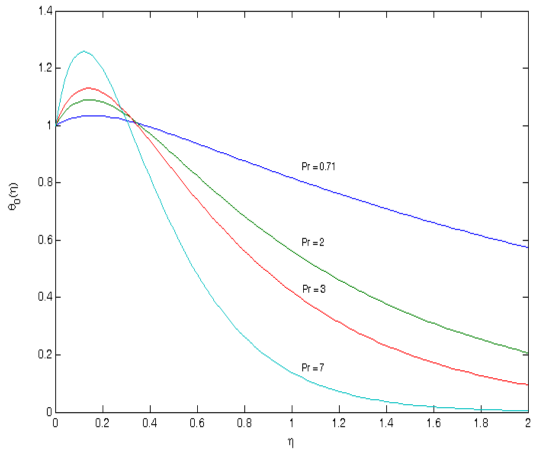

Figure 18: Variations of $\theta_0(\eta)$ for Pr when $M^2 = 5$, $K^2 = 5$, $S = 1$, $\beta = 0.5$ and $n = 4$.

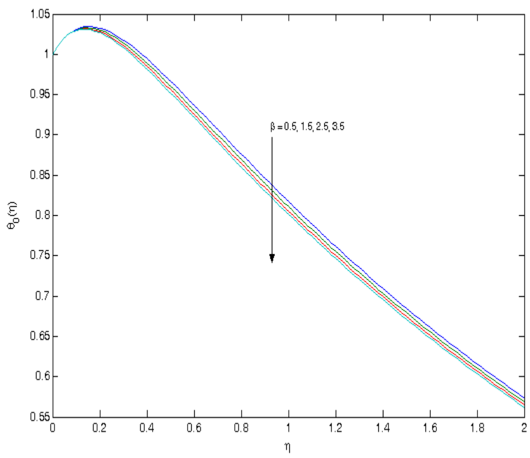

Figure 19: Variations of $\theta_0(\eta)$ for β when $M^2 = 5$, $K^2 = 2$, $S = 1$, $n = 4$ and $Pr = 0.71$.

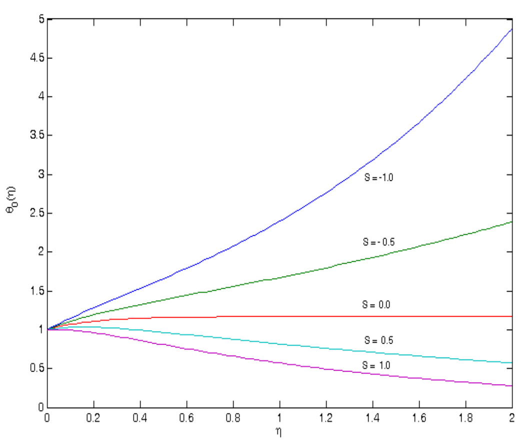

Figure 20: Variations of $\theta_0(\eta)$ for n when $M^2 = 5$, $K^2 = 2$, $n = 4$, $\beta = 0.5$ and $Pr = 0.71$.

The rate of heat transfer $\theta'(0)$ at the plate $\eta = 0$ is given by

$$\theta'(0) = \theta_0'(0) + 2EcPr\,R_3\cos(2n\tau + \psi), \quad (39)$$

where

$$R = d_r^2 + d_i^2,\ X = d_r(\alpha_3 - \alpha_4) + d_i(\beta_3 - \beta_4),$$
$$Y = d_r(\beta_3 - \beta_4) - d_i(\alpha_3 - \alpha_4),$$

$$R_3^2 = \left(\frac{c_r X - c_i Y}{R}\right)^2 + \left(\frac{c_i X + c_r Y}{R}\right)^2,\ \tan\psi = \frac{c_i X + c_r Y}{c_r X - c_i Y}. \quad (40)$$

and

$$\theta_0'(0) = -SPr + Ec\,Pr\left(\frac{A_1}{S + 2\alpha_1} + \frac{A_2}{S + 2\alpha_2}\right), \quad (41)$$

whether $S + 2\alpha_1 \neq SPr$ and $S + 2\alpha_2 \neq SPr$ or $S + 2\alpha_1 = SPr$ and $S + 2\alpha_2 \neq SPr$ or $S + 2\alpha_1 \neq SPr$ and $S + 2\alpha_2 = SPr$.

The variation of amplitude of the rate of heat transfer R_3 and tangent of the phase angle of the rate of heat transfer oscillations $\tan\psi$ are shown in Tables 1 and 2 against K^2 for different values of Pr and n. It is seen from Table 1 that the amplitude R_3 increases with an increase in either rotation parameter K^2 or Prandtl number Pr while it decreases with an increase in frequency parameter n for fixed values of K^2. Farther, it shows from Table 2 that the magnitude of tangent of the phase angle $\tan\psi$ decreases with an increase in either rotation parameter K^2 or Prandtl number Pr while it increases with an increase in frequency parameter n for fixed values of K^2.

Table 1: Amplitude of the rate of heat transfer oscillations $10^{-3} R_3$ at the plate $\eta = 0$ for $M^2 = 5$, $n\tau = \frac{\pi}{4}$, $S = 0.5$, $\beta = 0.5$ and $Ec = 1$.

	Pr with $n = 10$				n with $Pr = 0.71$			
K^2	2	3	5	7	2	3	4	5
2	0.73404	1.30662	2.62981	4.09991	0.08581	0.07295	0.06333	0.05243
4	0.86068	1.66219	3.56860	5.72213	0.22280	0.15159	0.11029	0.09011
6	1.09687	2.37460	5.54698	9.21527	0.61287	0.42904	0.30145	0.21324
8	1.23906	3.10512	8.01130	13.85063	1.40193	1.03917	0.77118	0.56806

Table 2: The magnitude of tangent of the phase angle of the rate of heat transfer oscillations $\tan\psi$ at the plate $\eta = 0$ for $M^2 = 5$, $n\tau = \frac{\pi}{4}$, $S = 0.5$, $\beta = 0.5$ and $Ec = 1$.

	Pr with $n = 10$				n with $Pr = 0.71$			
K^2	5	7	9	11	2	3	4	5
2	0.63530	0.54181	0.50338	0.48579	0.38746	0.67024	0.98702	1.29909
4	0.36693	0.27152	0.23503	0.21896	0.12769	0.27384	0.47076	0.74449
6	0.23659	0.12526	0.08797	0.07288	0.02516	0.09501	0.1816	0.30022
8	0.20780	0.06192	0.02223	0.00807	0.01479	0.02450	0.06783	0.12187

5. CONCLUSION

The unsteady MHD flow and heat transfer of an incompressible viscous fluid bounded by an infinite heated porous flat plate have been studied. It is found that with an increase in either magnetic parameter M^2 or suction parameter S the primary velocity u_1 and the magnitude of secondary velocity w_1 of fluid decrease at a particular point in flow filed. The primary velocity u_1 and the magnitude of secondary velocity w_1 increase with an increase in either accelerated parameter β or frequency parameter n or time τ. The mean wall temperature $\theta_0(\eta)$ as well as the rate of heat transfer $\theta'(0)$ is also obtained. It is found that with an increase of magnetic field intensity the mean temperature $\theta_0(\eta)$ increases at a particular point in flow filed. Further, it is found that the magnitude of tangent of the phase angle of the rate of heat transfer oscillations $\tan\psi$ decreases with an increase in either rotation parameter K^2 or Prandtl number Pr while it increases with an increase in frequency parameter n for fixed values of K^2. The effect of transverse magnetic field on a viscous incompressible conducting fluid is to suppress the velocity field which in turn causes the enhancement of the temperature field.

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