Image Enhancement and De-noising by Diffusion based Singular Value Decomposition

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ABSTRACT
This paper proposes an approach for enhancement and de-noising of the images having fine edges and homogeneous smooth regions by using singular value decomposition filtering technique on the diffused image subspaces. The existing singular value decomposition based image de-noising technique faces the problem of selecting the optimum threshold parameter for separation of noise subspace and noise-free image subspace. The proposed approach is a two stage process in which the diffused versions of the image are generated in the first stage using partial differential equation based linear isotropic diffusion to smooth the homogeneous regions and the inverse heat diffusion method for enhancement of the edge features. In the next stage, singular value decomposition is applied on the two oppositely featured diffused versions of the image with fixed threshold individually to remove noise. Experimental results were compared with respect to recently developed singular value decomposition method with minimum energy model and traditional block based singular value decomposition filtering method in terms of signal to noise ratio which shows that the proposed method is efficient for image enhancement as well as de-noising.

General Terms
Algorithms, Experimentation.

Keywords
Anisotropic diffusion, Isotropic diffusion, Singular value decomposition, Image enhancement, Image de-noising.

1. INTRODUCTION
In image pre-processing, enhancement as well as de-noising of images is an important challenging issue. A comprehensive approach have been proposed by many researchers to enhance the important features of the image such as sharp edges, lines and boundaries with the elimination of noise. Recently, image enhancement based on partial differential equation (PDE) based nonlinear diffusion methods and singular value decomposition (SVD) based de-noising methods are becoming popular. Till now various image enhancement and de-noising algorithms based on above two methods have been independently developed [5][21].

Wavelet transform based SVD techniques [1][2][3] have recently gained a lot of attraction for enhancement and de-noising of scientific images. Foisal et al. [2] developed SVD filtering in contourlet transform [2] for the images containing mostly fine textures and contours. Zujun Hou [3] proposed an approach for image de-noising by performing SVD filtering in detailed sub-bands of discrete wavelet domain which is quite suitable for the regions having abrupt changes. The electronic nose sensor array data analysis proposed by Jha and Yadava [4] demonstrated the usefulness of the SVD de-noising procedure. They showed that elimination of the lowest singular value components by rank reduction of the image matrix produces better results. However, in all of the above methods, the problem is to optimize the threshold parameter which is used to discriminate the noise free image subspace with that of the noisy image subspace in SVD which may not be properly achievable with a fixed threshold for the entire image subspace. In addition to this, the computational complexity for the operation to perform SVD on an $n \times n$ matrix is of $O(n^3)$ [8]. However, in order to have cheaper implementation of the SVD algorithm, block based SVD have been developed by Konstantinides et al. [9] dividing the whole image matrix and filtering the noise from each sub-matrix by SVD individually. In order to obtain the optimal threshold, Yong-le et al. [6] adjusted the threshold parameter for SVD using mathematical relation between threshold and the empirical value of signal to noise ratio of the images. A particular method of sparse representation framework has been proposed by Priyam Chatterjee [7] using K-SVD algorithm which effectively removes the noise from the image.

PDE based nonlinear diffusion methods [12][13][14][15] are the effective mathematical processes which has received much interest since the early work of Perona and Malik [10][11] in which image details are preserved by adding a high order nonlinear diffusive term to control the smoothing process near edge structures. Many researchers have investigated to develop improved version of Perona-Malik diffusion model such as a generalized diffusion model [12], a non-linear anisotropic diffusion equation based model [13] and a ramp preserving Perona-Malik model [14] etc. The diffusion coefficients are the functions of gradient magnitude and dynamic threshold of the image driven by a Poisson equation [11][15]. However, to generate a consistent diffusivity function and dynamic threshold to achieve better results was difficult.

In order to obtain better image enhancement and de-noising, some approaches [16][17][18] utilizing the operation of SVD with PDE based diffusion methods has been proposed recently. Zhang et al. [16] presents a PDE based minimum energy model for selecting the proper singular values for signal and discarding those which represents noise in SVD process. Nikapour et al. [17] proposed a time domain approach and use of PDEs in time series for noise reduction in FM signals. The signal data matrix is divided into signal subspace and noise subspace using the SVD based approach introduced in [8] and then the noise from the singular vectors (SVs) is reduced by PDEs. Our initial study [22] was based on Nikapour's approach [17]. But, the idea of taking a global threshold parameter in SVD for entire image is not quite effective for highly textured image data values.

In the proposed approach, we further investigate the relation between PDE based nonlinear diffusion techniques and SVD filtering method to develop an efficient image enhancement method with maximum noise reduction and cheaper implementation. This can be accomplished by adopting the idea of smoothing and sharpening diffusion coefficient used in nonlinear anisotropic diffusion model [13]. The input noisy image is diffused by smoothing diffusion coefficient and sharpening diffusion coefficient to get smoothed version and sharpened (edge enhanced) version of the input image respectively and perform SVD filtering on each image version individually. Finally, the two SVD filtered images are linearly added to get the enhanced as well as noise reduced image.

The rest of the paper is organized as follows. In Section 2, we briefly discuss the SVD method. Section 3 describes the proposed image enhancement and de-noising method based on PDE based diffusion and SVD filtering. The experimental results and performance comparisons are shown in Section 4. Finally, a conclusion is made in Section 5.

2. SVD BASED IMAGE DE-NOISING

SVD of a matrix is one of the most efficient mathematical tool used for solving the inverse problems [20]. In order to discriminate the noise from the signal using SVD [8][20], the image is converted into an observational data matrix and then the values of the matrix are decomposed into smaller matrices. For example, the SVD of matrix H with size $m \times n$ is of the form:

$$M = UWV^t = \sum_{i=1}^{n} \propto_i \overrightarrow{u_i}\, \overrightarrow{v_i^t} \qquad (1)$$

where U and V are orthogonal matrices and W is an $r \times r$ diagonal matrix whose elements are known as singular values of M with components $\propto_{ij} = 0$ if $i \neq j$ and $\propto_{ij} > 0$ if $i = j$. The columns of the orthogonal matrices U and V are called as the left and right singular vectors respectively. The noisy subspace and noise-free subspace separation introduced by Mohsen et al. [17] which is briefly expressed below:

$$M = UWV^t = (U_s \;\; U_n)\begin{bmatrix} W_s & 0 \\ 0 & W_n \end{bmatrix}\begin{pmatrix} V_s^T \\ V_n^T \end{pmatrix}$$

$$F_s = U_s U_s^t M = M V_s V_s^t$$

$$Q_n = U_n U_n^t M = M V_n V_n^t$$

where W_s and W_n represents the noise-free image subspace and noisy image subspace respectively. A threshold point T_h is to be determine in the W matrix as can be seen from (1), where the lower singular values from the point can be categorized as noisy subspace and hence, should be set to zero [8][19]. This threshold point can be determined by calculating the gradient of the image at each pixel position. Based on the perturbation theory and statistical hyperthesis testing [19], the threshold was bounded by Konstantinides [9, 20] due to random noise as shown below:

$$\sqrt{c}\sigma \;\leq\; T_h \;\leq\; \sqrt{mn}\sigma$$

where c is a parameter determined from the statistics of signal and noise, σ is the standard deviation of the noise. Konstantinides [20] demonstrated that the simple threshold $T_h = 3\sigma$ perform most stably under different noise levels.

3. PROPOSED IMAGE ENHANCEMENT AND DE-NOISING METHOD

The proposed image enhancement and de-noising method is a two stage process. In the first stage, diffusion process has been carried out and in the second stage SVD filtering has been followed. During the first stage, the input noisy image corrupted by additive Gaussian noise is diffused by using anisotropic diffusion technique which was first proposed by Perona-Malik [10][11] for scale space description of images. The nonlinear anisotropic diffusion [12][13] is given as:

$$\frac{\partial}{\partial t} f_t(x,y) = \mathrm{div}\big(c_t(x,y)\nabla f_t(x,y)\big) \qquad (2)$$

where $f_t(x,y)$ is a two dimensional image $f(x,y)$ at iteration t with $f_0(x,y)$ be the original image. $c_t(x,y)$ is the diffusion coefficient at iteration t, ∇ is the gradient operator operator and div is the divergence operator. The idea of anisotropic diffusion is to vary the diffusion coefficient $c_t(x,y)$ in different iterations at each pixel position so that the noisy homogeneous regions can be smoothed out with preservation of edges and fine structures. The diffusion coefficient $c_t(x,y)$ is the function of image gradient magnitude $\nabla f(x,y)$ at different pixel positions which is given as:

$$c_t(x,y) = c_t[\nabla f(x,y)] = \frac{1}{1+\left(\frac{|\nabla f|}{k}\right)^2} \qquad (3)$$

which implies that the diffusion coefficient is a decreasing function of the image gradient magnitude results in slow diffusion in high gradient regions like edges and fine structures whereas fast diffusion in low gradient homogeneous regions. The parameter k acts as a threshold between the two gradient strengths whose high value over-diffuse the entire image and results in a blurred image or otherwise if its value is kept low then the diffusion process will stop and restored the original image as an output image. It can be observed that for a fixed value of k in (3), the corresponding diffusion function $c_t(x,y)$ falls continuously and approaches to approximately zero in accordance with increasing gradient magnitudes ∇f of the image. In our experimentation, we observed that, at the portions of the image having lot of fine boundaries and edge structures, the gradient magnitude attains its maximum position with $\nabla f > 5k$ where the diffusion stops to recover the high gradient features of the image and at the position when $|\nabla f| = k$, a fast diffusion occurs which gives maximum smoothing effect because the diffusion function $c_t(x,y)$ in (3) gets changed to a constant diffusion parameter c_t =1/2 as shown below:

$$c_t(x,y) = c_t[\nabla f(x,y)] = \frac{1}{1+(1)^2}$$

$$c_t(x,y) = c_t[\nabla f(x,y)] = \frac{1}{2} \text{ or } c_t = \frac{1}{2}$$

which gives the linear isotropic diffusion equation with non-homogeneous diffusivity as shown below:

$$\frac{\partial}{\partial t} f_t(x,y) = \mathrm{div}\big(c_t \, \nabla f_t(x,y)\big) \quad (4)$$

Such type of diffusion is equivalent to convolving the image with Gaussian smoothing filter. This linear non-homogeneous isotropic diffusivity suits our requirement of getting the first diffused image in the form of smoothed version of the image $f_s(x,y)$. Then, we need to have an edge enhanced image as the second diffused image which we achieved from inverse heat diffusion equation [13] which is as follows:

$$\frac{\partial}{\partial t} f_t(x,y) = -\Delta f = -(f_{xx} + f_{yy}) \quad (5)$$

where Δ is the Laplacian operator which incorporates f_{xx} and f_{yy} which are the second derivative of the image function at horizontal and vertical direction respectively. The inverse heat diffusion equation [13] enhances the edges and fine structures of the image but also enhances the flat regions of the image. Thus, it is not in favour of detecting and extracting the edges and fine structures. In order to get more sharp edge enhanced image, either we may use the Canny edge detection operator [23] on the image diffused by inverse heat diffusion equation [13] or simply we may subtract the smoothed version of the image (first diffused image) from the original image. For better sharpness of edges and fine boundaries, we utilizes Canny edge detection operator [23] on the image diffused by inverse heat diffusion equation [13] which fulfilled our requirement of getting the second diffused image in the form of sharp edge enhanced image $f_e(x,y)$. The two diffused images are having just opposite features to each other. One is having smooth area of very low gradient values and other is having edge structures with high gradient values. After diffusion process, some undesirable blurred region and edges were still found in the two diffused images respectively. To further de-noise and enhance the diffused images, SVD filtering is applied to the two diffused images individually in the next stage.

Traditionally, the effectiveness of SVD filtering algorithm depends on the accuracy of the estimate of the fixed threshold T_h. To perform SVD on image matrix, the singular values lower than predefined threshold T_h are set to zero which represents noise. If we choose very small T_h, then some noise will be removed but most of them will be retained or otherwise, if we choose very high T_h, then most of the singular values will set to zero and entire image will be smooth out. To overcome this drawback of SVD filtering, we fixed the threshold T_h to a high value for the first diffused image (smoothed version) $f_s(x,y)$ so that the singular values lower than the threshold representing the over-smoothed portion of the image will get cancel out and gives a flat smoothed image with homogeneous structures. Reversibly, we set a low threshold T_h for the second diffused image (edge enhanced image) $f_e(x,y)$ so that the high gradient edges become more sharp and low gradient undesirable spots will be removed. In this way, we have two similar but totally opposite featured images $f_s'(x,y)$ and $f_e'(x,y)$ whose linear combination gives us a well enhanced and noise reduced image as follows:

$$f_o(x,y) = p f_s'(x,y) + q f_e'(x,y) \quad (6)$$

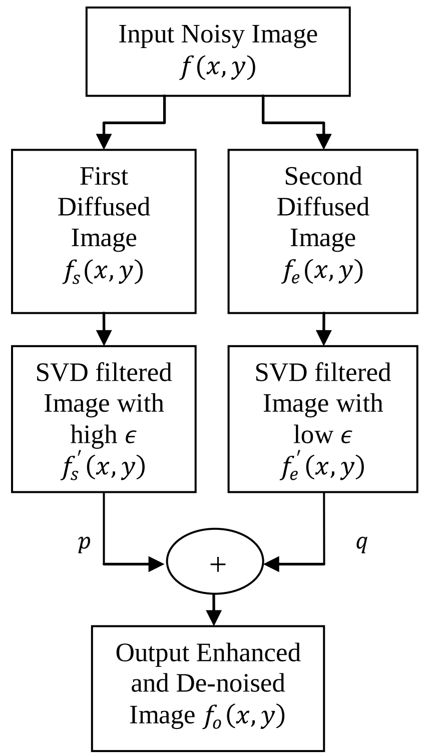

Fig 1: The functional block diagram of the proposed method

where $f_o(x,y)$ is the resultant image. The best result can be obtained by choosing the optimized values of the two constants p and q in (6). We compared the results on various test images for different combinations of the two constant values in accordance with increasing noise variance in terms of signal to noise ratio whose observation is shown in Section 4.

The complete functional block diagram of the proposed image enhancement and de-noising method by diffusion based SVD filtering is shown in Fig. 1. The functional steps of proposed diffusion based SVD filtering algorithm for image enhancement as well as de-noising are as follows:

1) Diffuse the input noisy image $f(x,y)$ into flat smoothed image $f_s(x,y)$ by using linear non-homogeneous isotropic diffusion equation (4).
2) Diffuse the image $f(x,y)$ into sharp edge enhanced image $f_e(x,y)$ by using inverse heat diffusion equation (5) followed by Canny edge detection operator.

3) Apply SVD filter to the image $f_s(x,y)$ with a high threshold ϵ and set the singular values to zero which are smaller than the threshold T_h to get the image $f_s^{'}(x,y)$.
4) Apply SVD filter to the image $f_e(x,y)$ with a low threshold ϵ and set the singular values to zero which are smaller than the threshold T_h to get the image $f_e^{'}(x,y)$.
5) Linearly combine the two SVD filtered images $f_s^{'}(x,y)$ and $f_e^{'}(x,y)$ as shown in (6) to get the enhanced and de-noised ouput image $f_o(x,y)$.

4. EXPERIMENTAL RESULTS

The proposed method for image enhancement and de-noising using diffusion based SVD filtering (DSVD) has been experimentally analyzed and validated by applying it on various test images. Matlab version 2009b has been used for implementation of the proposed method. The two representative images of moon and cameraman taken from image processing toolbox of Matlab version 2009b are shown in Fig. 2 for demonstration. Out of these two images, the image of moon is having a large homogeneous background with sharp moving curve where as the image of cameraman is highly textured with homogeneous background of sky and blurred ground region. The two images are of size 512 × 512 pixels and are represented by 8-bits per pixel. Both the images are corrupted by additive Gaussian noise $N(0,v)$ with mean = 0, variance v varying from 0.005 to 0.03. The performance of the proposed method is very much dependent on the two constants p and q in (6). During the experimentation, we calibrated different combinations of the two constants p and q in (6) for different values of signal to noise ratio at different noise levels for the two test images of moon and cameraman which are shown in Table 1 and Table 2 respectively.

Fig 2: The two original test images of moon and cameraman

Table 1: SNR of restored images of moon at different combinations of p, q and noise variance v = 0.01, 0.02 and 0.03 respectively.

p	q	$v=0.01$	$v=0.02$	$v=0.03$
1	1	29.14	28.81	28.43
1	1.5	32.13	31.08	30.55
2	2.5	34.67	33.55	33.07
2	3.5	36.42	36.13	35.47
3	**4.5**	**41.20**	**39.89**	**39.05**
4	4.5	38.80	37.68	37.09
4	5	38.97	37.88	37.13

Table 2: SNR of restored images of cameraman at different combinations of p, q and noise variance v = 0.01, 0.02 and 0.03 respectively.

p	q	$v=0.01$	$v=0.02$	$v=0.03$
1	1	26.32	26.11	25.23
1	1.5	28.03	27.78	27.15
2	2.5	32.77	32.05	31.47
2	3.5	35.20	34.32	33.57
3	**4.5**	**38.60**	**37.79**	**37.02**
4	4.5	37.80	37.27	36.49
4	5	37.57	36.98	36.15

This observation signifies that the effectiveness of the proposed method is based on the following three features:

(a) If $p=q$, then both the SVD filtered images $f_s^{'}(x,y)$ and $f_e^{'}(x,y)$ irrespective of the threshold T_h will be linearly superimposed over each other to give the output image $f_o(x,y)$ having high smoothing effect and edge blurring with low SNR value.

(b) If $p>q$, then the first SVD filtered image $f_s^{'}(x,y)$ with high threshold ϵ will be superimposed over second SVD filtered image $f_e^{'}(x,y)$ with low threshold to give the output image $f_o(x,y)$ with high smoothing effect and poor sharpening of the edges because a high value of p is weighted with first SVD filtered image $f_s^{'}(x,y)$ as compared to the low value of q for the second SVD filtered image $f_e^{'}(x,y)$. Conclusively, SNR of the output image $f_o(x,y)$ falls because of the high smoothing effect and poor sharpening of the edges with reduced Gaussian noise.

(c) If $p<q$, then the first SVD filtered image $f_s^{'}(x,y)$ with high threshold ϵ will be superimposed over second SVD filtered image $f_e^{'}(x,y)$ with low threshold to give the output image $f_o(x,y)$ with low smoothing effect and better sharpening of the edges because a low value of p is weighted with first SVD filtered image $f_s^{'}(x,y)$ as compared to the high value of q for the second SVD filtered image $f_e^{'}(x,y)$. Conclusively, SNR of the output image $f_o(x,y)$ increases because of the low smoothing effect and better sharpening of the edges with reduced Gaussian noise.

During the experimental observation, we varied the value of the two constants p and q in (6) from 1 to 5. This observation gives the optimal combination for p = 3 and q = 4.5 and shown in the bold fonts in Tables 1 and Table 2.

The another important parameter in the proposed method is the optimum choice of the constant diffusion coefficient c_t in (4) as a function of gradient magnitude $\nabla f(x,y)$ at each pixel positions for the generation of first diffused (smoothed version) image. The graph in Fig 3 shows the variation of the diffusion function $c_t(x,y)$ with respect to the ratio of the gradient magnitude $|\nabla f|$ to the edge threshold parameter k as shown in (3)

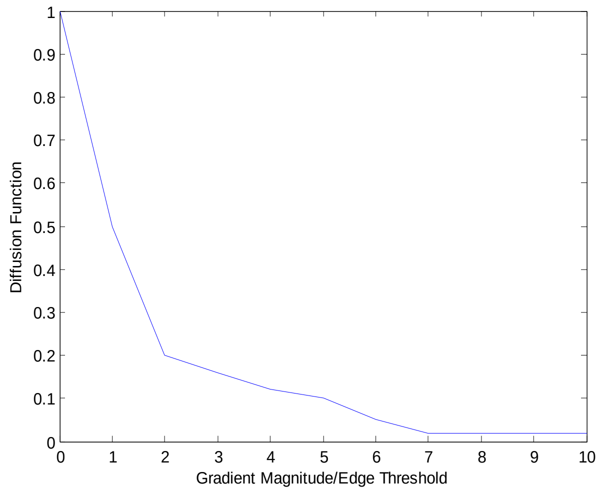

Fig 3: Graph between noise diffusion function $c_t(x, y)$ and the ratio of gradient magnitude $|\nabla f|$ to the edge threshold k

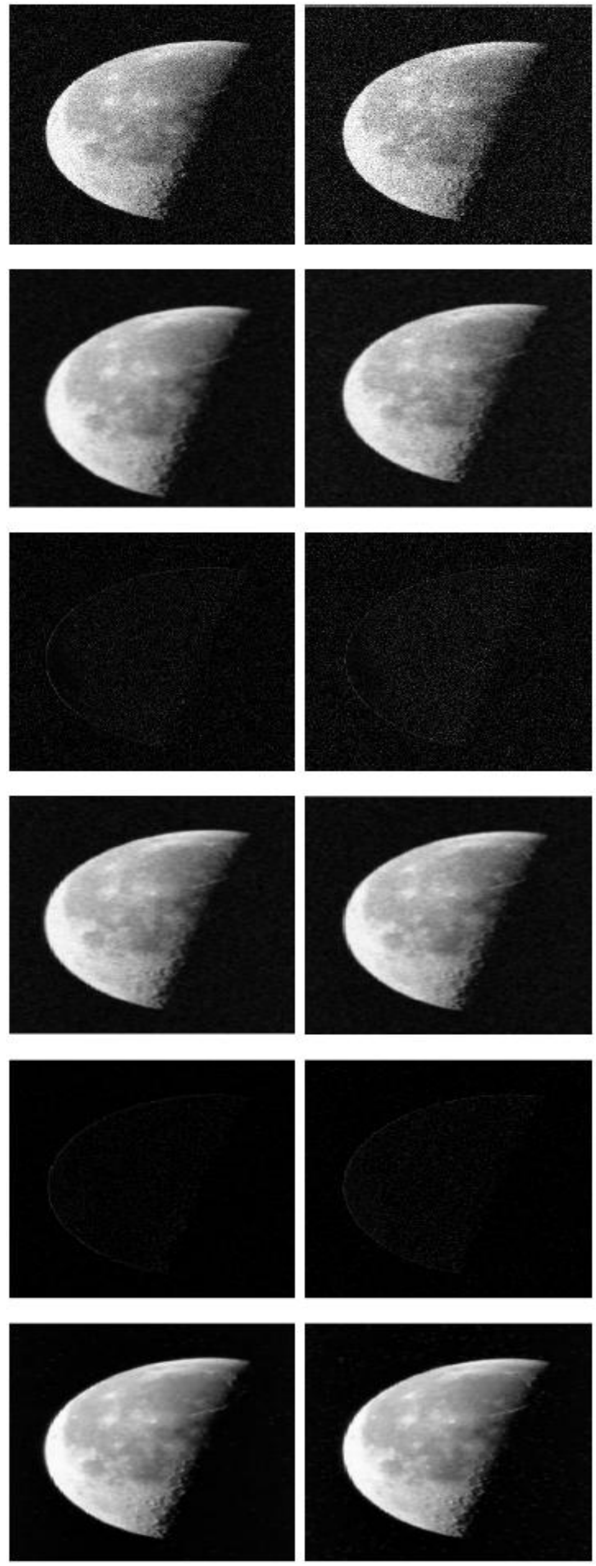

Fig 4: 1st row: Noisy images of moon with v = 0.01 and 0.02 respectively; 2nd row: First diffused images respectively; 3rd row: Second diffused images respectively; 4th row: SVD filtered images of first diffused images respectively; 5th row:

SVD filtered images of second diffused images respectively; 6th row: Output images respectively.

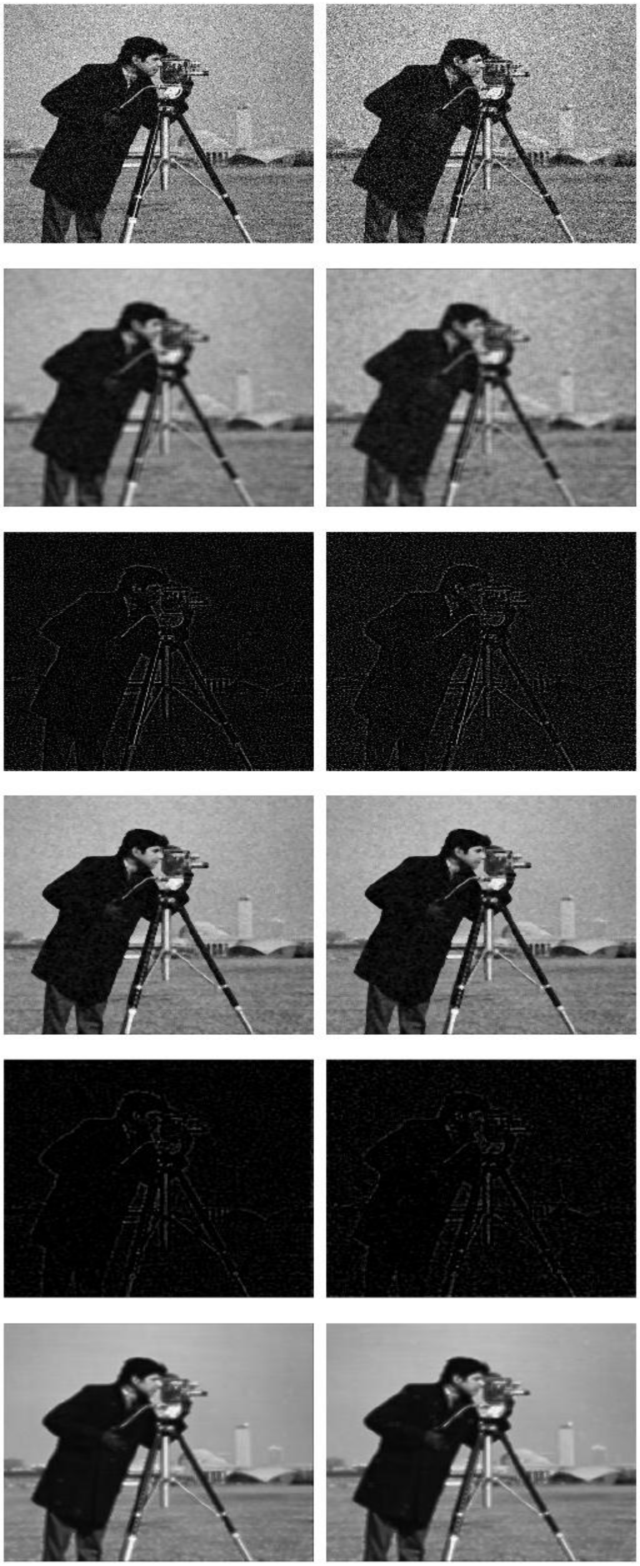

Fig 5: 1st row: Noisy images of cameraman with v = 0.01 and 0.02 respectively; 2nd row: First diffused images respectively; 3rd row: Second diffused images respectively; 4th row: SVD filtered images of first diffused images respectively; 5th row: SVD filtered images of second diffused images respectively; 6th row: Output images respectively.

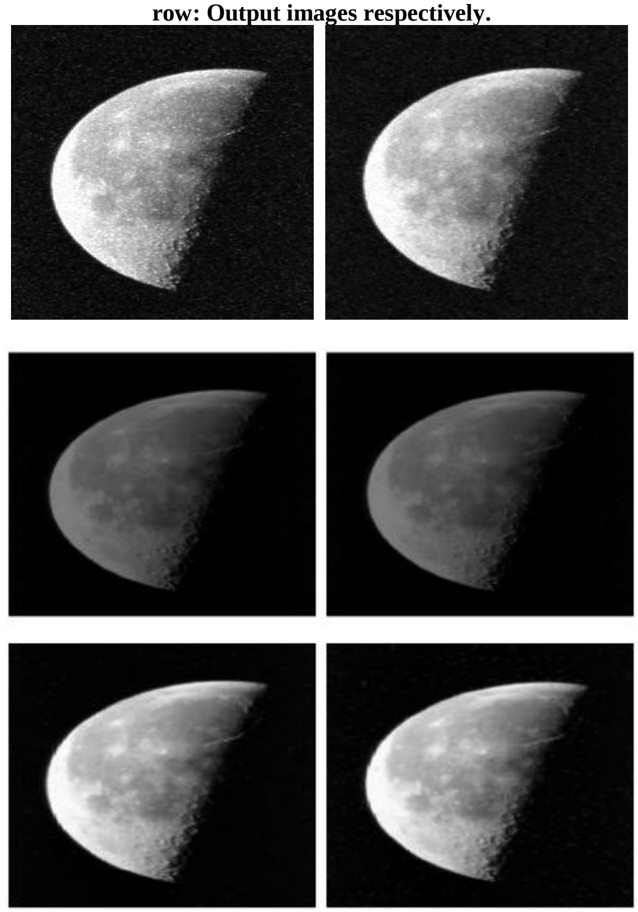

Fig 6: 1st row: Output moon images by BSVD at noise variance v = 0.01 and 0.02 respectively; 2nd row: Output moon images by SVDMEM at noise variance v = 0.01 and 0.02 respectively; 3rd row: Output moon images by DSVD at noise variance v = 0.01 and 0.02 respectively.

Table 3: SNR of restored images of moon by the three methods: BSVD, SVDMEM and DSVD

Noise Variance(v)	BSVD[9]	SVDMEM[16]	DSVD
0.005	38.22	39.42	41.23
0.010	37.52	38.32	40.12
0.015	37.14	37.54	39.03
0.020	36.86	36.97	38.44
0.025	35.29	36.06	37.94
0.030	35.02	35.11	37.67

Fig 7: 1st row: Output cameraman images by BSVD at noise variance v = 0.01 and 0.02 respectively; 2nd row: Output cameraman images by SVDMEM at noise variance v = 0.01 and 0.02 respectively; 3rd row: Output cameraman images by DSVD at noise variance v = 0.01 and 0.02 respectively.

Table 4: SNR of restored images of cameraman by the three methods: BSVD, SVDMEM and DSVD

Noise Variance(v)	BSVD[9]	SVDMEM[16]	DSVD
0.005	36.30	36.69	38.81
0.010	36.04	36.14	38.27
0.015	35.37	35.68	38.11
0.020	34.83	35.03	37.67
0.025	34.13	34.45	37.58
0.030	33.72	34.07	37.21

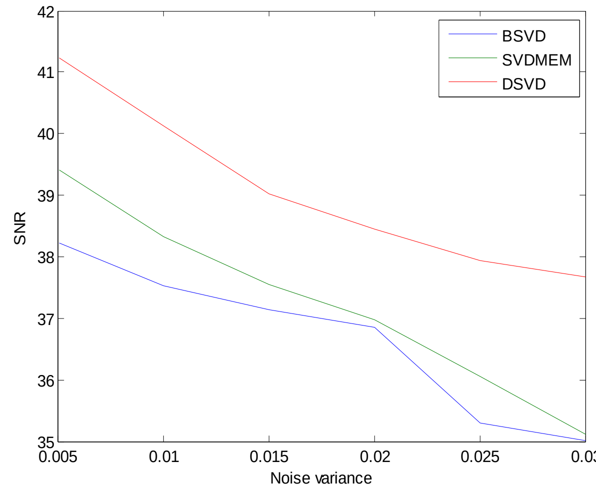

Fig 8: Graph between noise variance and SNR for moon image

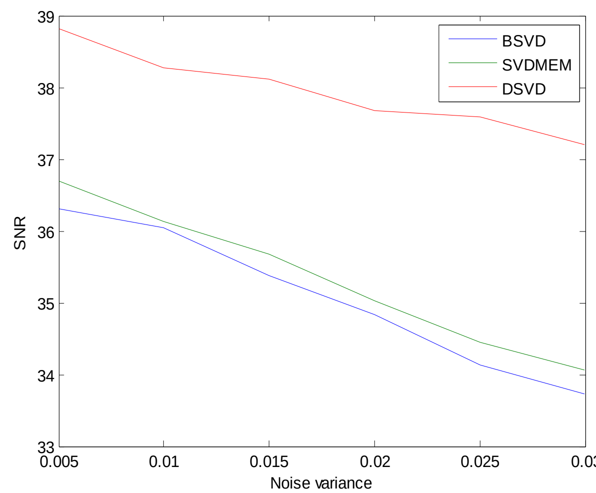

Fig 9: Graph between noise variance and SNR for cameraman image

which gives the optimal value of constant diffusion parameter $d = 1/2$ in (4) for the two representative images.

The noisy images of moon and cameraman with noise variance v = 0.01 and 0.02 and their restored images at each step of DSVD with p = 3 and q = 4.5 are shown in Fig. 4 and 5 respectively. It is observed from the experimental results that, DSVD performs well in homogeneous regions but slightly blurs the edges and fine structures for both the test images. On increasing the noise variance v beyond 0.03, the blurriness in the output image increases because of the use of high threshold during SVD filtering in the smoothed version (1st diffused image) of the noisy image which tends to make output image more smooth. However, the low threshold in the edge enhanced image (2nd diffused image) with high value of q is advantageous to get more sharpened edge enhanced image as in the case of cameraman image which have lot of edge structures. The method gives acceptable results for low edge abundant image like moon image but with the adjustment of the two constants p and q, diffusion coefficient c_t in addition with proper selection of thresholds for the two SVD filter may give a better results for high edge abundant images.

4.1 Performance Comparison

The proposed method (DSVD) has been experimentally compared with traditional block based SVD (BSVD) [9] and SVD based image de-noising with minimum energy model (SVDMEM) [16] in terms of signal to noise ratio (SNR). The comparison images are shown in Fig. 6 and 7. Fig. 6 shows the resultant images obtained by the three methods (BSVD, SVDMEM and DSVD) of noisy moon image with variance v = 0.01 and 0.02 respectively. Fig. 7 shows the same for cameraman image. For better noise reduction algorithm, the value of SNR should be very high. Table 3 and Table 4 respectively summarize the SNR values of restored images by the three methods for the two test images used in this validation. The graphs in Fig. 8 and 9 show the variation of SNR for the three methods with respect to the variation in noise variance of the two images respectively. It is observed from Fig. 6-7, Table 3-4 and the graphs of Fig. 8 and 9 that, the proposed method has attained maximum SNR as compared to othere two methods. In SVDMEM [16], the edges become slightly more blurred before the noise variance reaches to 0.03 in both of the test images where as in the case of BSVD [9], there is a sudden variation of SNR for noise variance varying between 0.02 to 0.025 in moon image. Furthermore, it is visible from Fig. 5 and 6 that the proposed method (DSVD) works as better noise limiter as well as edge enhancement filter as compared to SVDMEM [16] and BSVD [9] in low noisy conditions of the images having a large homogeneous area with edge structures.

5. CONCLUSION

The paper proposed a method of image enhancement and de-noising by applying traditional singular value decomposition on partial differential equation based diffused images. Linear non-homogeneous isotropic diffusion and inverse heat diffusion method has been applied in the first stage of the proposed process to generate the diffused versions of the original noisy image in the form of smoothed image and edge enhanced image respectively. Singular value decomposition is then applied to each of the two diffused versions of the image with a fixed threshold individually which leads to the enhancement of sharp features of the image as well as noise reduction. The proposed method may be considered as the solution of the problem of optimized threshold in singular value decomposition. The method performs well in low noisy images when compared with recently developed singular value decomposition method with minimum energy model and traditional block based singular value decomposition in terms of signal to noise ratio.

Some undesirable edge blurring effect in highly edge abundant images is the problem when high accuracy is important which needs to be improve in the future work with taking the account of proper selection of the parameters involved in the method.

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