Thermal Radiation, Buoyancy and Heat Generation Effects on Flow and Heat Transfer in Porous Medium

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ABSTRACT

An analysis is carried out to study free convective flow and heat transfer of an viscous incompressible electrically conducting fluid over a stretching sheet. Using the similarity variable, the partial differential equations are reduced to ordinary differential equations by using R-K Gill method along with shooting technique. Numerical results of the local of the local skin friction coefficient and the local Nusselt number as well as the velocity and temperature profiles are presented through graphs for different physical parameters, such as the Prandtl number(Pr), Grashof number(Gr), permeability parameter(AP) and ratio of the free stream velocity to parallel wall parameter(□) and the radiation parameter(R).

Keywords

Heat transfer, free convection, Rosseland approximation, porous medium.

1. INTRODUCTION

There are many transport processes which occur naturally and artificially in which flow is modified or driven by density differences caused by temperature, chemical composition differences and gradients and material or phase constitution. The problem of steady flow and heat transfer over a stretching surface could be very practicable in many applications in the polymer technology and metallurgy. In particular, many metallurgical processes involve the cooling of continuous strips or filaments by drawing them though a quiescent fluid and that in the process drawing, these strips are sometimes stretched. Accurate knowledge of the overall convection heat transfer is important in many fields, including heat exchangers, hot water and stream pipes heaters, refrigerators and electrical conductors. Because of its industrial importance, this class of heat transfer has been the subject of many experimental and analytical studies (Bassam and Abu-Hijleh, 2002)

Chaudhary and Merkin (1994) discussed the free convection boundary layer flow on a vertical surface which results when there was an exothermic catalytic chemical reaction on that surface. The system was seen to be governed by the two dimensionless chemical parameters □ □ and □ □ which were measures of the activation energy and heat of reaction respectively, as well as the Prandtl and Schmidt numbers. A series solution was obtained valid near the leading edge of the plate and this was continued downstream by numerical solutions of the full equations. The numerical solutions indicate the criticality of the system by local rapid increases in reaction rate □ □ and □ □ were small. Asymptotic solutions valid at large distances downstream were obtained and these we shown to be essentially different in character between the cases when □ □ = 0 and when □ □ □ □ 0. A singularity was seen to developed at a finite distance downstream when both □ □ = 0 and □ □ = 0 and this was analyzed. Gupta and Mahapatra (2003) analyzed stagnation point flow towards a stretching surface. They reported in their research work that a boundary layer is formed when stretching velocity is less than the free stream velocity. As the stretching velocity exceeds the free stream velocity is less than an inverted boundary layer is formed. Kafoussians (1989) studied heat transfer flow through a very porous medium bounded by a semi infinite horizontal plate. He observed that when the permeability parameter k increased the temperature of the fluid increases.

Sharma and Singh (2008) investigated the effects of variable thermal conductivity and heat source/sink on flow of a viscous incompressible electrically conducting fluid in the presence of uniform transverse magnetic field and variable free stream near a stagnation point on a non-conducting stretching sheet. The equations of continuity, momentum and energy were transformed into ordinary differential equations and solved numerically using shooting method. The velocity and temperature distributions were discussed numerically and presented through graphs. Skinfriction coefficient and the Nusselt number at the sheet were derived, discussed numerically and then numerical values of physical parameter were presented through tables. It was observed that fluid velocity decreases due to increase in the Hartmann number for □ □ < 1 while reverse effect is observed when □ □ > 1 and there was boundary layer formation when □ □ = 1. Prasad and Kulachi (1984) discussed numerical solutions for two-dimensional steady, free convection for rectangular cavity with constant heat flux on one vertical wall, the other vertical wall being isothermally cooled. The horizontal walls were insulted. Results were presented in terms of streamlines and isotherms, local and average Nusselt numbers at the heated wall and the local heat flux at the cooled wall flow patterns were observed to be quite different from those in the case of a cavity with both vertical walls at constant temperatures. Specifically, symmetry in the flow field is absent and any increase in applied heat flux was not accompanied by linearly proportional increase in the temperature on the heated wall. Also, for low Prandtl number, the heat transfer rate based upon the mean temperature difference is higher as compared to experimental results for the isothermal case. Heat transfer results, further indicate that the average Nusselt number is correlated by a relation of the form Nu = constant $R_a^m A^n$ where R_a^m is the Rayleigh number and A the height to-width ratio of the cavity. Mahanti and Gaur (2009) investigated the effects of linearly varying viscosity and thermal conductivity on steady free convective flow of a viscous incompressible fluid along an isothermal vertical plate in the presence of heat sink. The governing equations of continuity, momentum and energy were transformed into coupled and non-linear ordinary differential equations using similarity transformation and then solved using Runge-Kutta fourth order method with shooting technique. They showed that the velocity and temperature of the fluid decrease with the increase in Prandtl number. Skin-friction coefficient decrease while rate of heat transfer increases with the decrease in the heat sink. Mahapatra and Gupta (2001) reported MHD stagnation point flow towards isothermal stretching sheet and pointed that velocity decreases/increases with the increase in magnetic field intensity when free stream velocity is smaller/greater respectively than the stretching velocity. The radiation effect takes place at high temperature. Free convection heat transfer with radiation effect near the isothermal stretching sheet and over a flat sheet near the stagnation point have been investigated by Pop et al (2004). They found that a boundary layer thickness increases with radiation. The radiative effect on the heat transfer from an arbitrary stretching surface with non-uniform surface temperature in a porous medium has been studied by Rashad (2007).

2. GOVERNING EQUATIONS

Consider the steady free convective flow and heat transfer of a viscous, incompressible and electrically conducting fluid past a stretching surface coinciding with the plane y = 0. Keeping the origin fixed two equal and opposite forces are applied along the x-axis which results in stretching of the sheet and hence, the flow is generated. Fluid is in a porous medium in the presence of buoyancy effect and volumetric rate of heat generation /absorption
with radiation effect. The x-axis is taken along the wall and y-axis is transverse to the parallel walls. The temperature of the ambient fluid is T and that of the stretching surface is Tw(x). The fluid properties are assumed to be constant except the density in the buoyancy terms of the linear momentum equation which is approximated according to the Boussinesq's approximation. Under the above assumptions, the boundary layer form of the governing equations can be written as

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \quad (1)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\upsilon\frac{\partial^2 u}{\partial y^2}+g\,\beta(T-T\infty)-\frac{\upsilon}{K}u \quad (2)$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y^2}+\frac{Q}{\rho c_p}(T-T_\infty)-\frac{1}{\rho c_p}\frac{\partial q_r}{\partial y} \quad (3)$$

Here q_r is approximated by Rosseland approximation, which gives

$$q_r=-\left(\frac{4\sigma_1}{3k_1}\right)\frac{\partial T^4}{\partial y} \quad (4)$$

It is assumed that the temperature difference within the flow is so small that T^4 can be expressed as a linear function of T_∞. This can be obtained by expanding T^4 in a Taylor series about T_∞ and neglecting the higher order terms. Thus we get

$$T^4 \cong T_\infty^4+4(T-T_\infty)T_\infty^3=4T_\infty^3 T-3T_\infty^4 \quad (5)$$

Using (4) and (5), Equ. (3) reduces to

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y^2}+\frac{Q}{\rho c_p}+\frac{16\sigma_1 T_\infty^3}{3\rho c_p k_1}\frac{\partial^2 T}{\partial y^2} \quad (6)$$

In free-stream velocity $u_\infty=u(x)=bx$ where b is the free stream velocity parameter. Equation (2) becomes

$$-\frac{1}{\rho}\frac{\partial p}{\partial x}-\frac{\upsilon}{K}u_\infty=u_\infty\frac{du_\infty}{dx} \quad (7)$$

Eliminating $\frac{\partial p}{\partial x}$ from equ. (2). Using equ. (7), we get

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=u_\infty\frac{du_\infty}{dx}+g\,\beta(T-T\infty)+\frac{\upsilon}{K}(u_\infty-u) \quad (8)$$

The boundary conditions of the problem under consideration are

$$u=0, v=0, T=T_\infty \ \ at\ y=0$$
$$u\to u_\infty, T\to T_\infty \ \ at\ y\to\infty \quad (9)$$

where u, v are the velocity components in the x and y direction respectively, υ is the kinematic viscosity, p is the pressure, ρ is the density of the fluid, g is the acceleration due to gravity, β is the volumetric coefficient of thermal expansion, T and T_∞ are the temperature of the fluid inside the thermal boundary layer and the fluid temperature in the free stream respectively, K is the permeability of the porous medium, k is the thermal conductivity, ρ is the density of the fluid, c_p is the specific heat capacity at constant pressure, Q is the volumetric heat flux, σ_1 is the Stefan-Boltzman constant, k_1 is the mean absorption coefficient.

Introducing the stream function ψ(x,y) as defined by $u=\frac{\partial\psi}{\partial y}$ and $v=-\frac{\partial\psi}{\partial x}$ (10)

Such that the continuity equ. (1) is satisfied automatically. To avoid the fluid properties appearing explicitly in the coefficient of the equations we have the following similarity transformation

$$\eta=\left(\frac{c}{\upsilon}\right)^{\frac{1}{2}} y \text{ and } \psi(x,y)=(c\upsilon)^{\frac{1}{2}} x f(\eta)$$
(11)

$$\theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty}$$
(12)

Substituting equs. (10), (11), (12) into equs. (6) and (8) we finally obtain a system of non-linear ordinary differential equation with appropriate boundary conditions:

$$f^{'''}+f f^{''}-\left(f^{'}\right)^2+Gr\theta+A(\lambda-f^{'})+\lambda^2=0$$
(13)

$$\left(\frac{3+4R}{3}\right)\theta^{''}+\Pr f\theta^{'}+\Pr S\theta=0$$
(14)

The corresponding boundary conditions are

$$At\,\eta=0:\ f^{'}(0)=0,\ f(0)=0,\ \theta(0)=1$$

$$As\,\eta\to\infty;\ f^{'}(\infty)=\lambda,\ \theta(\infty)=0$$

Where primes denote differentiation with respect to η, $Gr=\frac{g\beta(T_w-T_\infty)}{c^2x}$ is the Grashof number, $A=\frac{1}{Kc}$ is the permeability parameter, λ=b/c is the ratio of the free stream velocity parameter to parallel wall parameter, $\Pr=\frac{c_p\mu}{k}$ is the prandtl number, $R=\frac{4\sigma_1T_\infty^3}{kk_1}$ is the radiative parameter and $S=\frac{Q}{c\rho c_p}$ is the heat source/sink parameter.

The quantities of physical interest in this problem are the local skin friction coefficient and the local Nusselt number, which are defined by

$$c_f=\frac{\tau_w}{\rho c(c\upsilon)^{\frac{1}{2}}}=x f^{''}(0) \quad \text{where } \tau_w=\mu\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)_{y=0} \text{ is}$$

the shear stress at the wall

$$Nu_x=\frac{x q_w}{k(T_w-T_\infty)\left(\frac{c}{\upsilon}\right)^{\frac{1}{2}}}$$

where $q_w=-\left(k\frac{\partial T}{\partial y}+\frac{16\sigma_1T_\infty^3}{3k_1}\frac{\partial T}{\partial y}\right)_{y=0}$

$$Nu_x=-\left(\frac{3+4R}{3}\right)\theta^{'}(0)$$

2.1 Numerical Methods for Solution

Eqs. (13) and (14) constitute a highly non-linear coupled boundary value problem of third and second order. So we develop most effective numerical shooting technique with R-K Gill method. To select η_∞ we begin with some initial guess value and solve the problem with some particular set of parameters to obtain $f^{''}(0)$ and $\theta^{'}(0)$. The solution process is replaced with another larger value of η_∞ until two successive values of $f^{''}(0)$ and $\theta^{'}(0)$ differ only desired digit signifying the limit of the boundary along η. The last value of η_∞ is chosen as appropriate value for that particular set of parameters. Eqs. (13) and (14) of third order in f and second order in θ has been reduced to a system of five simultaneous equations of first order for five unknowns following the method of superposition [13]. To solve this system we require initial conditions whilst we have only two initial conditions $f^{'}(0)$ and $f(0)$ on f, one initial condition on θ. Still there are two initial conditions $f^{''}(0)$ and $\theta^{'}(0)$ which are not prescribed. Now, we employ numerical shooting technique where these two ending boundary conditions are utilized to produce two known initial conditions at η = 0. In this calculation, the step size Δη = 0.001 is used while obtaining the numerical solution with η_{max} = 13 and four decimal accuracy as the criterion for convergence.

2.2 Results and Discussion

The system of similarity Eqs.(13) and (14) with boundary conditions are solved numerically by using shooting technique with R-K Gill method similar to that described by Na[13]. In order to get a clear insight of the physical problem, the velocity $f^{'}$ and the temperature θ have been discussed by assigning numerical values to the parameters encountered in the problem. To be realistic, the values of Prandtl number are chosen as 0.71,1,5,10 (especially for air Pr = 0.71 at which represents 20°C and one atmospheric pressure). Due to free convection problem local Grashof number for heat transfer takes value 0.001, 0.002, 0.003 and the permeability parameter takes value as 0.1,0.2,0.3, the ratio of free stream velocity parameter to parallel wall parameter λ = 0.1,0.2,0.3 and the radiation parameter takes value as 1,2,3.

Table 1. Results of $f''(0)$ and $-\theta'(0)$ for various values of Gr (Pr= 0.71, λ = 0.1, AP=0.1, R=0.05, S=0.05)

Gr	$f''(0)$	$-\theta'(0)$
0.001	0.0422	0.0740
0.002	0.0461	0.0701
0.003	0.0499	0.0663

Table 2. Results of $f''(0)$ and $-\theta'(0)$ for various values of Pr (Gr= 0.001, λ = 0.1, AP=0.1, R=0.05, S=0.05)

Pr	$f''(0)$	$-\theta'(0)$
1	0.0421	0.0766
5	0.0417	0.0338
10	0.0416	-0.0635

Table 3. $f''(0)$ and $-\theta'(0)$ for various values of λ (Pr= 0.71, Gr = 0.001, AP=0.1, R=0.05, S=0.05)

λ	$f''(0)$	$-\theta'(0)$
0.1	0.0422	0.0740
0.2	0.1134	0.0633
0.3	0.2059	0.0420

Table 4. $f''(0)$ and $-\theta'(0)$ for various values of AP (Gr=0.001, Pr= 0.71, λ = 0.1, R=0.05, S=0.05)

AP	$f''(0)$	$-\theta'(0)$
0.1	0.0518	0.0584
0.2	0.0607	0.0521
0.3	0.0684	0.0442

Table 5. $f''(0)$ and $-\theta'(0)$ for various values of R (Gr=0.001, Pr= 0.71, λ = 0.1, AP=0.1, S=0.05)

R	$f''(0)$	$-\theta'(0)$
1	0.0684	0.0809
2	0.0684	0.0940
3	0.0684	0.0999

Table 1-5; exhibit the behavior of the skin friction coefficient $f'(0)$ and the local Nusselt number $-\theta'(0)$ for representative values of different controlling parameters. From table1, it is observed that the Grashof number (Gr > 0) tends to increase the local skin friction. The local Nusselt number decreases as the Grashof number increases. From table 2, it is seen that the increase in the value of Prandtl number results in decreases the skin friction as well as the Nusselt number. From table 3, it is observed that the increase in the ratio of free stream velocity parameter to parallel wall parameter increases the skin friction but the Nusselt number decreases. Table 4, predicts that increase in permeability parameter results increase in skin friction but it decreases the Nusselt number. From table 5, it is observed that the radiation parameter has no significant influence on the skin friction whereas the increase in radiation parameter increases the rate of heat transfer.

Figs. 1-2 depict the velocity and the temperature profiles for different values of Buoyancy parameter. It is observed that the velocity component decreases by increasing the buoyancy parameter and also the temperature profile decreases with increase in Buoyancy parameter. Figs. 3-4 depict the velocity and the temperature profiles for different values of Prandtl number. The increase in Prandtl number for this problem has no noticeable effect on the entire velocity profile and thermal boundary layers growth. Increase in values of Prandtl number produces decrease in the temperature of the fluid. From Fig. 5, it is observed that the increase in the ratio of free stream velocity parameter to parallel wall parameter decreases the temperature distribution. Figs. 6 and 7 depict the velocity and the temperature profiles for different values of permeability parameter. The increase in the permeability parameter decreases the fluid velocity and the temperature of the fluid. Figs. 8 and 9 depict the velocity and the temperature profiles for different values of radiation parameter. This parameter has no significant influence on the velocity distribution as well as on the temperature distribution.

2.3 Conclusion

In this paper, we discuss thermal radiation, Buoyancy and heat generation effects on flow and heat transfer over a stretching sheet in porous medium. The set of governing equations and the boundary conditions are reduced to ordinary differential equations with appropriate boundary conditions. Furthermore, the similarity equations are solved numerically by using R-K Gill method along with shooting technique. Effects of Grashof number Gr, Prandtl number Pr, the ratio of free stream velocity parameter to parallel wall parameter λ, permeability parameter AP and the radiation parameter on free convective flow and heat transfer have been examined and discussed in detail. From the present numerical investigation we conclude that:

- The fluid velocity decreased as either of the Grashof number, permeability parameter, ratio of free stream velocity parameter to parallel wall parameter are increased
- The fluid temperature decreased as either the Grashof number, permeability parameter, ratio of free stream velocity parameter to parallel wall parameter are increased
- Skin friction increases owing to an increase in the Grashof number, the ratio of free stream velocity parameter to parallel wall parameter, Prandtl number and the permeability parameter
- Buoyancy parameter, Prandtl number, permeability parameter and ratio of the free stream velocity parameter to wall parameter has significant impact in controlling the rate of heat transfer in the boundary layer.

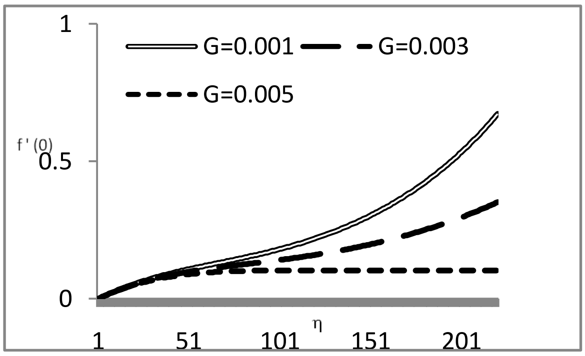

Fig 1. Velocity profile versus η when λ=0.1, AP=0.01, R=0.05, S=0.05

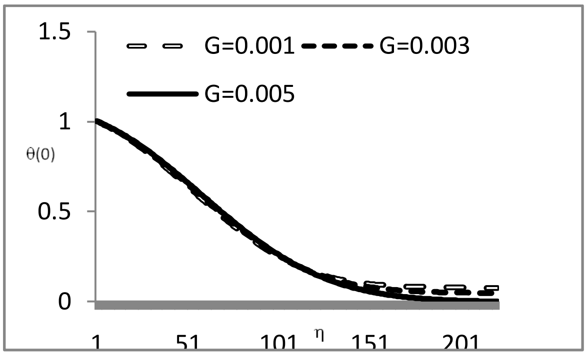

Fig 2. Temperature distribution versus η when λ=0.1, AP=0.01, R=0.05, S=0.05

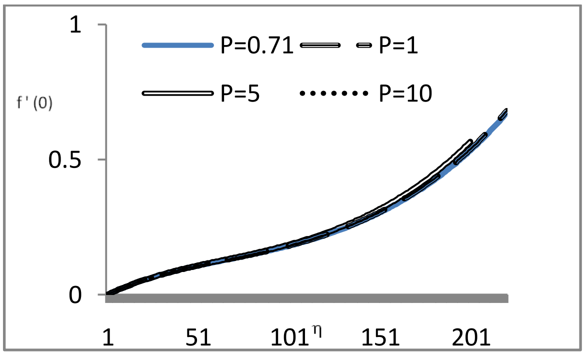

Fig 3. velocity distribution versus η when λ = 0.1, AP=0.01, Gr=0.001, R=0.05, S=0.05

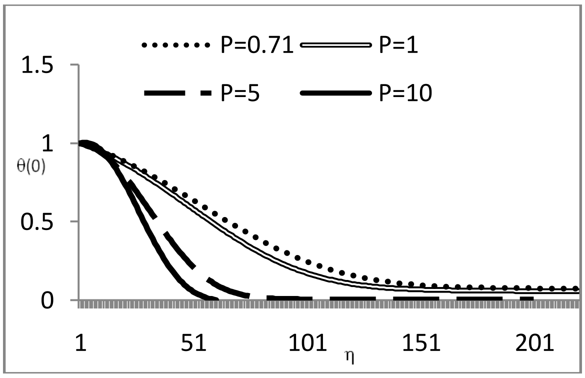

Fig 4. Temperature distribution versus η

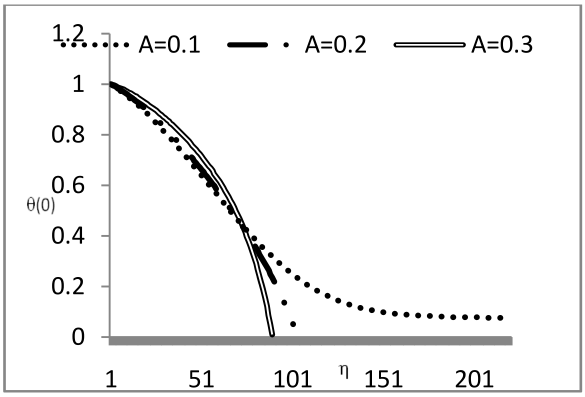

Fig 5. Temperature distribution versus η

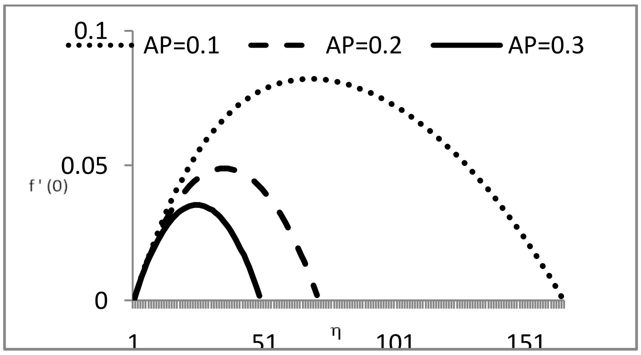

Fig 6. Velocity distribution versus η

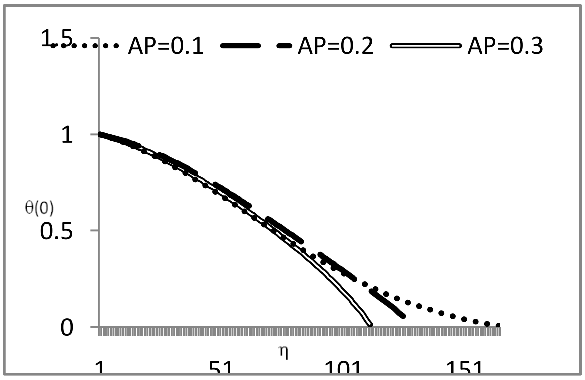

Fig 7. Temperature distribution versus η

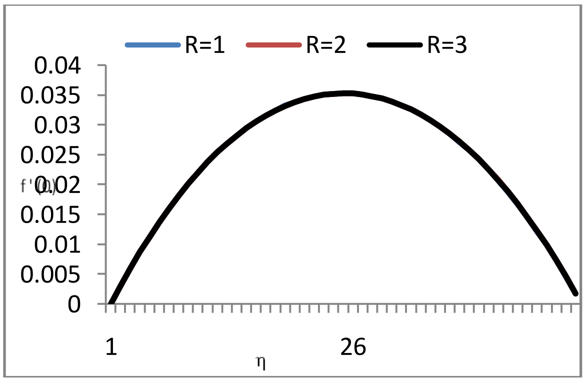

Fig 8. Velocity distribution versus η

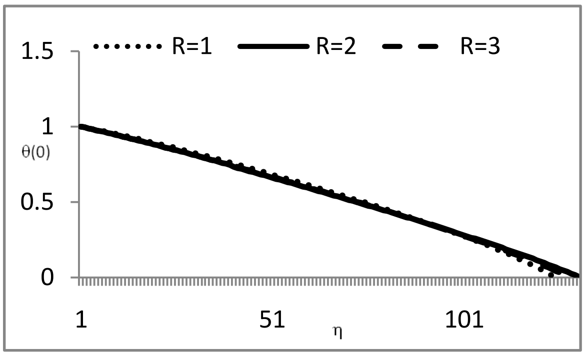

Fig 9. Temperature distribution versus η

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