A Scheduling Based Backlog Reduction Method in CONWIP Production Systems

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ABSTRACT
This article develops a scheduling orientation backlog area reduction methodfor customer production planning in a CONWIP controlled production, considering a make to order (MTO) production system. This method can be applied in order to minimize the backlog area by changing the delivery sequences of orders. Changing in sequences causes changes in some order delivery attributes, such delivery lead time of a customer order that can prevent the tardiness in delivery lead time. This changes must be happened in a special period of time called work ahead window (WAW) to insure the production ability of orders that must be delivered in that period of time. One case study has represented in last section of the paper.

General Terms
Algorithms, Scheduling.

Keywords
Scheduling, CONWIP, Customer order production planning Work-Ahead-Window, Market driver production planning.

1. INTRODUCTION

The market is forcing production towards shorter lead-times to ensure shorter delivery times. The favored production system is a make to order (MTO) system with a production lead time shorter than the customer required delivery time.There are several strategies to decrease the production lead time. Kosonen and Buhanist (1995) [1] for instance the change of a factory into a customer focused lean production system is discussed or Wisner (1995) [2] showed the advantage s of process oriented manufacturing in order to meet customer requirements. The approach developed in this paper is applied to a CONWIP system to minimize the backlog area caused by the lateness in customer order delivery. Framinan et al. (2003) [3] present a good overview of operation, application and comparison of CONWIP presented. Evaluation of a CONWIP system, especially concerning quality, is discussed in Duri et al. (2000) [4]. Chen and Wan (2005) [5] Compared two competing MTO firms concerning capacity and short Delivery times.H. Jodlbauer (2007) [6] presented a method that could be applied in order to determine the WIP cap and the work-ahead-window of a CONWIP controlled production.The main idea of his paper was a focus on a combination of the customer buying behavior with the plant capacity. He considered the model of two viewpoints. First the capacity or production view and then the delivery lead time or customer view and by combination of them, he calculated the control parameters of the CONWIP production policy include the backlog area caused by demand fluctuations.Peter S. Fader & Bruce G.S. Hardie (2009) [7] presented some probability models for customer-base analysis. Forecasting the customer behavior is necessary in a competitive market to get a good familiarity in customer expectations. AlenaAudzeyeva and others (2011) [8] proposed a novel approach to the estimation of Customer Lifetime Value (CLV).Jinquan li and others (2011) [9] investigated how to sequence jobs with fuzzy processing times and predict their due dates on a single machine such that the total weighted possibilistic mean value of the weighted earliness-tardiness costs is minimized. Cheng & Hsiang-Liu(2010) [10] proposed a coordinated scheduling of customer orders (CSCO) system, with the purpose of improving customer order flow time for the order-based production system.ErdalErel& Jay B. Ghosh (2007) [11] presented the algorithms and complexity of Customer order scheduling on a single machine with family setup times.Sheng-Yuan Hsu & C.-H. Liu (2009) [12] worked on improving the delivery efficiency of the customer order scheduling problem in a job shop. The focus of that paper was about customer order scheduling (COS) problem. Guoqing Wang & T.C. Edwin Cheng (2005) [13] considered the problem of scheduling customer orders on multiple facilities to minimize the weighted order completion time. In that paper each customer order consisted of several jobs of different types, which was to be processed on m facilities. Each facility was dedicated to the processing of only one type of jobs and all jobs of an orderhad to be delivered to the customer at the same time. They developed a heuristic to tackle that NP-hard problem.

In this article we will model the customer orders received and constant production ability as a scheduling model and try to minimize the weighted tardiness ($\overline{T}_w$) as an objective function in scheduling problem. Because of the special structure of optimizing the $\overline{T}_w$ problem and its complexity, there were no important advances in solving that objective function so usually it uses the general and heuristic algorithms to solve that problems. many improvements have done to develop some heuristic algorithms in order to optimization of that objective.A. Volgenant (2009) [14] compared the advantages of four heuristic methods include ATD & EDD & WPT & WSPT in computing the total weighted tardiness. Chou (2008) [15] presented an experienced learning genetic algorithm to solve the single machine total weighted tardiness scheduling problem. Wang (2008) [16] developed a population-based variable neighborhood search (PVNS) for the single machine total weighted tardiness problem and compared that with the basic variable neighborhood search (VNS).The focus of our paper is on how changing in delivery sequences of customer orders affects the total weighted tardiness of jobs that is equivalent by the backlogs therefore we use the VNS algorithm to solve our problem because it is a faster and simpler method to solve the problems in comparison with other algorithms such as DP, B&B and others which are methods for optimization. So at first we explain the original customer order production planning model with FCFS or FIFO sequences of delivery in order to allocate the products to customers and then, we try to apply a new sort of sequences for order deliveries, to show that's effects on total weighted tardiness and backlog quantities.

2. MODEL DESCRIPTION

The market is characterized by multi-item customer orders and by fluctuation of the delivery lead time required by the customer.The production environment is multi-level with predetermined sequential routing using a CONWIP pull production system. So we consider some different products are produced in a specific routing and constant work in process. The factory receives the orders from the customers in variant quantity for each final product.to be more specific, we discus s a production with constant production in each period of time and j=1 , … , n final product type and k=1 , … , m machines in job shop. other parameters are defined as:

X_{ij} : number of items of the i th customer order for the product type j

t_{ij} : total time needed for preparation of i th customer order for the product type j

d_{ij} : due date of i th customer order for the product type j

τ_{ij} : order date of i th customer order for the product type j

r_j : production ratio for the product type j

C_j :maximum capacity of system to produce the product type j

To be more comprehended, we assume that we just have one type of products. In this model, the customer orders are the jobs, and because we have a constant ratio for production of each item, we can suppose all of the production system as a single machine to produce that type of products. to be more obvious, we present a simple example to allocate the products to the customer order for a predetermined type of product. to be simpler, we assume the weights equal 1 for all jobs so the objective function alters to minimize the no weighted average of tardiness.

Table 1. Order Quantities During Six Days

Day	1	2	3	4	5	6
Orders	180	10.50	30	40	150,9	110

Suppose the constant production quantity for this type of product equals 80 items per a day and the production chart will be as following:

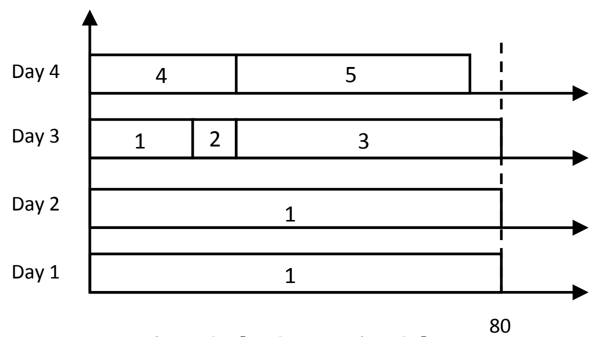

Fig 1: Order Sequencing Scheme

This delivery sequence is in FCFS order, it means the job with previously arrival time is sequenced earlier than others and must deliver sooner to the customer. The results have shown as tardiness and completion times of each job in table2.

Table 2. Calculation of Tardiness In FIFO Sequence

Job	Arrival	Completion	Due Date	Tardiness
1	1	3.25	4	0
2	2	3.375	4	0
3	2	4	3	1
4	3	4.375	4	0.375
5	4	4.875	4	0.875

When tardiness happens, it means we can't deliver the orders on time. So tardiness is equivalent by backlogs. Each changes in sequences of jobs, leads to a change in some tardiness values. So choosing the best sequencing of jobs is essential in order to facing the less tardiness and backlogs.

In real world we can consider the emphasis of all jobs are not equivalent because many customers may have different reflexes in front of lateness. On the other side, satisfaction of permanent customers are very important. Because of the mentioned, we must have different weights for each job that help us to consider some jobs more important of others. So we consider sequencing and scheduling objective function as minimizing total weighted tardiness.

$$Minimize \sum W_j . T_j$$

We can alter the FIFO (FCFS) order to another sequence of jobs that cause some changes in tardiness in order to reduce the total weighted tardiness. we can apply the SPT rule for example considering the arrival date of jobs, to observe the changes in total tardiness. in SPT order, the production chart alters as follow :

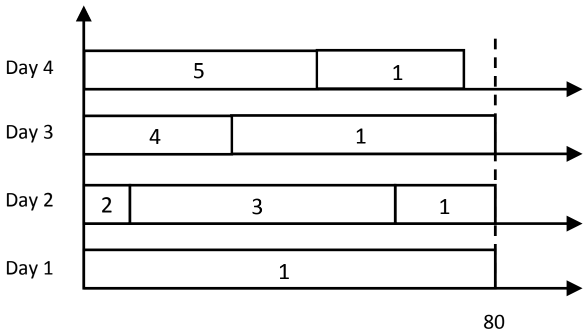

Fig 2: New Order Sequencing Scheme In SPT Sequence

According to that sequence of jobs, the parameters changes as follow:

Table 3. Calculation of Tardiness InSPT Sequence

Job	Arrival	Completion	Due Date	Tardiness
1	1	4.875	4	0.875
2	2	2.125	4	0
3	2	2.75	3	0
4	3	3.375	4	0
5	4	4.5	4	0.5

By this new sort of jobs, we could decrease total tardiness from 2.25 in FIFO sort of jobs to 1.625 in SPT sort of jobs. It equals 38% decreasing in total tardiness.

We can see one difference between this problem and a normal scheduling problem. it is possible to use the rest of capacity each day which doesn't consume to satisfy the orders to allocate another job. So it causes this problem solving to be different with a routine scheduling solving method. On the other hand, we can allocate the products to the order which not produced just to satisfy that order, so we can assume one job (order) even before its order date and upon its arrival date, we can deliver. In other words, we can connivance the order dates (ready times) and in this case we can have the orders without waiting times. Therefore we can sequence the jobs disregarding the ready times and when a job are placed before its order date, we will deliver. This means we can sort the jobs consecutive without any interruption of any jobs. We use the VNS that's a simple general method to solve the problem.VNS is not an optimum method but it can give us an acceptable solution near optimum. Applying the VNS to this example we would have following results.

The best sequence of jobs using VNS results as 3-2-4-5-1.

Table 4. Calculation of Tardiness In Custom Seqeuence

Job	Arrival	Completion	Due Date	Tardiness
1	1	4.875	4	0.875
2	2	1.75	4	0
3	2	1.625	3	0
4	3	2.125	4	0
5	4	2.625	4	0

Total tardiness will be 0.875 in this sort of jobs, that means orders of 2, 3, 4, and 5 will be completed sooner than their arrival. Only job 1 has a few tardiness equals 0.875 day to be satisfied. We could add the weights for each job to be nearer to real, and solve that by mentioned VNS method.

3. PRODUCTION CAPACITY

What is unseen in this problem is our production capacity. in last example we don't mention the emphasis of production capacity because we were sure that we can satisfy all of orders by available constant production. Total order items planned to delivery for four days were 310 whereas we could produce 320 items in that period of times. If we took the orders of 2 last days in to account, we didn't have enough capacity to face the orders. So it is very necessary to take a period of time in which available capacities are bigger or equal to requested products in other words we are looking for a period of time called work-ahead-window (WAW) in which the average of orders during that period must be less than available production capacity per day. Using the past horizon data it is possible to calculate the minimum WAW. We use 'h' as symbol for WAW parameter ; the planning horizon is defined by [0, T [which is divided into T sub time periods [0,1 [, [1,2 [, … ,[T-1, T[. The past horizon is set by [-T1, -T2 [. For practical usage the sub time periods maybe determined one day and the whole planning horizon is 1 month while the past horizon is 1 year. The minimum possible work-ahead-window h which ensures there is no excess capacity can be determined by the statistical behavior of the orders

$$\mu_j(h) = \frac{1}{T}\sum_{t=0}^{T-1}\frac{1}{h}\sum_{\substack{i,j \\ t_{ij}\in[t,t+h]}} x_{ij} \quad \textit{averege demand per subperiod of jth product type} \tag{1}$$

The variance of the demand per sub period of the jth product type will be calculated by the following formula:

$$\sigma_j^2(h) = \frac{1}{T}\sum_{t=0}^{T-1}\left(\frac{1}{h}\sum_{\substack{i,j \\ t_{ij}\in[t,t+h]}} x_{ij} - \mu_j\right)^2 \tag{2}$$

Assuming statistical independence, the equation for the mini mum possible work-ahead-window h is yielded.

$$F_{\mu_j(h),\delta_j{}^2(h)}\left(\bar{C_j}\right) = 0.99 \qquad (3)$$

$F_{\mu_j(h),\delta_j{}^2(h)}$ Statistical distribution function of the capacity needed for product type j.

$\bar{C_j}$ maximum available capacity for product j.

The value 0.99 means that overcapacity is avoided with a probability of 99%. Of course every other value instead of 99% can be used.

Obviously the following relationships hold true:

$$\mu_j(h) = \mu_j(1) = \mu_j \qquad (4)$$

$$\sigma_j{}^2(h) = \frac{\sigma_j{}^2(1)}{\text{h}}$$

The first equation says that the average value does not depend on the work- ahead-window. However, the variance decreases for longer WAW. For the case of a normal distribution E1. (3) can be easily solved by applying (4).

$$\bar{C_j} = \mu_j(1) + \frac{2.33\delta_j(1)}{\sqrt{\text{h}_\text{j}}}$$

$$h_j = \left(\frac{2.33\delta_j(1)}{\bar{C_j} - \mu_j(1)}\right)^2$$

The work-ahead-window h_j ensures that the capacity needed is less than the available capacity. The value 2.33 represents the 99% quartile of the normal distribution. Fig.1 illustrates the required capacity an d the effect of the time average .

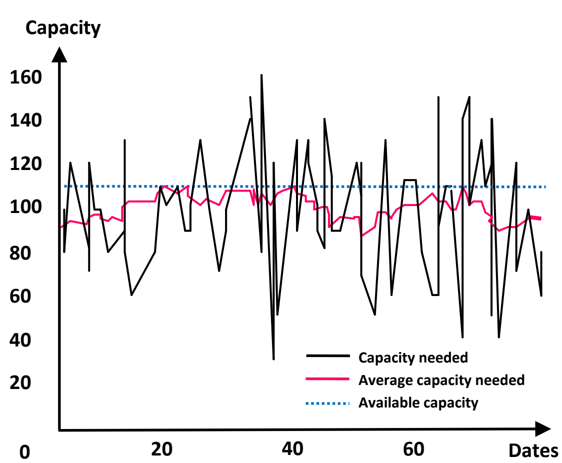

Fig 3: Capacities and Demand Fluctuations

4. COMBINATION OF WAW & JOB ALLOCATIONS

With combination of the production capacity and delivery order sequencing we can have a method that give us insurance about decreasing total weighted tardiness or our backlog area where changing the sequences of jobs doesn't cause any capacity problem. That is we have to categorize the orders in to groups and apply our sequencing in each group. Each group is made by calculating the h and dividing orders to the groups according to their due dates in each period of h.

5. CASE STUDY

Here we used the model of delivery order sequencing developed in this article to a real world study. We evaluated the production plans of TOPCO automotive company and tried to optimize the delivery order sequences by our method. TOPCO Automotive Company is located in southern countryside of Saveh City, Iran , produces Fiat Siena automobile under Fiat's license in Italy. The company's suppliers are Tofas Company in turkey and Fiat Company in Italy. The produced automobiles wouldn't be exported and they are producing usually evenly to cover the received orders from nationwide. We collected the order quantity data during 1 month from 09/22/2008 until 10/21/2008. We ignored the color of requested cars. Factory's sale agencies are in 13 cities include Tehran, Saveh , Isfahan , Shiraz , Qom, Tafresh, Kermanshah, Orumia , Sharhroud , Rasht , Babdar-abbas , Arak , Mashad. All online Purchasing registered orders have added to statistic according to nearest agency. For first step we calculated the work-ahead-window during planning horizon. The optimum value for h obtained equal to 6 days. This means if we change the first sort of orders, there will be no problem in production capacity. After division of all orders into 6 days period of time, we assume the orders independent in each range from other ranges, so in each period we use the VNSalgorithms to find the best delivery sequences of orders.

$$h^* = 6\ day \qquad \&\bar{C} = 24\ cars\ per\ day$$

Table 5.Order Received From The Sale Agencies

day	**Mon**	**Tue**	**Wed**	**Thi**	**Fri**	**Sat**
date	**09/22**	**09/23**	**09/24**	**09/25**	**09/26**	**09/27**
Tehran	12			7		5
Isfahan			6			
Tabriz		5				
Mashhad	6			6		7
Saveh	8	3			7	
Qom			11			6
Arak		6			5	
Bandarabbas						
Rasht				5		
Shahroud	11				5	
Orumia						
Kermanshah						
Tafresh		5				
shiraz		6				
Sum of Demands	**37**	**25**	**23**	**18**	**23**	**18**

There are 20 jobs in first 6 days period of time. If we suppose the weight of jobs equals 1 the consequences of FIFO sort of jobs are as following:

Table 6.Calculation of Tardiness in FIFO Sequencing

FIFO sorting of jobs					
Jobs	**Time**	**Ready T**	**Due T**	**Comp T**	**Tardiness**
J1,1	0.5	1	2	1.50	
J10,1	0.4583333	1	2	1.96	
J5,1	0.3333333	1	2	2.29	**0.29**
J4,1	0.25	1	2	2.54	**0.54**
J5,2	0.125	2	3	2.67	
J13,2	0.2083333	2	3	2.88	
J3,2	0.2083333	2	3	3.08	**0.08**
J14,2	0.25	2	3	3.33	**0.33**
J7,2	0.25	2	3	3.58	**0.58**
J2,3	0.25	3	4	3.83	
J6,3	0.4583333	3	4	4.29	**0.29**
J1,4	0.2916667	4	5	4.58	
J4,4	0.25	4	5	4.83	
J9,4	0.2083333	4	5	5.04	**0.04**
J5,5	0.2916667	5	6	5.33	
J7,5	0.2083333	5	6	5.54	
J10,5	0.2083333	5	6	5.75	
J1,6	0.2083333	6	7	5.96	
J6,6	0.25	6	7	6.21	
J4,6	0.2916667	6	7	6.50	

We can see if we apply the FIFO rule to the jobs it will result 2.17 day as total tardiness in jobs j5,1 , j4,1, j14,2 , j3,2 , j6,3 , j9,4 and j7,2.

By applying the VNS algorithm to solve the problem we could see by only a simple substitution of two jobs j1,1& j4,1, we can have a 11.5% decreasing in total tardiness. Total tardiness in this sort of job is the same as 1.92. During solving the problem by VNS this solution was minimum in total tardiness but it wasn't a unique solution. There were some answers for the problem by the value 1.92 and different sequences. So we can exert a secondary objective function as maximum of tardiness or number of tardy jobs.

Table 7. Calculation of Tardiness In Custom Sequencing

Custom sorting of jobs using VNS					
Jobs	**Time**	**Ready T**	**Due T**	**Comp T**	**Tardiness**
J4,1	0.25	1	2	1.25	
J10,1	0.4583333	1	2	1.71	
J5,1	0.3333333	1	2	2.04	**0.04**
J1,1	0.5	1	2	2.54	**0.54**
J5,2	0.125	2	3	2.67	
J13,2	0.2083333	2	3	2.88	
J3,2	0.2083333	2	3	3.08	**0.08**
J14,2	0.25	2	3	3.33	**0.33**
J7,2	0.25	2	3	3.58	**0.58**
J2,3	0.25	3	4	3.83	
J6,3	0.4583333	3	4	4.29	**0.29**
J1,4	0.2916667	4	5	4.58	
J4,4	0.25	4	5	4.83	
J9,4	0.2083333	4	5	5.04	**0.04**
J5,5	0.2916667	5	6	5.33	
J7,5	0.2083333	5	6	5.54	
J10,5	0.2083333	5	6	5.75	
J1,6	0.2083333	6	7	5.96	
J6,6	0.25	6	7	6.21	
J4,6	0.2916667	6	7	6.50	

Because of constant production speed, any changes in total tardiness lead to variation in backlog quantities. Because we use a general method for solving the problem it is convenient to apply weights for each job.

The results for all periods have illustrated by figure. We can see the effect of FIFO and new custom order delivery sequences. Duration of each period is h days(s).

Table8. Tardiness Changes Caused By Changes in Deviation of Due Dates and Process Times

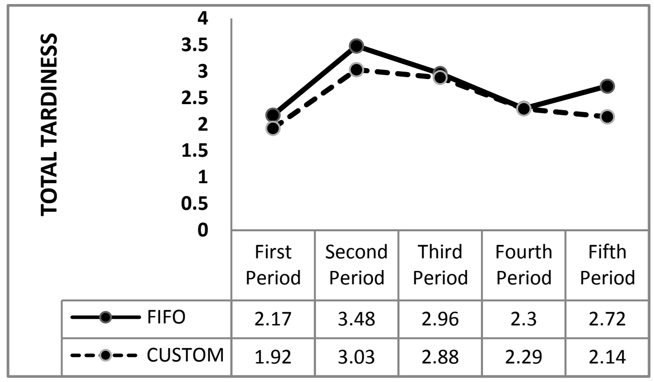

Fig4: Tardiness Comparison between FIFO & Custom Sequence of Jobs

Total tardiness thriftiness yielded by the custom sort of job is equal to 1.37 days that equals 110 items could be backlog but new sequencing of jobs prevented it. This quantity of discount brings the customer satisfaction .its quantity is related to smoothness of data. Whatever fluctuations of data are high; more tardiness changes are anticipated by changing the sequences. So we studied the 36 samples of data with different count of jobs and the same value of due dates and compared their changes in tardiness have originated of their data deviation. We can see when the deviation of process times or the due dates is greater than the previous data, we will face more discount in total tardiness. As we see by increasing the due dates and process time's deviations, total tardiness discounts values will increase. The discount values have shown in fig 2.

Table 9.Standard Deviation of Due Dates

StDev of Process Times	Standard Deviation of Due Dates					
	1.8	**2.6**	**4.1**	**8.3**	**12.8**	**20.4**
0.096	0.11	0.12	0.15	0.18	0.20	0.22
0.150	0.16	0.17	0.21	0.24	0.25	0.26
0.230	0.20	0.21	0.25	0.28	0.29	0.30
0.510	0.30	0.30	0.35	0.37	0.37	0.39
0.720	0.35	0.36	0.40	0.43	0.42	0.45

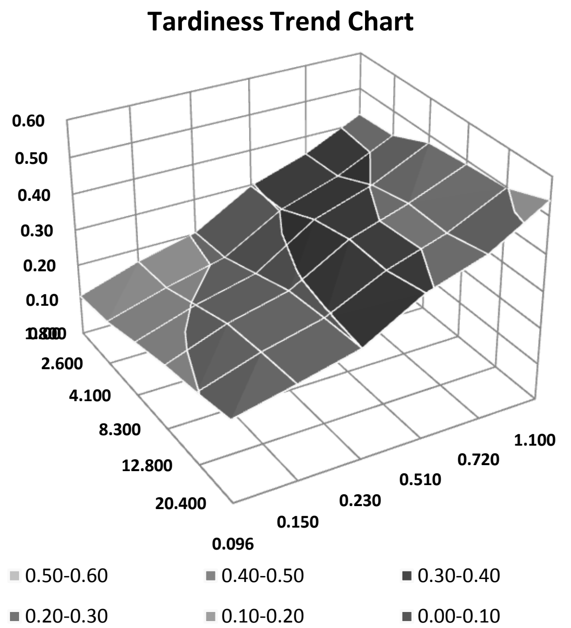

Fig 5: Tardiness Trend Chart

6. CONCLUSION & FUTURE STUDY

Many factories are producing the goods in a constant process manufacturing system and face many groups of order items to be produced and delivered to customers. The available production capacity is limited. A routine strategy in order delivery is FIFO or FCFS order that have a predefined sequence. But maybeit's not a optimum sequence so changing the order delivery may cause the tardiness. It is obvious that many order quantities are so few that it shouldn't be waited after a huge quantity of another order. In this case, another matter that is very important is regarded to the difference between two jobs in their emphasis. So the objective is minimizing the total weighted tardiness in order to minimizing the total backlogs. On other hand the problem complexity in changing the sequences of orders with the total weighted tardiness is NP-hard so it is necessary to use a good heuristic method to solve the problem. Because of the purpose of this article was showing the effect of changing in sequences of orders, on decreasing the total backlog, we used a VNS that's a simpler and faster general method to solve the problem knowing that VNS was not an exact method to give us a global solution. In future, comparison of other heuristic methods to have a more exact solution would be very necessary and testing the simple sequencing rule such SPT or EDD and others to find a good first point in heuristics to have a less iteration to reach the optimum point.

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