Study of g-α-irresolute Functions in the Special Class of Generalized Topological Space

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ABSTRACT
It has been studied that, the fine-irresolute mapping introduced in [13] includes all mappings defined by Császár (cf. [5]) in a fine space which is a special case of generalized topological space. It has been also noted that g-α-irresolute function defined by Bai et al [1] is also included in the same fine-irresolute mapping.

Keywords : Fine open set, g-open set, g-α-open set, g-β-open set, g-preopen set, g-semiopen set, g-α-continuity, fine-continuity, fine space, generalized topological space.

AMS SUBJECT CLASSIFICATION
Primary – 54XX, Secondary- 54CXX.

1. INTRODUCTION
Recently, Bai and Zuo [1] have initiated an interesting class of functions known as g-α-irresolute functions in generalized topological spaces and studied their several properties. Császár [5] (see also [3]) initiated various classes of functions. In particular, (gx, gx')-continuous, (α, gx')-continuous, (σ, gx')-continuous, (β, gx')-continuous functions. Weakly (gx, gx')-continuous and almost (gx, gx')-continuous functions were introduced by Min in [9] (see also, [2], [7], [8], [10], [12]).

The authors have initiated a fine space generated by a given topological space (see [13]). It is very interesting to note that this fine space is a special case of the generalized topological space (cf. [1]). In the present paper, we have investigated that all members of our fine collection are satisfying the conditions of g-semi-open, g-pre-open, g-α-open, g-β-open sets which are defined in [1]. Hence, we conclude that under this special case of generalized topological space all these above stated sets of [5] reduce to single class of fine sets only. In view of aforesaid, we have noticed that the following classes of functions are included in our definition of fine-irresolute mapping which we have defined in [13]:

I. (gx, gx')-continuous
II. (α, gx')-continuous
III. (β, gx')- continuous
IV. (σ, gx')-continuous
V. (p, gx')-continuous
VI. g-α-irresolute

2. PREREQUISITES
We need the following definitions for our study:

2.1 Definition
Let X be a nonempty set and g be a collection of subsets of X. Then g is called a generalized topology on X if $\phi \in g$ and $G_i \in g$ for $i \in I \neq \emptyset$ implies $G = \bigcup_{i\in I} G_i \in g$. We say g is strong if $X \in g$, and we call the pair (X, g) a generalized topological space on X (cf. [3]).

2.2 Remark
The elements of g are called g -open sets and the complements are called g- closed sets.

2.3 Definition
Let $A \subseteq X$, then the generalized closure of A is defined by the intersection of all g- closed sets containing A and is denoted by $c_g(A)$ (cf. [3]) .

2.4 Definition
Let $A \subseteq X$, then the generalized interior of A is defined by the union of all g-open sets contained in A and is denoted by $i_g(A)$ (cf. [3]). Then, we have

$$i_g(A) = C\text{-}c_g(X\text{-}A) \text{ and } c_g(A) = X\text{-}i_g(X\text{-}A) .$$

2.5 Definition
Let (X, gx) be a generalized topological space and $A \subseteq X$. Then, A is said to be

(1.) g-semi-open [4] if $A \subseteq c_g(i_g(A))$.

(2.) g-pre-open [4] if $A \subseteq i_g(c_g(A))$.

(3.) g-α-open [4] if $A \subseteq i_g(c_g(i_g(A)))$.

(4.) g-β-open [4] if $A \subseteq c_g(i_g(c_g(A)))$.

The complement of a g-semi-open (resp. g-pre-open, g-α-open, g-β-open) set is called g- semi-closed (resp. g- pre-closed, g-α-closed, g-β-closed) set. We denote by σ(g) (resp. p(g), α(g), β(g)) the class of all g-semi-open sets (resp. g-pre-open, g-α-open, g-β-open). It may be noted that $g \subseteq \alpha(g) \subseteq \sigma(g) \subseteq \beta(g)$ and $\alpha(g) \subseteq p(g) \subseteq \beta(g)$.

2.6 Definition
Let (X, gx) and (X', gx') be generalized topological spaces. Then a function $f: X \rightarrow X'$ is said to be (gx, gx')-continuous if $f^{-1}(V)$ is a g-open set in X for every g-open set V in X' (cf. [3]).

2.7 Definition

Let (X, gx) and (X', gx') be generalized topological spaces. Then a function f:X→ X' (cf. [5]) is said to be

1. (α, gx')- continuous if $f^{-1}(V)$ is g- α- open in X for every g-open set V in X'.
2. (σ, gx')-continuous if $f^{-1}(V)$ is g-semi-open in X for every g-open set V in X'.
3. (p, gx')-continuous if $f^{-1}(V)$ is g-pre- open in X for every g-open set V in X'.
4. (β, gx')-continuous if $f^{-1}(V)$ is g-β-open in X for every g-open set V in X'.

2.8 Definition

Let (X, gx) and (X', gx') be generalized topological spaces. Then a function f:X→ X' is said to be **g-α-irresolute** if $f^{-1}(V)$ is g-α-open in X for every g-α-open set V of X' (cf. [1]).

2.9 Definition

Let (X, gx) and (X', gx') be generalized topological spaces. Then a function f:X→ X' is said to be weakly (gx, gx')-continuous if for each x∈ X and each g-open set V containing f(x) , there exists a g-open set U containing x such that $f(U)\subseteq c_g(V)$ (cf. [10]).

3. FINE SPACE

In this section, we define a fine space which is generated with the help of the given topological space. We also define the fine-irresolute map.

3.1 Definition

Let (X, τ) be a topological space, we define

$$\tau(A_\alpha) = \tau_\alpha \text{ (say)} = \{G_\alpha (\neq X): G_\alpha \cap A_\alpha \neq \emptyset, \text{ for } A_\alpha \in \tau \text{ and } A_\alpha \neq \emptyset, X, \text{ for some } \alpha \in J, \text{ where J is the index set.}\}$$

Now, we define

$$\tau_f = \{\emptyset, X, \cup_{\alpha \in J}\{\tau_\alpha\}\}$$

The above collection τ_f of subsets of X is called the fine-collection of subsets of X and (X, τ, τ_f) is said to be the fine-space of X , generated by the topology τ on X .

3.2 Example

Let X = {a, b, c} be a topological space with the topology τ = { ∅, X, {a}, {b}, {a, b}} ≅ {∅, X, A_α, A_β, A_γ} (say) where, A_α = {a}, A_β= {b}, A_γ= {a, b}. In view of Definition 3.1, we have

$$\tau_\alpha = \tau(\{a\}) = \{\{a\}, \{a, b\}, \{a, c\}\},$$

$$\tau_\beta = \tau(\{b\}) = \{\{b\}, \{a, b\}, \{b, c\}\},$$

$$\tau_\gamma = \tau(\{a, b\}) = \{\{a\}, \{b\}, \{a, b\}, \{a, c\}, \{b, c\}\}$$

then, the fine-collection is

$$\tau_f = \{\emptyset, X, \cup \tau_\alpha\} = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{b, c\}\}$$

3.3 Definition

A subset U of a fine space X is said to be a fine-open set of X, if U belongs to the collection τ_f and the complement of every fine-open set of X is called the fine-closed set of X and we denote the collection by F_f.

3.4 Example

Let X = {a, b, c} be a topological space with the topology τ = { ∅, X, {a }, {b}, {a,b}} then, the collection of fine-open sets τ_f is given by

$$\tau_f = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{b, c\}\}$$

and the collection of fine-closed sets is given by

$$F_f = \{\emptyset, X, \{b, c\}, \{a, c\}, \{c\}, \{b\}, \{a\}\}$$

3.5 Definition

A function f : $(X, \tau, \tau_f) \to (Y, \tau', \tau'_f)$ is called fine-irresolute (or f-irresolute) if $f^{-1}(V)$ is fine-open in X for every fine-open set V of Y .

3.6 Example

Let X= {a, b, c} with the topology τ = {∅, X, {a}, {a, b}}, τ_f = {∅, X, {a}, {a, b}, {a, c}, {b}, {b, c}} and Y= {1, 2, 3} with the topology τ' = {∅, Y, {1}, {2}, {1, 2}}, τ'_f = { ∅, Y, {1},{1, 2}, {1,3}, {2}, {2, 3}}.

We define a mapping f: X → Y such that f(a) = 1, f(b) = 2, and f(c) = 3.

- It may be verified that the function f is not a continuous function.
- It may be checked that the pre-images of fine-open sets of Y viz. {1}, {1, 2}, {1, 3}, {2}, {2, 3} are {a}, {a, b}, {a, c}, {b}, {b, c} respectively, which are fine-open in X. Therefore, f is fine-irresolute.

4. FINE SPACE AS A SPECIAL CASE OF GENERALIZED TOPOLOGICAL SPACE

In this section, we study certain properties of fine space and able to conclude that it is a special case of generalized topological space.

4.1 Theorem

Let (X, τ, τ_f) be the fine space then, arbitrary union of fine-open sets in X is fine open set in X.

Proof

Let $\{G_\alpha\}_{\alpha \in J}$ be the collection of fine-open sets of X

⇒ $G_\alpha \cap A_\alpha \neq \emptyset$, ∀ α ∈ J and $A_\alpha \neq (\emptyset, X)$ in τ.

Claim: ⇒ $\cup_{\alpha \in J} G_\alpha$ = G is fine-open.

It is enough if we show that $G \cap A_\beta \neq \emptyset$ for $A_\beta \neq (\emptyset, X)$ in τ. Now,

$$(\cup_{\alpha \in J} G_\alpha \cap A_\beta) = (G_\alpha \cap A_\beta) \cup (G_\beta \cap A_\beta) \cup \ldots$$

⇒ there exists an index β ∈ J such that $G_\beta \cap A_\beta \neq \emptyset$ (since, $G_\beta \in \tau_f$). Hence, $(\cup G_\alpha) \cap A_\beta \neq \emptyset$

⇒ G is fine-open.

4.2 Remark

The intersection of two fine-open sets need not be a fine-open set as the following example shows. **Thus, the collection of fine-open sets in a space X do not form a topology. Hence, it is not a topological space but it is a generalized topological space.**

4.3 Example

Let X = {a, b, c} be a topological space with the topology τ = {∅, X, {a}, {b}, {a, b}} then,

τ_f = {∅, X, {a}, {b}, {a, b}, {a, c}, {b, c}}

We can easily check that the above collection τ_f is not a topology since, {a, c}∩{b, c } = {c } which is not a member of τ_f. But τ_f is a generalized topological space (cf. Definition 2.1.).

4.4 Remark

- It is interesting to note that the fine space is the special class of generalized topological space, since the union of fine-open set is fine-open, as above Theorem 4.1. shows.
- It may be noted that each fine space is associated with one topological space where as the generalized topological space is not associated with any topological space in general.

5. PROPERTIES OF g-SEMI OPEN, g-PRE-OPEN, g-α-OPEN, g-β-OPEN SETS IN FINE SPACE

5.1 Theorem

Let (X, τ, τ_f) be a fine space and A be any arbitrary subset of X then, $c_g(A) \subseteq cl(A)$.

Proof

Let $A \subseteq X$. Consider, $cl(A) = F_\alpha \in F$ where, F is the family of closed subsets of X. Then, there may exist $F_{\alpha f} \in F_f$ (since, $F \subseteq F_f$) such that $A \subseteq F_{\alpha f} \subseteq F_\alpha \Rightarrow F_{\alpha f} = c_g(A) \subseteq cl(A)= F_\alpha$. Hence, $c_g(A) \subseteq cl(A)$.

5.2 Example

Let X = {a, b, c} with the topology τ = {∅, X, {a}, {a, b}}, F = {∅, X, {b, c}, {c } }, and τ_f = {∅, X, {a}, {a, b}, {a, c}, {b}, {b, c}}, F_f = {∅, X, {b, c}, {c}, {b}, {a, c}, {a}}.

Let A= {a, c}, consider cl(A) = X and $c_g(A)$ = {a, c }. Hence, $c_g(A) \subset cl(A)$.

5.3 Remark

Let (X, τ, τ_f) be a fine space and A be any arbitrary subset of X . Then, it may be observed that $Int(A) \subseteq i_g(A)$.

5.4 Theorem

Let (X, τ, τ_f) be the fine space with respect to the topological space (X, τ). Let $F \subset X$, if $F \notin \tau_f$ then, **F is not**

(i) g- semi-open and g-α-open.

(ii) g-pre-open and g-β-open.

Proof

(i) Consider, $F \subset X$ and $F \notin \tau_f$. We shall show that F is not g-semi-open and not g-α-open. By the definition of τ_f, we have

$$F \cap A_\alpha = \emptyset \; \forall A_\alpha \; (\neq \emptyset, X) \in \tau \qquad (5.1.)$$

Claim: $i_g(F) = \emptyset$

Let if possible $i_g(F) = A(say) \subseteq F$. By the definition of generalized interior i_g, $A \in \tau_f$.

Again, applying Definition of τ_f $\exists$ some $A_\beta \in \tau$ such that

$$A \cap A_\beta \neq \emptyset \text{ for some } \beta \in J \qquad (5.2.)$$

Since, (5.1) holds for all $A_\alpha \in J$ hence, (5.1) holds for A_β also.

$$A \cap A_\beta = \emptyset \text{ for } A_\beta \in \tau \text{ (since, } A \subseteq F) \qquad (5.3.)$$

Since, (5.2) is a contradiction to the hypothesis (5.3), hence $A \notin \tau_f$.

Since, $F \neq X \Rightarrow i_g(F) \neq X$. Thus, finally $i_g(F) = \emptyset$ which implies $c_g(i_g(F)) = \emptyset \Rightarrow F \nsubseteq c_g(i_g(F))$. Hence, F is not g-semi-open and hence we may also conclude that $F \nsubseteq i_g(c_g(i_g(F)))$ and therefore F is not g- α-open.

(ii) Consider, $F \subset X$ and $F \notin \tau_f$. We shall show that F is not g- pre-open and not g-β-open in X. It is given that $F \notin \tau_f$. By the Definition of τ_f, we have

$$F \cap A_\alpha = \emptyset \; \forall A_\alpha \; (\neq \emptyset, X) \text{ in } \tau \text{ and } \alpha \in J \qquad (5.4)$$

$\Rightarrow F \subset C(A_\alpha)$ (where C stands for the complement).

By Theorem 5.1 and the Definition of the closure in topological space, we have

$$F \subseteq c_g(F) \subseteq cl(F) \subseteq C(A_\alpha)$$

Since,

$$A_\alpha \cap C(A_\alpha) = \emptyset \Rightarrow c_g(F) \cap A_\alpha = \emptyset \qquad (5.5)$$

We know,

$$i_g(c_g(F) \; (=A, \text{ say}) \subseteq c_g(F)$$

By using (5.5), we have

$$A \cap A_\alpha = \emptyset \; \forall A_\alpha \text{ in } \tau, \alpha \in J \qquad (5.6)$$

Claim: $i_g(c_g(F)) = \emptyset$

Let if possible $i_g(c_g(F)(=A) \in \tau_f$, there exists some $A_\beta \in \tau$ such that

$$A \cap A_\beta \neq \emptyset \qquad (5.7)$$

By hypothesis, $A \cap A_\beta = \emptyset$ holds for some $\beta \in J$ (cf. relation (5.6)).

Since, (5.7) is a contradiction to the hypothesis (5.6), hence,

$$A \cap A_\beta = \emptyset \Rightarrow A \notin \tau_f$$

Thus, $i_g(c_g(F)) = \emptyset \Rightarrow F \nsubseteq i_g(c_g(F))$, hence F is not g- pre-open and we may further conclude that $F \nsubseteq c_g(i_g(c_g(F)))$. Hence, F is not g-β-open.

5.5 Example

Let X = {a, b, c} be a topological space with the topology τ = { ∅, X, {a}} then, the fine-collection is

τ_f = {∅, X, {a}, {a, b}, {a, c}}

F_f = {∅, X, {b, c}, {c}, {b}}

We can easily check that, the sets {b, c}, {c}, {b} are not the member of τ_f, they are not g- α-open, g-β-open, g-semi-open and g-pre-open, but they are fine-closed sets.

5.6 Theorem

In the fine space each fine-open set is

(i) g-semi-open (ii) g-α-open

(iii) g-pre-open (iv) g-β-open

Proof

Let (X, τ, τ_f) be the fine space. Let $S \subset X$ and S in τ_f.

Case 1. If S is both g-open and g-closed then, $i_g(S) = c_g(S) = S$ and hence (i), (ii), (iii) and (iv) follows directly.

Case 2. $S \in \tau_f$ but S is not both g-open and g-closed.

(i) We first show that S is g-semi-open.

Since, $S \in \tau_f$, we have

$$S = i_g(S) \tag{5.8}$$

By the Definition 2.2, we have

$$S \subseteq c_g(S) \tag{5.9}$$

Using (5.8) and (5.9), we get

$$S \subseteq c_g(i_g(S)) \tag{5.10}$$

Thus, S is g-semi-open.

(ii) We next show that S is g-α-open.

It is well known that $A \subseteq B \Rightarrow i_g(A) \subseteq i_g(B)$. Applying this property of interior in (5.10) , we get

$$i_g(S) \subseteq i_g(c_g(i_g(S))) \tag{5.11}$$

Thus, S is g-α-open, when we appeal to (5.8).

(iii) We show that S is g-pre-open.

Since, $S \in \tau_f$, by using (5.8) and (5.9), we get

$$S = i_g(S) \subseteq i_g(c_g(S)) \tag{5.12}$$

Thus, S is g-pre-open.

(iv) We next show that S is g-β-open.

We know that if $A \subseteq B \Rightarrow c_g(A) \subseteq c_g(B)$. Applying this property of closure in (5.12), we have

$$c_g(S) \subseteq c_g(i_g(c_g(S))) \tag{5.13}$$

From (5.9) and (5.13) we have,

$$S \subseteq c_g(i_g(c_g(S))) \tag{5.14}$$

Thus, S is g-β-open.

Hence, we finally conclude that every $S \in \tau_f$ is g-semi-open, g-α-open, g-pre-open and g-β-open.

5.7 Example

Let X = {a, b, c} be a topological space with the topology $\tau = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}$ then,

$$\tau_f = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{b, c\}\},$$

$$F_f = \{\emptyset, X, \{b, c\}, \{a, c\}, \{c\}, \{b\}, \{a\}\}.$$

Let A = {a}, consider $i_g(c_g(i_g(A))) = \{a\} \Rightarrow A$ is g-α-open. Again, consider $c_g(i_g(c_g(A))) = \{a\} \Rightarrow A$ is g-β-open. We may easily check that A is g-pre-open, g-semi-open. Similarly we may check that all fine-open sets are satisfying the conditions of g-semi-open, g- pre-open, g-α-open, and g-β-open sets.

5.8 Theorem

Let $f:(X, \tau, \tau_f) \rightarrow (Y, \tau', \tau'_f)$ be the fine-irresolute mapping. Then, it is (gx, gx')- continuous, (α, gx')-continuous, (σ, gx')-continuous, (p, gx')-continuous, (s, gx')-continuous and g- α-irresolute mapping.

Proof

In view of Theorem 5.4 and 5.6 we conclude that in case of fine space which is a special case of generalized topological space each member of the collection τ_f is g-semi-open, g-pre-open, g-α-open, g-β-open set and those sets which are not in τ_f are not g-semi-open, g-pre-open, g-α-open, g-β-open sets.

We notice that all special sets defined in the Definition 2.5 are the fine open sets only. Hence, all types of continuities specified in Definitions 2.6, 2.7 and 2.8 reduce to the single continuity viz. fine-irresolute map.

5.9 Remark

We may summarized our Theorem 5.9 as follows :

	⇒ (gx, gx')-continuous
	⇒ (α, gx')-continuous
Fine-irresolute	⇒ (β, gx')-continuous
	⇒ (σ, gx')-continuous
	⇒ (p, gx')-continuous
	⇒ g- α-irresolute

5.10 Example

We construct an example of a function which is fine-irresolute and therefore it is (gx, gx')-continuous, (α, gx')- continuous, (σ, gx')-continuous, (p, gx')-continuous, (s, gx')-continuous and g-α-irresolute.

Let X = {a, b, c} with the topology $\tau = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}$, $\tau_f = \{\emptyset, X, \{a\}, \{a, b\}, \{a, c\}, \{b\}, \{b, c\}\}$ and Y = {1, 2, 3} with the topology $\tau' = \{\emptyset, Y, \{1, 2\}\}$, $\tau'_f = \{\emptyset, Y, \{1\}, \{1, 2\}, \{1, 3\}, \{3\}, \{2, 3\}\}$.

We define a mapping $f: X \rightarrow Y$ such that f(a) = 1, f(b) = 2, and f(c)= 3.

- It may be checked that pre-images of fine-open sets {1}, {1, 2}, {1, 3}, {3}, {2, 3} of Y are fine-open in X , therefore f is fine-irresolute.
- It may be checked that pre-images of g-open sets {1}, {1, 2}, {1, 3}, {3}, {2, 3} of Y are g-open in X, therefore f is (gX, gx')-continuous.
- It may be seen that for the given g-open sets {1}, {1, 2}, {1, 3}, {3}, {2, 3} of Y, their respective pre-images {a}, {a, b}, {a, c}, {b}, {b, c} are g-α-open in X. Therefore, f is (α, gx')-continuous.
- It may be checked that g-open sets {1}, {1, 2}, {1, 3}, {3}, {2, 3} of Y, their respective pre-images {a}, {a, b}, {a, c}, {b}, {b, c} are g-β-open in X. Therefore, f is (β, gx')-continuous.
- It may be checked that g-open sets {1}, {1, 2}, {1, 3}, {3}, {2, 3} of Y, their respective pre-images {a}, {a, b}, {a, c}, {b}, {b, c} are g-semi-open in X. Therefore, f is (σ, gx')- continuous.
- It may be checked that g-open sets {1}, {1, 2}, {1, 3}, {3}, {2, 3} of Y, their respective pre-images {a}, {a, b}, {a, c}, {b}, {b, c} are g-pre-open in X. Therefore, f is (p, gx')-continuous.
- It may be checked that g-α-open sets {1}, {1, 2}, {1, 3}, {3}, {2, 3} of Y, their respective pre-images {a}, {a, b}, {a, c}, {b}, {b, c} are g-α-open in X. Therefore, f is g- α-irresolute.

5.11 Note

It may be easily checked that no set other than the member of τ_f is g-semi-open, g- pre-open, g-α-open, g-β-open.

5.12 Remark

If f:(X, τ, τ_f) → (Y, τ', τ'_f) is fine-irresolute mapping then, it is weakly (gx, gx')-continuous (see Proposition 3.8 of [1]). By using the concept of fine-irresolute mapping, we have defined fine-irresolute homeomorphism in [13] (see also [6]).

6. CONCLUSION

The concept of fine-irresolute mapping has been studied in the special class of generalized topological space. This concept may have an extensive applicational values in quantum physics (cf. [11]).

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