Production Planning for Perishable Products with Partial Postponement Strategy

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ABSTRACT
This work addressed the tactical production planning for perishable products with partial postponement strategy in which the cost is minimized subject to a set of constraints such as labor working time and machine time. For a specific final configuration some portion of demand has less demand fluctuation and we can use make-to-stock strategy to respond demand. But for other portion of demand which has high degree of uncertainty, we use concept of postponement and it will be beneficial. We compute the level of postponement capacity and finished goods inventory by developing a mixed integer nonlinear programming model. We have analyzed the behavior of the model to make managerial insight under different scenarios of fill rates and finished-goods inventory costs. Our finding shows the finished goods inventory and fill rates can decrease the expected total cost and increases postponement capacity.

General Terms
Production

Keywords
Production management, mass customization, partial postponement, perishable

1. INTRODUCTION
In this age of globalization, rapid technological advancement, increased economic uncertainty and demanding customers lead to short product cycle and shorter lead times which require quick response to highly customized customers' needs, better product and service quality. Postponement strategy is considered an effective way to responding environmental uncertainty and achieving mass customization. With today's market characteristics uncertain nature should be viewed as an opportunity and not a problem. In general, postponement is a strategy which is used to determine the efficient manner to make products and services available to end user. Postponement, also known as end of line configuration, late customization or delayed product differentiation, is now defined as an organizational concept whereby changing, timing and/or placing some of the processes in the supply chain (such as purchasing, designing, manufacturing, production, packaging, labeling and marketing) would take place after some key information about the customer's specific needs or requirements is revealed. The goal of postponement is to supply desirable products to customers at a relatively low cost in a responsive way. In this paper we develop a general tactical production planning model for perishable products that have significant setup costs.

Perishable products regarded products that cannot be stored for a long time because they rot or can no longer be used. Another type of perishable products are computers and mobile phones which sale volumes drop dramatically and their value decreases quickly with the lapse of time due to constant improvement of technology. Seasonal products like high fashion apparel, Christmas gifts and calendars are another type of these products. These are sold only below full price after a day or a season. In order to deal with the production planning under limited resources for perishable products we adopt partial postponement strategy. Postponement in production refers to generic intermediate products being manufacture in early stage and according to differentiation option final activity such as packaging; final assembly is postponed to later until customer order information received. Partial postponement means to stock products both in generic form and customized form so that customers demand satisfied either from customized goods inventory or generic goods inventory. Partial postponement is a special form of form postponement. It has been proved it's not beneficial from any amount of postponement from service level-inventory trade off perspective [1]. Partial postponement uses effectively some portion of the demand for product which has less demand fluctuation and is known with certainty. It is indicate less expensive make-to-stock sequence of operation would be used to produce this portion of finished-goods inventory.

This work mainly motivated by dairy production companies that products different flavored dairy production and dependent products demand such as yoghurt. This paper organized as follow. In section 2 related literatures are introduced. After that problem description and model formulation is presented. In section 4 we provide numerical illustration. In section 5 sensitive analyses of fill rates and finished-goods holding cost are presented. Section 5 contains a summary, conclusion and futures works.

2. LITERATURE
In general, cost models for analyzing postponement strategies can be classified into four categories. They are *descriptive models*, *deterministic models*, *stochastic models* and *heuristic models*. According to Hiller and Lieberman [2] stochastic models is defined as an inventory model where demand in any period is random. Many authors describe various approaches to handle uncertainty in their model. This train starts with landmark work of Zinn and Bowersox [3] by using a normative cost model and discriminate analysis. They concluded product value is the most important variables that justify postponement. Iyer et al., [4] developed a two-stage capacity planning problem

model under demand postponement. The main idea behind the model is that by preempting stockouts through demand postponement we reduce the total cost. And this is achievable by postponing a fraction of the demand in a period to permit capacity to be procured to fulfill demands later. Their approach is to formulate a two-stage stochastic dynamic program with updating of demand information. Recent stochastic postponement Bish et al., [5] study the advantages of two perishable indivisible and flexibility resources structure under demand forecast uncertainty and demand variability in a firm that to satisfy two demand types. The main goal was to delay resource allocation decision until more information on customer demand is achieved Al-Salim and Choobineh [6] propose two nonlinear binary optimization models for determining the optimal stage for differentiating multi-product production system. Their two developed models maximize the expected value of profit and the value of options to postpone product differentiation respectively. Their results show the profit model recommends a later product differentiation than the option model. If coefficient of variation of product is low demand correlation is not important and negative correlation among product demand is better than positive correlation. Recent work, Song and Kusiak [7] present a general framework for finding optimal modules in a delayed product differentiation scenario. The main goal was to minimize the mean number of assembly operation and pre-assembly cost. Anupindi and Jiang [8] presents a duopoly model where firms' decisions on capacity, production, and price under demand uncertainty are made in a three-stage decision making framework. In their model the firm always decides about capacity before and price after the demand realization, while there is a difference in the timing of the production decision. Their study show that the optimal capacity of the flexible resource is increasing in demand variability and that the value of flexibility is mainly significant if the demand levels are highly variable. Graman and Sanders [9] modeled the tradeoff between postponement capacity and forecast accuracy. Their funding shows postponement capacity and forecast accuracy resulting in reduced inventory levels while maintaining a constant service level. Their study uses inventory reduction as the only benefit of improving forecast accuracy and increasing postponement capacity. The value of a product is a factor sets limits on the benefit that can be achieved by increasing postponement capacity and improving forecast accuracy. Bish et al., [10] in their paper study the effect of two main driver product, substitution and operational postponement, on the firms optimized capacity decision in a monopolistic setting. They consider price and quantity (PQ) and price only as a two type of postponement strategy. They formulate both postponement strategies as two-stage stochastic programming problems Tong [11] presents a two-stage supply chain model with a single new product manufacturer and a single retailer. The manufacturer organizes his production after taking orders from the retailer. He show for products featuring an exponential demand with Gamma prior, if the demand forecast improves over time and the manufacturer sets the wholesale price, the expected order size increases when the order is placed later.

Recently partial postponement has become an increasingly attractive research subject. Swaminathan and Tayur [12] analyze a final assembly process with production capacity constraint where inventory is stored in the intermediate form called vanilla boxes. This model compares performance of both assemble to order (where components are stocked and products assembled from the components after demand is realized) and make-to-stock (where inventory is carried in finished form only) semi-finished assembly process. Their approach allows for multiple points of differentiation. They develop a stochastic integer program to determine the optimal types of vanilla boxes as well as their inventory levels which minimize the expected holding and penalty costs in single and multi-period settings. A single-period, multi-product, capacitated inventory model considered by Graman and Magazine [13] where inventory can be stored in an intermediate form. To satisfy the demand subjected to a capacity constraint the finished goods are used first and then the semi-finished product. They used an inventory- service level approach without costs to determine the levels of finished goods inventory for a specified level of capacity. Their approach minimizes total inventory subject to meeting a specified service level rather than considering shortage costs. Chopra and Mindl [14] described a strategy of producing the amount of products that is very likely to sell using the lower cost production. He called that tailored postponement. Silver and Minner [15] developed a partial postponement strategy for a fast food industry using a newsvendor-style approach. They help a pizza shop determine the quantities of each type of finished and unfinished pizzas. Graman [1] developed a nonlinear programming including a single-period, two product, order-up-cost model to aid in setting the levels of finished goods inventory and postponement capacity. He developed previous work of Graman and Magazine [13] by incorporating a decision cost model to setting the levels of inventory. He found out the expected total cost decreases and postponement capacity increases as the value of the postponement common item and the cost of postponement decrease also the cost of packaging becomes greater than the assembly cost.

Recently in production planning literature using postponement strategy we can refer to Leung and Ng [16] which define a two-stage stochastic model for determining the production loading planning for perishable products in particularly holiday-themed toys. The main deference between this work and our paper is that we use a continuous demand distribution while leung and Ng use a demand under four different scenarios in each period. They found out with postponement strategy a saving about 7% in total cost (including operational cost, inventory cost, and hiring cost and lay-off cost) is made in comparison with nonpostponement strategy. In another work they also [17] use a preemptive goal programming model to deal with the aggregate production planning problem for that firm. Demand for different period is considered Deterministic and time-sensitive. Three major objectives with target values are optimized hierarchically. Their finding indicate in order to meet the dramatically increase in demand some semi-finished products recommended to be produced in the earlier planning horizon.

Previous work by Graman [1] provides a basis for our integration partial postponement strategy into tactical production planning while we have limited resources and significant setup cost.

3. PROBLEM DESCRIPTION AND MODEL DEVELOPMENT

In this study the tactical production planning problem for perishable products is investigated. We developed single period, two product order up-to partial postponement under limited resources. In every period we determine how many finished products (direct production), generic products (master production) and transfer product (final activity) should be produce at the beginning of the period so that use better utilization of limited resources. Production starts from raw material to produce generic products. After generic products production some portion of generic product is not postponed and immediately final activity is carried out and stored as a finished goods inventory. Remain of generic products held as semi-finished goods to satisfy demand by combination with finished goods inventory. Demand in excess of the non-postponed inventory is met if possible through doing final activity of the generic inventory. Without loss of generality and better understanding of the model we assume one unit of generic item makes one unit of finished product.

3.1 Problem formulation

Notation:

Parameters:

$C_{SF,i}$ setup cost for production of direct finished product i from raw material

$C_{SS,i}$ setup cost for production of generic product i

$C_{ST,i}$ setup cost for production of finished product i from generic product

C_W labour cost

$C_{RF,i}$ regular-time unit production cost to produce one unit of direct finished product i

$C_{RS,i}$ regular-time unit production cost to produce one unit of common item from raw material

$C_{RT,i}$ regular-time unit production cost to produce one unit of finished product i from semi-finished products

$C_{OF,i}$ overtime unit production cost to produce one unit of finished product i from raw materials

$C_{OS,i}$ overtime unit production cost to produce one unit of common item from raw materials

$C_{OT,i}$ overtime unit production cost to produce one unit of finished product i from common item

H_p inventory holding cost for one unit of common item

H_{pp} the cost of postponement materials and activities for one unit of the common item

$H_{F,i}$ inventory holding cost for one unit of finished product i.

λ^W fraction of regular workforce available for over-time

δ regular working hours of labor in each period

γ_i the target number of stockouts for product i

λ^M fraction of regular machine capacity available for over-time use

M_t maximum regular time machine capacity

M a big number

$a_{F,i}$ man hours required to produce one unit of finished product i from raw materials

$a_{S,i}$ man hours required to produce one unit of common item product i from raw materials

$a_{T,i}$ man hours required to produce one unit of finished product i from semi-finished products

$b_{F,i}$ machining time required to produce one unit of finished product i from raw materials

$b_{S,i}$ machining time required to produce one unit of semi-finished product i from raw materials

$b_{T,i}$ machining time required to produce one unit of finished product i from common item products

Uncertain parameters

D_i random variable representing the demand for product i

Decision variables

continuous variable

W Number of workers

$Z_{RF,i}$ amount direct product i during regular time

$Z_{OF,i}$ amount direct product i during over-time

Cap_R the amount of postponement capacity produces in regular time

Cap_O the amount of postponement capacity produces in over-time

$P_{R,i}$ the amount of postponement capacity used to meet demand for product i in regular time

$P_{O,i}$ the amount of postponement capacity used to meet demand for product i in over-time

Binary variables

$$Y_{F,i} = \begin{cases} 1 & if\ Z_{RF,i} \geq 0 \\ 0 & otherwise \end{cases}$$

$$Y_{S,i} = \begin{cases} 1 & if\ Z_{RS,i} \geq 0 \\ 0 & otherwise \end{cases}$$

$$Y_{T,i} = \begin{cases} 1 & if\ Z_{RT,i} \geq 0 \\ 0 & otherwise \end{cases}$$

Expected value function

$E[TC_{PP}]$ the expected total cost of a partial postponement strategy (if CapR + CapO>0)

$E[TC_{Npp}]$ the expected total cost of a partial postponement strategy (if CapR + CapO=0)

$E[SO_i]$ the expected number of stock out of product i in terms of generic products.

3.2 Component of objective function

3.2.1 Final activity cost

The final activity such as packaging, labeling and final assembly occurs after the production of generic item. There are cost $C_{RT,i}$ and $C_{OT,i}$ associated with regular and over-time production cost to carry out final activity for product i. Now we compute this cost according to the different region of Figure 1. if demand for product one occurring in regions {1}, {2}, {3} or {13} and demand for product two occurring in regions {1}, {4}, {10} or {11} are met using $(Z_{RF,1} + Z_{OF,1})$ and $(Z_{RF,2} + Z_{OF,2})$ respectively. So, the expected total cost to occur final activity for both finished goods inventories is given by:

$$\sum_{i=1}^{2} C_{RF,i}.Z_{RF,i} + \sum_{i=1}^{2} C_{OF,i}.Z_{OF,i} \qquad (1)$$

If demand for product one occur in region {4}-{12} in addition to $\left(\sum_i Z_{RF,it} + \sum_i Z_{OF,it}\right)$ the postponed inventory will also transformed to the finished goods to met demand. The expected cost for both items will be:

$$\sum_{i=1}^{2} C_{RT,i}.E[P_{RT,i}] + \sum_{i=1}^{2} C_{OT,i}.E[P_{OF,it}] \qquad (2)$$

The expected number of units of the generic item to carry out final activity for product i is equals to:

$$E[P_{RT,i}] = \sum_K E\left[\left(P_{RT,i}\right)_K\right] \quad i = 1, 2 \tag{3}$$

$$E[P_{OT,i}] = \sum_K E\left[\left(P_{RT,i}\right)_K\right] \quad i = 1, 2 \tag{4}$$

The expression and limits integration for each region of interest can be found in Appendix A.

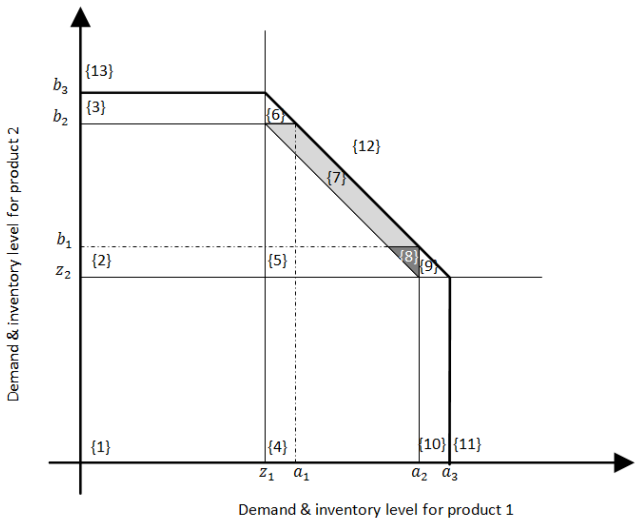

Figure 1: Graphical depiction of inventory levels under partial postponement strategy

$$a_1 = z_1 + Cap_O \quad a_2 = z_1 + Cap_R \quad a_3 = z_1 + Cap_R + Cap_O$$
$$b_1 = z_2 + Cap_O \quad b_2 = z_2 + Cap_R \quad b_2 = z_2 + Cap_R + Cap_O$$

3.2.2 *Holding cost of finished goods inventory*

If demand for product one occurs in regions {1}, {2}, {3} or {13} and demand for product two occurs in regions {1}, {4}, {10} or {11} it is possible to have some finished goods inventory. The total expected cost for period t is given by

$$E\sum_{i=1}^{2}\left[H_{F,i}\left(Z_{RF,i} + Z_{OF,i} - x_i\right)^+\right] \tag{5}$$

The expected quantity of finished goods left over of product one at the end of period is

$$E\left[\left(Z_{RF,1} + Z_{OF,1} - x_1\right)^+_{\{1,2,3,15\}}\right] = \int_0^\infty \int_0^{z_1} (Z_{RF,1} + Z_{OF,1} - x_1)\, f_t(x_1, x_2)\, dx_1 dx_2 \tag{6}$$

Where $\left(Z_{RF,i} + Z_{OF,i} - x_i\right)^+ = max\left(0, Z_{RF,i} + Z_{OF,i} - x_i\right)$

The expression for $f(x_1, x_2)$ was developed using Morgenstern's copula. Copula provides a method of constructing multivariate distributions from known marginal distributions.

The demand for product two will be solely from finished-goods inventory if it occurs in regions {1}, {4}, {10} or {11}. Expression for product two are similar to product one.

3.2.3 *Holding cost of postponed inventory*

At the beginning of the period an inventory of postponed items exist. It's given by

$$H_P.\left(Cap_R + Cap_O - E\left[\sum_{i=1}^{2} P_{RT,i} + \sum_{i=1}^{2} P_{OT,i}\right]\right) \tag{7}$$

3.2.4 *Shortage cost*

Demand in different regions are met by combination of $Z_{RF,it}$, $Z_{OF,it}$, $P_{RT,i,t}$ and $P_{OT,i,t}$. The expected number of stockouts for each product i is the sum of the expected number of stockouts for each region in which stockouts can occur:

$$E[SO_i] = \sum_K E[(SO_i)_K] \quad i = 1, 2 \tag{8}$$

The detail expression for the expected number of stockouts for product one in region k can be found in Appendix B.

Objective function: The objective function of the mixed-integer nonlinear program includes six terms. The first and the second term are setup cost and labour cost respectively. The third term is production cost of direct finished products cost associated with regular and over-time. The fourth term is production cost of generic product cost associated with regular and over-time. Final term is the expected total holding cost of finished-goods inventory.

$$\begin{aligned} min\, E[TC_{PP}] = &\sum_{i=1}^{n}\left(C_{SF,i}Y_{F,i} + C_{SS,i}Y_{S,i} + C_{ST,i}Y_{T,i}\right) + \sum_{t=1}^{T} C_W W \\ &+ \sum_{i=1}^{n}\left(C_{RF,i}Z_{RF,i} + C_{OF,i}Z_{OF,i}\right) \\ &+ \sum_{i=1}^{n}\left(\left(C_{RS,i} + H_{pp} + H_p\right)Cap_R \right.\\ &\left.+ \left(C_{OS,i} + H_{pp} + H_p\right)Cap_O\right) \\ &+ E\left[\sum_{t=1}^{T}\sum_{i=1}^{n}\left(\left(C_{RT,i} - H_p\right)P_{RT,i}\right.\right. \\ &\left.\left.+ \left(C_{OT,i} - H_p\right)P_{OT,i}\right)\right] \\ &+ E\left[\sum_{i=1}^{n} H_F\left(Z_{RF,i} + Z_{OF,i} - D_i\right)^+\right] \end{aligned} \tag{9}$$

$$\begin{aligned} min\, E[TC_{NONPP}] = &\sum_{i=1}^{n}\left(C_{SF,i}Y_{F,i} + C_{SS,i}Y_{S,i} + C_{ST,i}Y_{T,i}\right) \\ &+ \sum_{t=1}^{T} C_W W + \sum_{i=1}^{n}\left(C_{RF,i}Z_{RF,i} + C_{OF,i}Z_{OF,i}\right) \\ &+ E\left[\sum_{i=1}^{n} H_F\left(Z_{RF,i} + Z_{OF,i} - D_i\right)^+\right] \end{aligned} \tag{10}$$

Constraints:

$$E[SO_i] \le \gamma_i \qquad i = 1,2 \tag{11}$$

$$\left(Z_{RF,i} + Z_{OF,i}\right) + \left(Cap_R + Cap_O\right) \geq a_i \qquad i = 1,2 \qquad (12)$$

$$\left(Z_{RF,i} + Z_{OF,i}\right) + \left(Cap_R + Cap_O\right) \leq b_i \qquad i = 1,2 \qquad (13)$$

$$\sum_{i=1}^{n}\left(P_{RT,i} + P_{OT,i}\right) \leq Cap_R + Cap_O \qquad (14)$$

$$Cap_R + Cap_O \leq K \qquad (15)$$

$$\sum_{i=1}^{n}\left(a_{F,i}Z_{RF,i} + a_{Si}Cap_R + a_{T,i}P_{R,i}\right) \leq \delta W \qquad (16)$$

$$\sum_{i=1}^{n}\left(a_{F,i}Z_{OF,i} + a_{Si}Z_{OS,i} + a_{T,i}P_{O,i}\right) \leq {\lambda_t}^{W}\delta W \qquad (17)$$

$$\sum_{i=1}^{n}\left(b_{F,i}Z_{RF,i} + b_{Si}Z_{RS,i} + b_{T,i}P_{R,i}\right) \leq M \qquad (18)$$

$$\sum_{i=1}^{n}\left(b_{F,i}Z_{OF,i} + b_{S,i}Z_{OS,i} + b_{T,i}P_{O,i}\right) \leq \lambda^{M} M \qquad (19)$$

$$Z_{RF,i} + Z_{OF,i} \leq MY_{F,i} \qquad i = 1,2 \qquad (20)$$

$$Cap_R + Cap_O \leq MY_{S,i} \qquad i = 1,2 \qquad (21)$$

$$P_{R,i} + P_{O,i} \leq MY_{T,i} \qquad i = 1,2 \qquad (22)$$

$$Z_{RF,i}, Z_{OF,i} \geq 0 \qquad i = 1, 2 \qquad (23)$$

$$Cap_R, Cap_O, W, K \geq 0 \qquad (24)$$

$$P_{R,i}, P_{O,i} \geq 0 \qquad i = 1, 2 \qquad (25)$$

$$Y_{F,i} \in \{0, 1\} \qquad i = 1, 2 \qquad (26)$$

$$Y_{S,i} \in \{0, 1\} \qquad i = 1, 2 \qquad (27)$$

$$Y_{T,i} \in \{0, 1\} \qquad i = 1, 2 \qquad (28)$$

The objective functions minimize the expected total cost with and without postponement. Constraint (11) insures that the target fill rates for each product are met. Constraint (12) and constraint (13) are lower and upper bound of demand. Constraint (14) insures that the amount of the postponed inventory used by both products is less than or equal to the postponed inventory. Constraint (15) is used to creating the nonpostponement or some specific amount of postponed capacity. Constraints (16) and (17) limit the labour working hours during regular time and overtime respectively. Similarly, constraints (18) and (19) limit the machining time during regular time and overtime respectively. Constraints (20)–(22) ensure that setup costs will be incurred when the corresponding production activities started. Constraints (23)-(25) are the non-negative constraints. Constraints (26)-(28) are used for the setup indication of the product activity.

4. COMPUTATIONAL RESULTS

In this section to validate our proposed model we use a numerical example. The input parameters are given in table 1. to table 3. We assume the demand of two products are identical and uniform distributed in [480,1520]. Products demand are independent expect in the discussion of correlated demand. We use different measurement, number of stockouts for customer service level. The expected number of stockouts for product i is defined as bellow:

$$E[(SO)_i] \leq \gamma_i \qquad (29)$$

That γ_i is given by:

$$\gamma_i = (1 - \beta_i).\mu_i \qquad (30)$$

That β_i is fill rate for product i. the fill rates are set to $\beta_1 = \beta_2 = 0.975$.

Table 1: Production cost

		Product	Cost
Direct production	Regular time	1	60
		2	70
	Over-time	1	60
		2	70
Generic Production	Regular time	1	40
		2	40
	Over-time	1	40
		2	40
Transfer production	Regular time	1	35
		2	45
	Over-time	1	35
		2	45

Table 2: Operating and cost data

	Product	Direct	Generic	Transformed
Set up cost	1	2000	1000	1500
	2	2500	1000	2000
Labor time	1	0.5	0.35	0.15
	2	0.6	0.35	0.25
Machine time	1	0.5	0.4	0.1
	2	0.6	0.4	0.2

Table 3: Machine and workforce level

Maximum workforce level	1000
Maximum machine capacity	1600
Fraction of workforce available for over-time	0.3
Fraction of machine capacity available for over-time	0.4

The optimal solution for the base case under postponement strategy is $z_1 = z_2 = 818$, $P_{1,R} = 305$, $P_{2,R} = 366$, $P_{1,0} = 31$ and $P_{2,0} = 0$. Table 4. and table 5. present a comparison between two strategy (PP and NONPP). As we see applying postponement strategy lead to reduction in finished goods inventory and direct production cost and saving about 26% is made.

Table 4: Variables value (Base Case)

	Without PP		With PP	
	Product1	Product2	Product1	Product2
ZRF	1325	1318	818	818
ZOF	0	7	0	0
PRT	0	0	305	366
POT	0	0	31	0
YF	1	1	1	1
YS	0	0	1	1
YT	0	0	1	1
CapR	0		702	
CapO	0		0	

Table 5: Optimal Solution of Base Case (Demand Correlation=+0.33)

	Labor cost	Setup cost	D.production cost	G.& T. production cost	F.G. inventory cost	PP.cost	Total cost
With PP	196	10000	106294	56188	62437	3512	278639
Without PP	182	45000	172201	0	160187	0	351425

5. SENSITIVE ANALYSIS

In this section we study the effect of fill rate on postponement capacity and percent reduction in expected total cost compared to independent demand and nonpostponement case. Although Morgenster's Bivarate Uniform distribution limit the correlation coefficient to a range of -1/3 to +1/3. It's enable us to gain insight into the impact of demand correlation on expected total cost and postponement capacity.

5.1 Fill rate

The various levels of fill rates between 90% and 100% are considered to examine the behavior of different objective function with different demand correlation. These variations are shown in Figure 2. In this graph percent cost reduction compared to independent products demand and nonpostponement case are depicted. According to Figure 2. to Figure 5. we can conclude increasing the fill rates results in higher postponement capacity level, higher finished goods inventory and increasing percent reduction in the expected total cost of partial postponement compared to independent products demand and nonpostponement case. But in a very high level of fill rate the level of postponement capacity decreases while the finished goods inventory continue to increase in a more aggressive mode. We can see relationship between postponement capacity and finished goods inventory in Figure 2. to Figure 5. The fill rate level which percent reduction in expected total cost begins to decrease is different for various demand correlation. The lower demand correlation results to higher fill rate levels that percent reduction in expected total cost increases. i.e. for -0.33 correlation coefficient case percent reduction in expected total cost increases until to 99.87% fill rate level. For the higher level of fill rate beneficiary of partial postponement decreases in a very speed and make-to-stock strategy is better. In other side for +0.33 demand correlation case the percent reduction fill rate level is lower than 99.98% and equal to 99.68%.

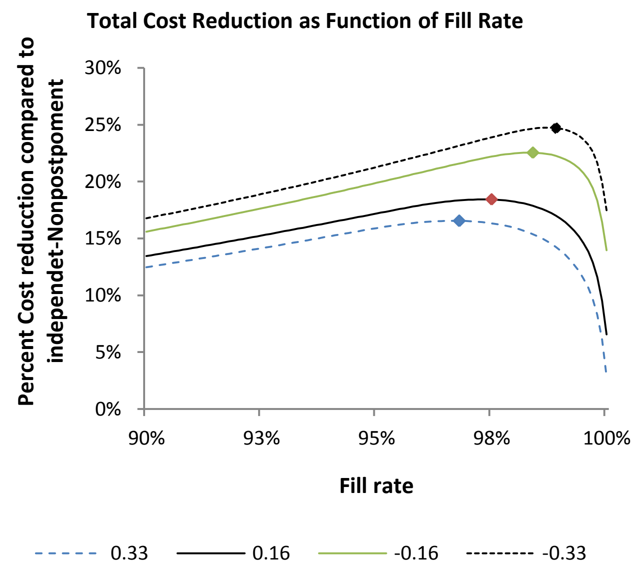

Figure 2: Effect of fill rate on the expected total cost for selected values of the coefficient of demand

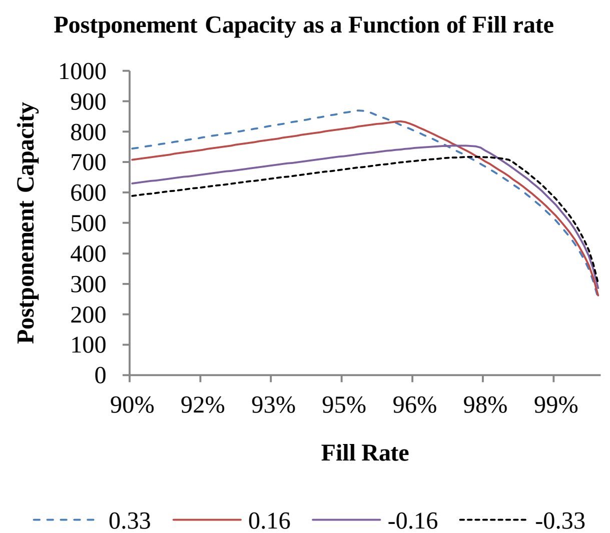

Figure 3: Effect of fill rate on the postponement capacity for selected values of the correlation of demand

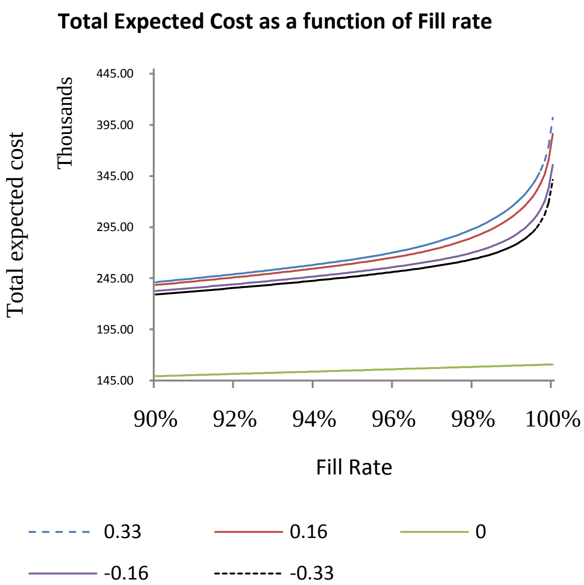

Figure 4: Effect of fill rate on the expected total cost for selected values of the coefficient of demand

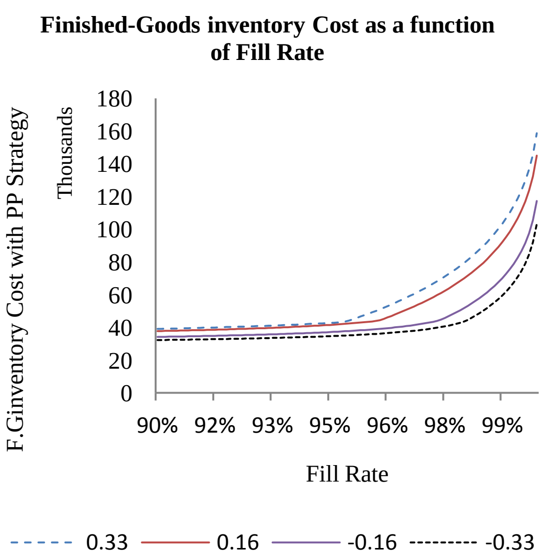

Figure 5: Effect of fill rate on the finished-goods inventory cost for selected values of the correlation of demand

5.2 Finished goods inventory cost

We now examine the impact of various value of finished goods inventory cost on expected total cost and postponement capacity in various demand correlation. The finished-goods holding cost for base case is 60. Graph in Figure 6. showed from -50% of holding cost of base case to +250% from base case. As holding cost increases thus the postponement capacity (CapR+CapO) increases. The effects of different demand correlation on total expected cost are the same as the ones used in previous section. Comparison of the graphs at each level of the demand correlation shows that the percent reduction of the total expected cost for negatively-correlated demands is greater than for positively-correlated demand. But for a postponement capacity as holding cost increases the postponement capacity increase in different pattern for different demand correlation. Other constraints in our model affect the level of postponement for different holding cost. These affects are illustrated in Figure 7.

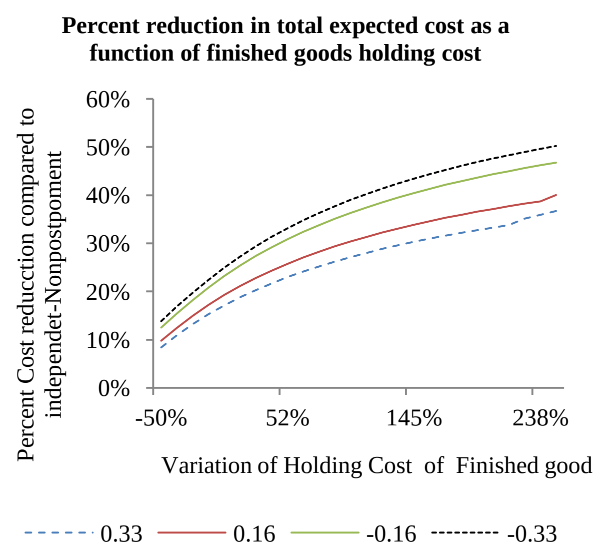

Figure 6: Total expected cost as a function of finished goods holding cost

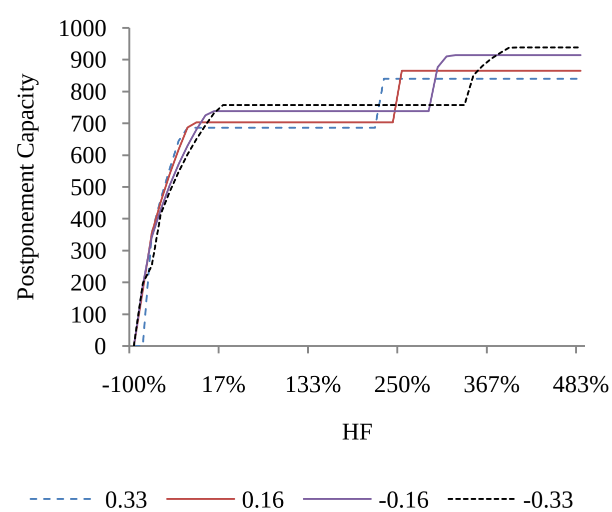

Figure 7: Percent reduction in total expected cost as a function of finished goods holding cost

6. SUMMARY AND FUTURE WORKS

In this study we developed a mixed integer nonlinear programming model for single period, two products production planning using a partial postponement strategy in which demands of products are dependent and uniformly-distributed to setting the level of finished goods inventory and postponement capacity. We derived expression for the expected number of stockouts and expected operational cost. The computational results show for dealing uncertainty partial postponement is a good strategy. We found out that expected total cost decrease and postponement increases as (1) the value of finished goods inventory cost increases (2) product demands became more negatively correlated. The expected total cost increases and capacity decreases as the fill rate approach 100%.

There is still room for improvement. First real data from companies can be used to validate our proposed model and to analysis its sensitivity to changes in production planning strategies. Second we developed the model for two products. However we are facing more than two products in real world. Third we developed a single period, short production planning. It's valuable to develop a model for intermediate and long term production planning.

7. APPENDIX.A

Expected of postponement capacity used by product 1

In below expression we assume:

$$z_i = Z_{RF,i} + Z_{OF,i} \quad i = 1,2 \qquad (A.1)$$

Regions {1, 2, 3, 15}

If demand for product one occurs in regions {1}, {2}, {3}, or {15} of Fig. 2, then none of the postponed items will be used to meet demand.

$$E\left[\left(P_{RT,1}\right)_{\{1,2,3,15\}}\right] = 0 \qquad (A.2)$$

$$E\left[\left(P_{OT,1}\right)_{\{1,2,3,15\}}\right] = 0 \qquad (A.3)$$

Region {4}

$$E\left[\left(P_{RT,1}\right)_{\{4\}}\right] = \int_0^{z_2}\int_{z_1}^{z_1+Cap_R}(x_1 - z_1)\, f_t(x_1,x_2)dx_1dx_2 (A.4)$$

$$E\left[\left(P_{OT,1}\right)_{\{4\}}\right] = 0 \qquad (A.4)$$

Region {5}

$$E\left[\left(P_{RT,1}\right)_{\{5\}}\right] = \int_{z_2}^{z_2+Cap_R}\int_{z_1}^{z_1+z_2+Cap_R-x_2}(x_1 - z_1)\, f(x_1,x_2)dx_1dx_2 \qquad (A.5)$$

$$E\left[\left(P_{OT,1}\right)_{\{5\}}\right] = 0 \qquad (A.6)$$

Region {6}

$$E\left[\left(P_{RT,1}\right)_{\{6\}}\right] = 0 \qquad (A.7)$$

$$E\left[\left(P_{OT,1}\right)_{\{6\}}\right] = \int_{z_2+Cap_R}^{z_2+Cap_R+Cap_O}\int_{z_1}^{z_1+z_2+Cap_R+Cap_O-x_2}(x_1 - z_1)\, f_t(x_1,x_2)dx_1dx_2$$

Region {7}

$$E\left[\left(P_{RT,1}\right)_{\{7\}}\right] = \int_{z_2+Cap_O}^{z_2+Cap_R}\int_{z_1+z_2+Cap_R-x_2}^{z_1+z_2+Cap_R+Cap_O-x_2} P_{RT,1} f(x_1,x_2)dx_1dx_2 \; (A.8)$$

$$E\left[\left(P_{OT,1}\right)_{\{7\}}\right] = \int_{z_2+Cap_O}^{z_2+Cap_R}\int_{z_1+z_2+Cap_R-x_2}^{z_1+z_2+Cap_R+Cap_O-x_2}(x_1 - z_1 - Cap_R)\, f(x_1,x_2)dx_1dx_2 \qquad (A.9)$$

Region {8}

$$E\left[\left(P_{RT,1}\right)_{\{8\}}\right] = \int_{z_1+Cap_R-Cap_O}^{z_1+Cap_R}\int_{z_1+z_2+Cap_R-x_1}^{z_2+Cap_O} P_{1R} f(x_1,x_2)dx_2dx_1 \qquad (A.10)$$

$$E\left[\left(P_{OT,1}\right)_{\{8\}}\right] = \int_{z_1+Cap_R-Cap_O}^{z_1+Cap_R}\int_{z_1+z_2+Cap_R-x_1}^{z_2+Cap_O}(x_1 - z_1 - Cap_R) f(x_1,x_2)dx_2dx_1 \qquad (A.11)$$

Region {9}

$$E\left[\left(P_{OT,1}\right)_{\{9\}}\right] = \int_{z_2}^{z_2+Cap_O}\int_{z_1+Cap_R}^{z_1+z_2+Cap_R+Cap_O-x_2}(x_1 - z_1 - Cap_R)\, f(x_1,x_2)dx_1dx_2 \qquad (A.12)$$

$$E\left[\left(P_{RT,1}\right)_{\{9\}}\right] = \int_{z_2}^{z_2+Cap_O}\int_{z_1+Cap_R}^{z_1+z_2+Cap_R+Cap_O-x_2} P_{1R}\, f(x_1,x_2)dx_1dx_2 \qquad (A.13)$$

Region {10}

$$E\left[\left(P_{OT,1}\right)_{\{10\}}\right] = \int_0^{z_2}\int_{z_1+Cap_R}^{z_1+Cap_R+Cap_O}(x_1 - z_1 - Cap_R) f(x_1,x_2)dx_1dx_2 \qquad (A.14)$$

$$E\left[\left(P_{RT,1}\right)_{\{10\}}\right] = \int_0^{z_2}\int_{z_1+Cap_R}^{z_1+Cap_R+Cap_O} P_{1R}\, f(x_1,x_2)dx_1dx_2 \qquad (A.15)$$

Region {11}

$$E\left[\left(P_{OT,1}\right)_{\{11\}}\right] = \int_0^{z_2}\int_{z_1+Cap_R+Cap_O}^{\infty} Cap_O f(x_1,x_2)dx_1dx_2 \qquad (A.16)$$

$$E\left[\left(P_{RT,1}\right)_{\{11\}}\right] = \int_{0}^{z_2}\int_{z_1+Cap_R+Cap_O}^{\infty} Cap_R f(x_1,x_2)dx_1dx_2 \qquad (A.17)$$

Region {12, 13, 14}

$$E\left[\left(P_{RT,1}\right)_{\{12\}}\right] = \int_{z_2}^{\infty}\int_{z_1}^{\infty} P_{RT,1} f(x_1,x_2)dx_1dx_2 - \int_{z_{2,t}}^{z_2+Cap_R}\int_{z_{1,t}}^{z_1+z_2+Cap_R-x_2} P_{RT,1}\, f(x_1,x_2)dx_1dx_2 - 0 - \int_{z_2+Cap_O}^{z_2+Cap_R}\int_{z_1+z_2+Cap_R-x_2}^{z_1+z_2+Cap_R+Cap_O-x_2} P_{RT,1} f(x_1,x_2)dx_1dx_2 - \int_{z_1+Cap_R-Cap_O}^{z_1+Cap_R}\int_{z_1+z_2+Cap_R-x_1}^{z_2+Cap_O} P_{RT,1} f(x_1,x_2)dx_2dx_1 - \int_{z_2}^{z_2+Cap_O}\int_{z_{1,t}+Cap_R}^{z_1+z_2+Cap_R+Cap_O-x_2} P_{RT,1}\, f(x_1,x_2)dx_1dx_2 \qquad (A.18)$$

$$E\left[\left(P_{OT,1}\right)_{\{12\}}\right] = \int_{z_2}^{\infty}\int_{z_1}^{\infty} P_{1O} f(x_1,x_2)dx_1dx_2 - 0 - \int_{z_2+Cap_R}^{z_2+Cap_R+Cap_O}\int_{z_1}^{z_1+z_2+Cap_R+Cap_O-x_2} P_{OT,1}\, f(x_1,x_2)dx_1dx_2 - \int_{z_2+Cap_O}^{z_2+Cap_R}\int_{z_1+z_2+Cap_R-x_2}^{z_1+z_2+Cap_R+Cap_O-x_2} P_{OT,1}\, f(x_1,x_2)dx_1dx_2 - \int_{z_1+Cap_R-Cap_O}^{z_1+Cap_R}\int_{z_1+z_2+Cap_R-x_1}^{z_2+Cap_O} P_{OT,1} f(x_1,x_2)dx_2dx_1 - \int_{z_2}^{z_2+Cap_O}\int_{z_1+Cap_R}^{z_1+z_2+Cap_R+Cap_O-x_2} P_{OT,1}\, f(x_1,x_2)dx_1dx_2 \qquad (A.19)$$

8. APPENDIX B.

Expected Stock out of Product one:

$$E[(SO_1)_{11}] = \int_{0}^{z_2}\int_{z_1+Cap_R+Cap_O}^{\infty} (x_1 - z_1 - Cap_R - Cap_O) f(x_1,x_2)dx_1dx_2 \qquad (B.1)$$

$$E\left[(SO_1)_{\{12,13,14\}}\right] = E[(SO_1)_\bullet] - \left(E[(SO_1)_5] + E[(SO_1)_6] + E\left[(SO_1)_{\{7,8,9\}}\right]\right) \qquad (B.2)$$

$$E[(SO_1)_\bullet] = \int_{z_2}^{\infty}\int_{z_1}^{\infty}\left(x_1 - z_1 - P_{RT,1} - P_{OT,1}\right) f(x_1,x_2)dx_1dx_2 \qquad (B.3)$$

$$E[(SO_1)_5] = \int_{z_2}^{z_2+Cap_R}\int_{z_1}^{z_1+z_2+Cap_R-x_2}\left(x_1 - z_1 - P_{RT,1}\right) f(x_1,x_2)dx_1dx_2 \qquad (B.3)$$

$$E[(SO_1)_6] = \int_{z_2+Cap_R}^{z_2+Cap_R+Cap_O}\int_{z_1}^{z_1+z_2+Cap_R+Cap_O-x_2}\left(x_1 - z_1 - P_{OT,1}\right) f(x_1,x_2)dx_1dx_2 \qquad (B.4)$$

$$E\left[(SO_1)_{\{7,8,9\}}\right] = \int_{z_2}^{z_2+Cap_R}\int_{z_1+z_2+Cap_R-x_2}^{z_1+z_2+Cap_R+Cap_O-x_2}\left(x_1 - z_1 - Cap_R - P_{OT,1}\right) f(x_1,x_2)dx_1dx_2 \qquad (B.5)$$

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