International Journal of Computer Applications (0975 – 8887)
Volume 30– No.7, September 2011

On Some Properties of Fuzzy Kernel with Thresholds and Homomorphism

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ABSTRACT

Wee introduced the concept of fuzzy automata following Zadeh in 1967. Yuan.et.al gave the definition of a fuzzy subgroup with thresholds which is a generalization of Rosenfeld's fuzzy subgroup and Bhakat and Das's fuzzy subgroup. Das introduced fuzzy kernel and fuzzy subsemiautomaton of a fuzzy semiautomaton. In this paper we have proved some results of fuzzy kernel with thresholds and also it's transformation under strong homomorphism.

General Terms

Switching Network.

Keywords

Fuzzy subgroup with thresholds, Fuzzy subsemiautomaton with thresholds, Fuzzy kernel with thresholds.

1. INTRODUCTION

L.A.Zadeh introduced fuzzy sets in 1965 [1]. Rosenfeld defined fuzzy subgroups in 1971 [6]. In 1982 Antony and Sherwood introduced T-fuzzy subgroups [8]. Mukherjee introduced fuzzy normal subgroups in 1984 [9]. The concept $\in,\in\vee q$ fuzzy was introduced by Bhakat and Das in 1992[7]. Also in 1997 [2] Das introduced fuzzy kernel and fuzzy subsemiautomaton of a fuzzy semiautomaton over a finite group using the notions of a fuzzy normal subgroup and a fuzzy subgroup of a group. This concept was generalized as fuzzy subgroup with thresholds by Yuan.et.al. in 2003 [3]. In this paper we have proved some results of fuzzy kernel with thresholds.

2. PRELIMINARIES

In this section we summarize the preliminary definitions and results that are required for developing main results.

Let $G,*$ denote a group. We sometimes write G for $G,*$ when the operation $*$ is understood.

2.1 Definition

Let X be a nonempty set. A fuzzy set A in X is characterized by its membership function $A: X \to 0,1$ and $A\; x$ is interpreted as the degree of membership of element x in fuzzy set A for each $x \in X$

2.2 Definition

A fuzzy subset λ of a group G is a fuzzy subgroup of G if for all $x, y \in G$

(i) $\lambda\; x*y \;\geq \lambda\; x \;\wedge \lambda\; y$

(ii) $\lambda\; x^{-1} \;\geq \lambda\; x$

2.3 Definition

A fuzzy subgroup λ of G is called a fuzzy normal subgroup of G if $\lambda\; x*y*x^{-1} \;\geq \lambda\; y$ for all $x, y \in G$

2.4 Definition

Let $\lambda, \mu \in 0,1$ and $\lambda < \mu$ .Let A be a fuzzy subset of a group G . A is called a fuzzy subgroup with thresholds of G if for all $x, y \in G$

(i) $A\; x*y \;\vee \lambda \geq A\; x \;\wedge A\; y \;\wedge \mu$

(ii) $A\; x^{-1} \;\vee \lambda \geq A\; x \;\wedge \mu$

2.5 Definition

Let $\alpha,\beta \in \; 0,1$ and $\alpha < \beta$. Let μ be a fuzzy subset of a group G . μ is called a fuzzy normal subgroup with thresholds of G if for all $x, y \in G$

$$\mu \;\; y^{-1} * x * y \;\; \vee \alpha > \mu \;\; x \;\; \wedge \beta$$

2.6 Definition

A fuzzy semiautomaton over a finite group $Q,*$ is a triple Q, X, μ where X is a finite set and μ is a fuzzy subset of $Q \times X \times Q$.

2.7 Definition

Let $S = \;\; Q, X, \mu$ be a fuzzy semiautomaton over a finite group G . A fuzzy subset λ of Q is called fuzzy kernel of S if the following conditions hold

(i) λ is a fuzzy normal subgroup of Q

(ii) $\lambda \;\; p * r^{-1} \;\; \geq \mu \;\; q * k, x, p \;\; \wedge \mu \;\; q, x, r \;\; \wedge \lambda \;\; k$
$\forall p, q, r, k \in Q, x \in X$

2.8 Definition

Let $S = \;\; Q, X, \mu$ be a fuzzy semiautomaton over a finite group G. A fuzzy subset λ of Q is called fuzzy subsemiautomaton of S if the following conditions hold

(i) λ is a fuzzy subgroup of Q

(ii) $\lambda \;\; p \;\; \geq \mu \;\; q, x, p \;\; \wedge \lambda \;\; q \;\; \text{for all } p, q \in Q, x \in X.$

2.9 Definition

Let $\alpha,\beta \in \; 0,1$ and $\alpha < \beta$. Let $S = \;\; Q, X, \mu$ be a fuzzy semiautomaton over a finite Group. A fuzzy subset λ of Q is called a fuzzy kernel of S with thresholds if

(i) λ is a fuzzy normal subgroup of Q with thresholds

(ii) $\lambda \;\; p * r^{-1} \;\; \vee \alpha \geq \mu \;\; q * k, x, p \;\; \wedge \mu \;\; q, x, r$
$\wedge \lambda \;\; k \;\; \wedge \beta, \forall\, p, q, r, k \in Q$

2.10 Definition

Let $\alpha,\beta \in \; 0,1$ and $\alpha < \beta$. Let $S = \;\; Q, X, \mu$ be a fuzzy semiautomaton over a finite Group. A fuzzy subset λ of Q is called a fuzzy subsemiautomaton of S with thresholds if the following conditions hold

(i) λ is a fuzzy subgroup of Q with thresholds

(ii) $\lambda \;\; p \;\; \vee \alpha \geq \mu \;\; q, x, p \;\; \wedge \lambda \;\; q \;\; \wedge \beta$ for all $p, q \in Q \; and \; x \in X$.

2.11 Definition

Let $S = \;\; Q, X, \mu$ and $T = \;\; Q_1, X_1, \mu_1$ be fuzzy semiautomatons over a finite group. A pair of functions f, g where $f : Q \to Q_1, g : X \to X_1,$ is called a strong homomorphism from S into T , written $f, g \;\; : S \to T$ if the following conditions hold:

(1) f is a group homomorphism

(2) $\mu \;\; p, x, q \;\; \leq \mu_1 \;\; f \;\; p \;\; , g \;\; x \;\; , f \;\; q$ for all $p, q \in Q \; and \; x \in X$

The pair f, g is called a strong homomorphism from S into T if it satisfies (1) of the definition and the added condition, $\mu_1 \;\; f \;\; p \;\; , g \;\; x \;\; , f \;\; q \;\; = \vee \left\{ \begin{array}{l} \mu \;\; p, x, r \;\; / \\ r \in Q, f \;\; r \;\; = f \;\; q \end{array} \right\}$ for all $p, q \in Q \; and \; x \in X$.

2.12 Definition

Let T = Q, X, δ be a fuzzy semiautomaton over a finite group $G, *$. An element $x_0 \in X$ is called an e- input if $\delta \;\; e, x_0, e \;\; > 0$.

3. MAIN RESULTS

3.1 Proposition

Let $\alpha, \beta \in \ 0,1$ and $\alpha < \beta$. A fuzzy normal subgroup λ of Q with thresholds α, β is a fuzzy kernel of $S = \ Q, X, \mu$ with thresholds α, β if and only if

$$\lambda \ \ p*r^{-1} \ \vee \alpha \ \ \geq \mu^* \ \ q*k, x, p \ \wedge \mu^* \ \ q, x, r \\ \wedge \lambda \ \ k \ \ \wedge \beta \\ for\ all\ p, q, r, k \in Q\ and\ x \in X^*$$

Proof

We prove the theorem by induction on $|x| = n$

Let λ be a fuzzy kernel of $S = \ Q, X, \mu$ with thresholds. Let $n = 0$ then $x = \Lambda$

If $p = q*k, and\ r = q$

$$\mu^* \ \ q*k, x, p \ \wedge \mu^* \ \ q, x, r \ \ \wedge \lambda \ \ k \ \ \wedge \beta = \\ \lambda \ \ k \ \ \wedge \beta \leq \lambda \ \ q*k*q^{-1} \ \ \vee \alpha$$

Since λ is a fuzzy normal subgroup with thresholds.

If $p \neq q*k, or\ \ r \neq q$ then

$$\mu^* \ \ q*k, x, p \ \wedge \mu^* \ \ q, x, r \ \ \wedge \lambda \ \ k \ \ \wedge \beta = 0 \leq \\ \lambda \ \ q*r^{-1} \ \ \vee \alpha$$

For $n = 0$ it is proved

Suppose the result holds for $y \in X^*$ where $|y| = n-1,$ $n > 0$

Let $x \in X^*$ be such that

$$x = ya\ ,\ y \in X^*, a \in X\ , |y| = n-1, n > 0$$

$$\mu^* \ \ q*k, x, p \ \ \wedge \mu^* \ \ q, x, r \ \ \wedge \lambda \ \ k \ \ \wedge \beta$$

$$= \ \vee \ \ \mu^* \ \ q*k, y, u \ \ \wedge \mu \ \ u, a, p / u \in Q \quad \wedge$$

$$\vee \ \ \mu^* \ \ q, y, v \ \ \wedge \mu \ \ v, a, r \ \ / v \in Q \quad \wedge \lambda \ \ k \ \ \wedge \beta$$

$$= \vee \left\{ \vee \begin{Bmatrix} \mu^* \ \ q*k, y, u \ \ \wedge \\ \mu \ \ u, a, p \ \ \wedge \mu^* \ \ q, y, v \ \ \wedge \mu \ \ v, a, r \\ \wedge \lambda \ \ k \ \ \wedge \beta / u \in Q \end{Bmatrix} / v \in Q \right\}$$

By induction

$$\leq \vee \left\{ \vee \begin{Bmatrix} \lambda \ \ u*v^{-1} \ \ \vee \alpha \ \ \wedge \mu \ \ u, a, p \ \ \wedge \mu \ \ v, a, r \ \ / \\ u \in Q \end{Bmatrix} / v \in Q \right\}$$

$$= \vee \left\{ \vee \begin{Bmatrix} \lambda \ \ v^{-1}*u \ \ \wedge \mu \ \ v*v^{-1}*u, a, p \ \ \wedge \mu \ \ v, a, r \ \ \vee \alpha / \\ u \in Q \end{Bmatrix} / v \in Q \right\}$$

$$\leq \vee \left\{ \vee \begin{Bmatrix} \lambda \ \ v^{-1}*u \ \ \wedge \mu \ \ v*v^{-1}*u, a, p \ \ \wedge \mu \ \ v, a, r \ \ \wedge \beta / \\ u \in Q \end{Bmatrix} / v \in Q \right\}$$

$$\leq \lambda \ \ p*r^{-1} \ \ \vee \alpha$$

3.2 Proposition

Let $\alpha, \beta \in \ 0,1$ and $\alpha < \beta$. Let $T = \ Q_1, X, \mu_1$ be a fuzzy semiautomaton over a finite group and let f be a homomorphism from $S\ onto T$. If λ is a fuzzy kernel of S with thresholds α, β then $f \ \ \lambda$ is a fuzzy kernel of T with thresholds α, β.

Proof

Let λ be a fuzzy kernel of S with thresholds α, β. Then λ is a fuzzy normal subgroup of Q with thresholds α, β .Since f is an epimorphism from Q onto Q_1 with thresholds α, β .

Let $p_1, q_1, r_1, k_1 \in Q_1$ *and* $x \in X$ .Then

$$\mu_1(q_1 * k_1, x, p_1) \wedge \mu_1(q_1, x, r_1) \wedge f(\lambda)(k_1) \wedge \beta$$

$$= \mu_1(q_1 * k_1, x, p_1) \wedge \mu_1(q_1, x, r_1) \wedge \beta \wedge \left(\vee\{\lambda(k) / k \in Q, f(k) = k_1\}\right)$$

$$= \vee \left\{ \begin{array}{l} \mu_1(q_1 * k_1, x, p_1) \wedge \mu_1(q_1, x, r_1) \wedge \lambda(k) \wedge \beta / \\ k \in Q, f(k) = k_1 \end{array} \right\}$$

Now let $p, q, r, k \in Q$ such that $f(p) = p_1, f(q) = q_1, f(r) = r_1$ and $f(k) = k_1$.

Then

$$\mu_1(q_1 * k_1, x, p_1) \wedge \mu_1(q_1, x, r_1) \wedge f(\lambda)(k_1) \wedge \beta$$

$$= \mu_1(f(q * k), x, f(p)) \wedge \mu_1(f(q), x, f(r)) \wedge \lambda(k) \wedge \beta$$

$$= \left(\vee\{\mu(q * k, x, a) / a \in Q, f(a) = f(p)\}\right) \wedge \left(\vee\{\mu(q, x, b) / b \in Q, f(b) = f(r)\}\right) \wedge \lambda(k) \wedge \beta$$

Since f is a strong homomorphism

$$= \vee \left\{ \begin{array}{l} \vee \left\{ \begin{array}{l} \mu(q * k, x, a) \wedge \mu(q, x, b) \wedge \lambda(k) \wedge \beta / \\ a \in Q, f(a) = f(p) \end{array} \right\} / \\ b \in Q, f(b) = f(r) \end{array} \right\}$$

$$\leq \vee \left\{ \begin{array}{l} \vee\, \lambda(a * b^{-1}) \vee \alpha / a \in Q, f(a) = f(p) \\ /b \in Q, f(b) = f(r) \end{array} \right\}$$

$$\leq f(\lambda)(p_1 * r_1^{-1}) \vee \alpha$$

Hence

$$f(\lambda)(p_1 * r_1^{-1}) \vee \alpha$$

$$\geq \vee \left\{ \begin{array}{l} \mu_1(q_1 * k_1, x, p_1) \wedge \mu_1(q_1, x, r_1) \wedge \lambda(k) \wedge \beta / \\ k \in Q, f(k) = k_1 \end{array} \right\}$$

$$= \mu_1(q_1 * k_1, x, p_1) \wedge \mu_1(q_1, x, r_1) \wedge f(\lambda)(k_1) \wedge \beta$$

Hence $f(\lambda)$ is a fuzzy kernel of $T = (Q_1, X, \mu_1)$.

3.3 Definition

Let $\alpha, \beta \in [0,1]$ and $\alpha < \beta$. Let $S = (Q, X, \mu)$ be a fuzzy semiautomaton. Let λ be a fuzzy subsemiautomaton of $S = (Q, X, \mu)$ with thresholds α, β . A fuzzy set ν of Q is called a fuzzy kernel of λ with thresholds if

(i) $\nu \subseteq \lambda$ and ν is a fuzzy normal subgroup of λ with thresholds α, β and

(ii) $\nu(p * r^{-1}) \vee \alpha \geq \mu(q * k, x, p) \wedge \mu(q, x, r) \wedge \nu(k) \wedge \beta$

for all $p, r, k \in Q$,

$q \in \text{supp}(\lambda), x \in X$

3.4 Definition

Let $\alpha, \beta \in [0,1]$ and $\alpha < \beta$. An fuzzy semiautomaton $T = (Q, X, \delta)$ over a finite group is called multiplicative with thresholds if there exists an e- input $x_0 \in X$ having the following properties

i. $\delta(q, x, p * r) \vee \alpha = \delta(q, x_0, p) \wedge \delta(e, x, r) \wedge \beta$

ii. $\delta(p_1 * p_2^{-1}, x_0, q_1 * q_2^{-1}) \vee \alpha = \delta(p_1, x_0, q_1) \wedge \delta(p_2, x_0, q_2) \wedge \beta$ for all $p_1, p_2, q_1, q_2 \in Q$

3.5 Proposition

Let $\alpha,\beta \in\ 0,1$ and $\alpha < \beta$. Let $T=\ Q,X,\delta$ be a multiplicative fuzzy semiautomaton with thresholds α,β over a finite group with e-input $x_0 \in X$. Let λ be fuzzy subsemiautomaton of $T=\ Q,X,\delta$ with thresholds α,β and ν be a fuzzy kernel of λ with thresholds α,β . If ν is a fuzzy normal subgroup of Q with thresholds α,β ,then ν is a fuzzy kernel of $T=\ Q,X,\delta$ with thresholds α,β .

Proof

$$\delta\ q*k,x,p\ \wedge\delta\ q,x,r\ \vee\alpha$$

$$=\delta\ q*k*e,x,p\ \wedge\delta\ q*e,x,r*e\ \vee\alpha$$

$$=\delta\ q*k,x_0,p\ \wedge\delta\ e,x,e\ \wedge\beta\wedge \quad \delta\ q,x_0,r\ \wedge\delta\ e,x,e\ \wedge\beta ,$$

By definition3.4

$$=\begin{pmatrix}\delta\ q*k,x_0,p\ \wedge\delta\ e,x,e\ \wedge \\ \delta\ q,x_0,r\ \wedge\beta\end{pmatrix}\vee\alpha$$

$$=\delta\ q*k,x_0,p\ \vee\alpha\wedge\delta\ e,x,e\ \wedge\delta\ q,x_0,r\ \wedge\beta$$

$$=\ \delta\ q,x_0,p\ \wedge\delta\ k^{-1},x_0,e\ \wedge\beta\ \wedge \quad \delta\ e,x,e\ \wedge\delta\ q,x_0,r\ \wedge\beta ,$$

By definition 3.4

$$=\delta\ q,x_0,p\ \wedge\delta\ k^{-1},x_0,e\ \wedge\delta\ e,x,e\ \wedge\delta\ q,x_0,r\ \wedge\beta$$

Now for any, $q\in Q$, $b\in$ supp λ

$q=b*\ b*q'^{-1}\ ^{-1}$ For some $q'\in Q$

$$\delta\ q*k,x,p\ \wedge\delta\ q,x,r\ \vee\alpha$$

$$=\delta\left(b*\ b*q'^{-1}\ ^{-1},x_0,p\right)\wedge$$
$$\delta\ k^{-1},x_0,e\ \wedge\delta\ e,x,e\ \wedge$$
$$\delta\left(b*\ b*q'^{-1}\ ^{-1},x_0,r\right)\wedge\beta$$

$$=\begin{pmatrix}\delta\left(b*\ b*q'^{-1}\ ^{-1},x_0,p\right)\wedge\delta\ k^{-1},x_0,e\ \wedge\delta\ e,x,e\ \wedge \\ \delta\left(b*\ b*q'^{-1}\ ^{-1},x_0,r\right)\wedge\beta\end{pmatrix}\vee\alpha$$

$$=\left(\delta\left(b*\ b*q'^{-1}\ ^{-1},x_0,p\right)\vee\alpha\right)\wedge$$
$$\delta\ k^{-1},x_0,e\ \wedge\delta\ e,x,e\ \wedge$$
$$\left(\delta\left(b*\ b*q'^{-1}\ ^{-1},x_0,r\right)\vee\alpha\right)\wedge\beta$$

$$=\delta\ b,x_0,p\ \wedge\delta\ b*q'^{-1},x_0,e\ \wedge\beta\wedge$$
$$\delta\ k^{-1},x_0,e\ \wedge\delta\ e,x,e\ \wedge$$
$$\delta\ b,x_0,r\ \wedge\ \delta\ b*q'^{-1},x_0,e\ \wedge\beta\ \wedge\beta$$

$$=\delta\ b,x_0,p\ \wedge\delta\ b*q'^{-1},x_0,e\ \wedge$$
$$\delta\ k^{-1},x_0,e\ \wedge\delta\ e,x,e\ \wedge\delta\ b,x_0,r\ \wedge\beta$$

Since ν is a fuzzy kernel of λ with thresholds

$$\nu\ p*r^{-1}\ \vee\alpha\ \geq\delta\ b*k,x,p\ \wedge\delta\ b,x,r\ \wedge\nu\ k\ \wedge\beta$$

$$\geq\ \delta\ b*k,x,p\ \wedge\delta\ b,x,r\ \wedge\nu\ k\ \wedge\beta\ \vee\alpha$$

$$\nu\ p*r^{-1}\ \vee\alpha\ \geq\ \delta\ b*k,x,p\ \vee\alpha\ \wedge \quad \delta\ b,x,r\ \vee\alpha\ \wedge\ \ \nu\ k\ \wedge\beta\ \vee\alpha$$

$$=\ \delta\ b*k,x_0,p\ \wedge\delta\ b,x_0,r\ \wedge\delta\ e,x,e\ \wedge\nu\ k\ \wedge\beta$$

By definition 3.

$$= \delta \; b, x_0, p \; \wedge \delta \; k^{-1}, x_0, e \; \wedge \delta \; b, x_0, r$$
$$\wedge \delta \; e, x, e \; \wedge \nu \; k \; \wedge \beta$$

By definition 3.4

$$= \; \delta \; q * k, x, p \; \wedge \delta \; q, x, r \; \vee \alpha \; \wedge \nu \; k$$

$$= \; \delta \; q * k, x, p \; \wedge \delta \; q, x, r \; \wedge \nu \; k \; \vee \alpha$$

$$\geq \; \delta \; q * k, x, p \; \wedge \delta \; q, x, r \; \wedge \nu \; k \; \wedge \beta \; .$$

4. CONCLUSION

In this paper we have proved many results using the definition of fuzzy subsemiautomata with thresholds and strong homomorphism. The fuzzy thresholds concept can be introduced to all the product concepts of fuzzy finite state machines.

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