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A Priority based Fuzzy Goal Programming to Multi-Objective Linear Fractional Programming Problem

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ABSTRACT
This paper deals with priority based fuzzy goal programming approach for solving multi-objective linear fractional programming problem. In the model formulation of the problem, we construct the fractional membership functions by determining the optimal solution of the objective functions subject to the system constraints. The fractional membership functions are then transformed into equivalent linear membership functions at the individual best solution point by first order Taylor series. In the solution process, fuzzy goal programming approach is used to solve problem by minimizing negative deviational variables. Then, sensitivity analysis is performed with the change of priorities of the fuzzy goals. Euclidean distance function is used to identify the appropriate priority structure in the decision-making situation. The efficiency of the proposed approach is illustrated by solving a numerical example.

General Terms
Multi-objective linear fractional programming.

Keywords
Fractional programming, Goal programming, Multi-objective linear fractional programming, Priority based fuzzy goal programming.

1. INTRODUCTION
In this paper, priority based fuzzy goal programming (FGP) is used to solve multi-objective linear fractional programming problem (MOLFPP). MOLFPP consists of multiple objectives, which are linear fractional in nature. The system constraints are linear functions.

Our main results in this paper are as follows: (i) a priority based FGP approach for solving MOLFPP is presented. (ii) We construct the fractional membership functions by determining individual optimal solution of the objective functions subject to the system constraints. Then we transform the fractional membership functions into equivalent linear membership functions by using first order Taylor series at the individual best solution point. (iii) Priority based FGP is used to solve the transformed MOLFPP. (iv) Sensitivity analysis with the change of the priority structure is performed and Euclidean distance function is used to obtain appropriate priority structure.

Rest of the paper is organized in the following way. In section 2, we present a brief literature review. Section 3 provides MOLFPP formulation. Subsection 3.1 describes fuzzy programming formulation to MOLFPP and subsection 3.2 discusses linearization of fractional membership functions by first order Taylor series. Subsection 3.3 presents priority based FGP model of MOLFPP. In section 4, selection of appropriate priority structure based on Euclidean distance function is presented. Section 5 provides priority based FGP algorithm to MOLFPP. In section 6, we provide a numerical example to show the efficiency of the proposed approach. Section 7 provides the concluding remarks.

2. LITERATURE REVIEW
In the area of multi criteria decision-making problems, the priority based goal programming (GP) is one of the powerful and robust techniques for solving decision problems with multiple and conflicting objectives. Ijiri [1] studied priority based GP at first. Ignizio [2], Lee [3], Steuer [4] and other researchers investigated priority based GP and applied to various types of practical problems. In the priority based GP, we group the objectives according to their priorities and then assign weight to the objectives [2]. The goals at the highest (first) priority level are taken to be infinitely more important than the goals of next (second) priority level. The goals at the second priority level are taken to be infinitely more important than the goals of third priority level and so on.

MOLFPP has been studied extensively in the literature by several researchers for the past several decades. MOLFPPs pose some computational difficulties, so MOLFPPs are converted into single objective linear fractional programming problem, and then solved by using the variable transformation method due to Charnes and Cooper [5] or by adopting the updating objective function method due to Bitran and Noveas [6]. Kornbluth and Steuer [7] discussed GP algorithm to MOLFPP. To overcome the computational difficulties for solving MOLFPPs, the concept of fuzzy sets has been introduced in the field of fractional programming [8, 9]. Linguistic variable concept of Zadeh [10-12] to fuzzy MOLFPP was introduced by Luhandjula [8]. Dutta et al. [9] extended Luhandjula's approach and solved MOLFPPs by fuzzy programming technique. Sakawa and Kato [13] presented interactive approach for solving MOLFPPs with block angular structure involving fuzzy numbers. Chakraborty and Gupta [14] discussed fuzzy set theoretic approach to MOLFPP by transformation of variables. Pal et al. [15] applied FGP procedure to MOLFPP by using the concept of variable change method. Minasian and Pop [16] pointed out certain shortcomings in the work of Dutta et al. [9] and presented the correct proof of theorem for obtaining the efficient solutions for MOLFPP. Sadjadi et al. [17] studied fuzzy inventory problem involving multi-objective linear fractional objectives. Guzel and Sivri [18] presented Taylor series solution approach to MOLFPP in crisp environment. Toksarı [19] used Taylor series for solving fuzzy MOLFPP. However, studies in this field are still not quite satisfactory. Development of efficient computational approach is one of the emerging areas of investigations.

In this paper, we transformed MOLFPP into multi-objective linear programming problem by first order Taylor series. Recently, Pramanik [20] modified FGP model of Pramanik and Roy [21, 22] and applied it to BLPP with fuzzy parameters.Then FGP approach due to Pramanik [20] is used for achieving highest degree of each of membership goals by minimizing negative deviational variables.

3. MOLFPP FORMULATION

The general formulation of MOLFPP can be formulated as:

$$\max Z_1(\bar{x}) = \frac{\bar{c}_1^T \bar{x} + \alpha_1}{\bar{d}_1^T \bar{x} + \beta_1}, \quad (1)$$

$$\max Z_2(\bar{x}) = \frac{\bar{c}_2^T \bar{x} + \alpha_2}{\bar{d}_2^T \bar{x} + \beta_2}, \quad (2)$$

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$$\max Z_k(\bar{x}) = \frac{\bar{c}_k^T \bar{x} + \alpha_k}{\bar{d}_k^T \bar{x} + \beta_k} \quad (3)$$

subject to

$$\bar{x} \in S = \left\{ \bar{x} \in \bar{R}^n \mid \bar{A}\bar{x} (\le, =, \ge) \bar{b} \right\} \quad (4)$$

Here, $\bar{c}_i^T, \bar{d}_i^T \in \bar{R}^n$, $\bar{A} \in \bar{R}^{m\times n}$, $\bar{b} \in \bar{R}^m$ and α_i, β_i (i = 1, 2, …, k) are constants. Here, S is assumed to be non empty, convex and compact in $\bar{R}^n$. Here, n is the total number of variables, m is the total number of constraints. The symbol T represents transposition.

3.1 Fuzzy programming formulation to MOLFPP

To formulate the fuzzy programming model of MOLFPP, the objective functions $Z_i(\bar{x})$ (i = 1, 2, …, k) would be transformed into fuzzy goals by assigning an imprecise aspiration level to each of the objectives.

Let us suppose that $Z_i^B = \max_{\bar{x}\in S} Z_i(\bar{x})$, $Z_i^W = \min_{\bar{x}\in S} Z_i(\bar{x})$ (i = 1, 2, …, k)

The fuzzy goal takes the form:

$$Z_i(\bar{x}) \underset{\sim}{\ge} Z_i^B, \text{ (i = 1, 2, …, k)}$$

Then, the fuzzy MOLFPP can be written as:

Find $\bar{x}$ (5)

so as to satisfy $Z_i(\bar{x}) \underset{\sim}{\ge} Z_i^B$, (i = 1, 2, …, k)

subject to

$$\bar{x} \in S = \left\{ \bar{x} \in \bar{R}^n \mid \bar{A}\bar{x} (\le, =, \ge) \bar{b} \right\}$$

Here, Z_i^B is the aspiration of the i-th objective function $Z_i(\bar{x})$ (i = 1, 2, …, k) and " $\underset{\sim}{\ge}$ " indicates the fuzziness of the aspiration level.

The membership function of i-th fuzzy objective goal can be formulated as:

$$\mu_i(\bar{x}) = \begin{cases} 1, & \text{if } Z_i(x) \ge Z_i^B \\ 1 - \dfrac{Z_i^B - Z_i(\bar{x})}{Z_i^B - Z_i^W}, & \text{if } Z_i^W \le Z_i(\bar{x}) \le Z_i^B \\ 0, & \text{if } Z_i(\bar{x}) \le Z_i^W \end{cases} \text{ (i = 1, 2, …, k)} \quad (6)$$

Here, Z_i^B and Z_i^W (i = 1, 2, …, k) are respectively the upper and lower tolerance limits of i-th fuzzy objective goal.

Then, the problem reduces to

$$\max \mu_i(\bar{x}) \text{ (i = 1, 2, …, k)} \quad (7)$$

subject to

$$\bar{x} \in S = \left\{ \bar{x} \in \bar{R}^n \mid \bar{A}\bar{x} (\le, =, \ge) \bar{b} \right\}.$$

3.2 Linearization of the fractional membership functions

Let, $\bar{x}_i^* = (x_{i1}^*, x_{i2}^*, ..., x_{in}^*)$ be the individual best solution of the fractional membership function $\mu_i(\bar{x})$ (i = 1, 2, …, k) subject to the system constraints, where n is the total number of variables of the system. Next, we linearize the fractional membership function $\mu_i(\bar{x})$ (i = 1, 2, …, k) at $\bar{x}_i^*$ into equivalent linear membership function by first order Taylor series. The transformed linear membership function can be written as:

$$\mu_i(\bar{x}) \cong \mu_i(\bar{x}_i^*) + [(x_1 - x_{i1}^*)\frac{\partial}{\partial x_1}\mu_i(\bar{x}_i^*) + (x_2 - x_{i2}^*)\frac{\partial}{\partial x_2}\mu_i(\bar{x}_i^*) + \ldots + (x_n - x_{in}^*)\frac{\partial}{\partial x_n}\mu_i(\bar{x}_i^*)] = \tilde{\mu}_i(\bar{x}) \; (i = 1, 2, \ldots, k) \tag{8}$$

3.3 Priority based FGP model of MOLFPP

The problem (7) discussed in subsection (3.1) reduces to the following problem:

$$\max \tilde{\mu}_i(\bar{x}) \; (i = 1, 2, \ldots, k) \tag{9}$$

subject to

$$\bar{x} \in S = \left\{ \bar{x} \in \bar{R}^n \mid \bar{A}\bar{x}(\le, =, \ge)\bar{b} \right\}.$$

Now, the highest degree of a membership goal is unity (one). So i-th membership goal with unity as the aspiration level can be formulated as:

$$\tilde{\mu}_i(\bar{x}) + d_i^- - d_i^+ = 1 \tag{10}$$

Here, $d_i^-(\ge 0)$ and $d_i^+(\ge 0)$ (i = 1, 2, …, k) represent the negative and positive deviational variables respectively. The maximum value of a membership goal is unity (one) so positive deviation is not possible here. Then due to Pramanik [20] only the negative deviational variable $d_i^-(\ge 0)$ (i = 1, 2, …, k) is required to minimize to get satisfycing solution. Therefore, (10) can be written as:

$$\tilde{\mu}_i(\bar{x}) + d_i^- = 1 \tag{11}$$

Then, the priority based FGP model for solving MOLFPP can be explicitly formulated as: find $\bar{x}$ so as to

$$\min \bar{\gamma} = [P_1(D^-), P_2(D^-), \ldots, P_r(D^-), \ldots, P_R(D^-)] \tag{12}$$

subject to

$$\mu_i(\bar{x}_i^*) + [(x_1 - x_{i1}^*)\frac{\partial}{\partial x_1}\mu_i(\bar{x}_i^*) + (x_2 - x_{i2}^*)\frac{\partial}{\partial x_2}\mu_i(\bar{x}_i^*) + \ldots + (x_n - x_{in}^*)\frac{\partial}{\partial x_n}\mu_i(\bar{x}_i^*)] + d_i^- = 1, \; (i = 1, 2, \ldots, k)$$

$$\bar{x} \in S = \left\{ \bar{x} \in \bar{R}^n \mid \bar{A}\bar{x}(\le, =, \ge)\bar{b} \right\},$$

$$d_i^- \ge 0 \; (i = 1, 2, \ldots, k).$$

Here, $\bar{\gamma}$ denotes the vector of R priority structure and $P_r(D^-)$ is a linear function of the weighted negative deviational variables.

Here, $P_r(D^-)$ can be written as:

$$P_r(D^-) = \sum_{i=i}^{k} w_{ri}^- d_{ri}^- \; (i = 1, 2, \ldots, k, \; r \le k) \tag{13}$$

Here, the negative deviational variable d_{ri}^- is renamed for d_i^- to represent it at the r-th priority level. w_{ri}^- $(0 \le w_{ri}^- \le 1)$ is the weight associated with d_{ri}^- at the r-th priority level. In the preemptive priority FGP, r-th priority P_r is preferred to the very next priority P_{r+1} regardless of weight associated with r + 1-th priority P_{r+1}.

The priority factors have the following relationships

$$P_1 >>> P_2 >>> \ldots >>> P_r >>> \ldots >>> P_R \tag{14}$$

The symbol '>>>' represent very much greater than i.e. the membership goals at the first priority level (P_1) are achieved to the extent possible before the set of membership goals at the next priority level (P_2) is considered. The process will be continued until the priority level P_R is considered.

However, if all the fuzzy goals are considered as equally important in a decision-making situation the priority based FGP model (11) will be transformed into the FGP model.

It may be noted that "too many" different priority structure can increase the computational burden to the decision maker. If R be the total priority levels, then there will be R! priority structure. However, in general two to five priority levels are important and the conflict of assigning priorities occurs at the most three priority levels in the decision making environment [2]. If some priority levels may provide infeasible solutions, then they are obviously discarded [23].

4. SELECTION OF APPROPRIATE PRIORITY STRUCTURE BASED ON EUCLIDEAN DISTANCE FUNCTION

Priorities are assigned to the goals based on the importance of achieving of the aspired levels of the goals. But, in the highly conflicting decision making environment, the decision maker feels confused with assigning proper priority structure for achieving the aspired goals.

In order to overcome such problem, the concept of Euclidean distance function is used for the proposed MOLFPP formulation. The concept of ideal point [24] has been widely used to several multi-objective decision-making to arrive at satisfactory solution [25-27].

In the present FGP formulation of the MOLFPP, since the aspired level of each of the membership goals is unity, the point comprising of the highest membership value of each of the goals would represent the ideal point. Let R be the total number of different possible priority structure. The Euclidean distance function can be defined as:

$$D_r = [\sum_{i=1}^{k}[1-\mu_i^r(\bar{x})]^2]^{1/2} \; (r = 1, 2, \ldots, R) \quad (15)$$

Here, $\mu_i^r(\bar{x})$ represents the achieved membership value of the t-th objective goal under the r-th priority structure. A decision can be considered as the most satisfactory decision for which the achieved membership values are found to be closest to the ideal point.

Let, $$\min_{r=1,2,\ldots,R} \{D_r\} = D_p,\; 1\le p \le R \quad (16)$$

Then, p-th priority structure can be identified as the appropriate priority structure.

5. PRIORITY BASED FGP ALGORITHM FOR MOLFPP

Following the above discussion, the proposed priority based FGP algorithm for solving MOLFPP can be presented as:

Step 1: Determine the individual best and worst solutions of each objective function $Z_i(\bar{x})$ (i = 1, 2, …, k) subject to the system constraints.

Step 2: Formulate the fractional membership function $\mu_i(\bar{x})$ (i = 1, 2, …, k) of i-th objective function $Z_i(\bar{x})$ (i = 1, 2, …, k).

Step 3: Find the individual best solution $\bar{x}_i^* = (x_{i1}^*, x_{i2}^*, \ldots, x_{in}^*)$ (i = 1, 2, …, k) of the fractional membership function $\mu_i(\bar{x})$ (i = 1, 2, …, k) subject to the system constraints.

Step 4: Transform the fractional membership function into equivalent linear membership function $\tilde{\mu}_i(\bar{x})$ (i = 1, 2, …, k) at the individual best solution point $\bar{x}_i^* = (x_{i1}^*, x_{i2}^*, \ldots, x_{in}^*)$ (i = 1, 2, …, k) by first order Taylor series approximation as given by (8).

Step 5: Formulate the priority based FGP model (12).

Step 6: Solve the problem (12).

Step 7: Euclidean distance function D_r to identify the most suitable priority structure.

Step 8: End.

6. NUMERICAL EXAMPLE

To illustrate the proposed priority based FGP, consider the following MOLFPP:

$$\max Z_1(\bar{x}) = \frac{x_1 + 2x_2 + 1}{x_1 + x_2 - 2},$$

$$\max Z_2(\bar{x}) = \frac{x_1 - 2x_2 + 2}{2x_1 - x_2 - 2},$$

$$\max Z_3(\bar{x}) = \frac{2x_1 - x_2 + 1}{2x_1 + x_2 - 3}$$

subject to

$-x_1 + 2x_2 \le 3$,

$3x_1 - x_2 \le 2$,

$x_1 + x_2 \ge 3$,

$x_1 \ge 0,\; x_2 \ge 0$.

The individual best solutions of the fractional objective functions subject to the system constraints are $Z_1^B = 6$ at (1, 2); $Z_2^B = 0.714$ at (1.4, 2.2); $Z_3^B = 1.4$ at (1.25, 1.75). The individual worst solutions subject to the constraints are $Z_1^W = 4.25$ at (1.4, 2.2); $Z_2^W = 0.2$ at (1.25, 1.75); $Z_3^W = 0.8$ at (1.4, 2.2).

Then, the fuzzy goals appear as follows:

$Z_1(\bar{x}) \gtrsim 6$, $Z_2(\bar{x}) \gtrsim 0.7143$, $Z_3(\bar{x}) \gtrsim 1.4$.

The fractional membership functions corresponding to the objective functions are as follows:

$$\mu_1(\bar{x}) = \frac{Z_1(\bar{x}) - 4.25}{6 - 4.25} = \frac{\frac{x_1 + 2x_2 + 1}{x_1 + x_2 - 2} - 4.25}{6 - 4.25},$$

$$\mu_2(\bar{x}) = \frac{Z_2(\bar{x}) - 0.2}{0.714 - 0.2} = \frac{\frac{x_1 - 2x_2 + 2}{2x_1 - x_2 - 2} - 4.25}{0.714 - 0.2},$$

$$\mu_3(\bar{x}) = \frac{Z_3(\bar{x}) - 0.8}{1.4 - 0.8} = \frac{\frac{2x_1 - x_2 + 1}{2x_1 + x_2 - 3} - 0.8}{1.4 - 0.8}$$

The fractional membership functions $\mu_1(x)$, $\mu_2(x)$ and $\mu_3(x)$ are maximal at the points (1, 2), (1.4, 2.2) and (1.25, 1.75) respectively subject to the system constraints.

Then, the fractional membership functions are transformed into linear membership functions at the individual best solution points by first order Taylor polynomial series as follows:

$$\tilde{\mu}_1(\bar{x}) = \mu_1(1, 2) + (x_1 - 1)\frac{\partial}{\partial x_1}\mu_1(1, 2) + (x_2 - 2)\frac{\partial}{\partial x_2}\mu_1(1, 2),$$

$\tilde{\mu}_1(\bar{x}) = 1 + (x_1 - 1) \times (-2.857) + (x_2 - 2) \times (-2.286)$,

$\tilde{\mu}_2(\bar{x}) = \mu_2(1.4, 2.2) + (x_1 - 1.4) \frac{\partial}{\partial x_1} \mu_2(1.4, 2.2) + (x_2 - 2.2) \frac{\partial}{\partial x_2} \mu_2(1.4, 2.2)$,

$\tilde{\mu}_2(\bar{x}) = 1 + (x_1 - 1.4) \times (0.595) + (x_2 - 2.2) \times (1.786)$,

$\tilde{\mu}_3(\bar{x}) = \mu_3(1.25, 1.75) + (x_1 - 1.25) \frac{\partial}{\partial x_1} \mu_3(1.25, 1.75) + (x_2 - 1.75) \frac{\partial}{\partial x_2} \mu_3(1.25, 1.75)$,

$\tilde{\mu}_3(\bar{x}) = 1 + (x_1 - 1.25) \times (-1.067) + (x_2 - 1.75) \times (-3.2)$.

Then priority based FGP formulation can be represented as:

Find $\bar{x}$ so as to

$\min \bar{\gamma} = [P_1(D^-), P_2(D^-), \ldots, P_r(D^-), \ldots, P_R(D^-)]$

subject to

$1 + (x_1 - 1) \times (-2.857) + (x_2 - 2) \times (-2.286) + d_1^- = 1$,

$1 + (x_1 - 1.4) \times (0.595) + (x_2 - 2.2) \times (1.786) + d_2^- = 1$,

$1 + (x_1 - 1.25) \times (-1.067) + (x_2 - 1.75) \times (-3.2) + d_3^- = 1$,

$-x_1 + 2x_2 \leq 3$,

$3x_1 - x_2 \leq 2$,

$x_1 + x_2 \geq 3$,

$x_1 \geq 0,\ x_2 \geq 0$.

The results, obtained by different priority structure are shown in the Table 1.

From the Table 1, we observe that the minimum Euclidean distance value is 0.78612. We also observe that the priority structure under the serial number 2, 4 & 5 are appropriate. The optimal solution set corresponding to the appropriate priority structure is given by $Z_1^* = 6,\ Z_2^* = 0.5,\ Z_3^* = 1$ at $x_1^* = 1,\ x_2^* = 2$.

The membership values are given by $\mu_1 = 1$, $\mu_2 = .583$, $\mu_3 = 0.333$

Note 1: All the solutions of the numerical example are obtained by the software, Lingo 6.0.

7. CONCLUSIONS

In this paper, priority based FGP approach is proposed to solve MOLFPP. Euclidean distance function is used to obtain a proper priority structure to reach the most satisfactory solution. The proposed approach can also be applied to production planning problems, agriculture planning problems, financial planning problems, transportation problems and other real world MOLFPPs. We hope that the concept presented here will open up new avenue of research in practical problems involving linear fractional objectives.

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Table1. Sensitivity analysis with the change of priority structure

Serial Number	Priority Structure	Solution Points	Objective values	Membership values	Euclidean distance
1	$[P^1(d_1^- + d_2^- + d_3^-)]$	1.25, 1.75	5.75, .2, 1.4	0.8571, 0, 1	1.0101
2	$[P^1(d_1^-), P^2(d_2^-), P^3(d_3^-)]$	1, 2	6, 0.5, 1	1, 0.5833, 0.3333	0.78612
3	$[P^1(d_3^-), P^2(d_1^-), P^3(d_2^-)]$	1.25, 1.75	5.75, .2, 1.4	0.8571, 0, 1	1.0101
4	$[P^1(d_1^- + d_2^-), P^2(d_3^-)]$	1, 2	6, 0.5, 1	1, 0.5833, 0.3333	0.78612
5	$[P^1(d_1^-), P^2(d_2^- + d_3^-)]$	1, 2	6, 0.5, 1	1, 0.5833, 0.3333	0.78612
6	$[P^1(d_2^-), P^2(d_1^- + d_3^-)]$	1.4, 2.2	4.25, .7143, .8	0, 0.9999, 0	1.4142

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