An Interactive Method for Multi Stage Fuzzy Lattice Decision Making Problems using Triangular Fuzzy Numbers

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ABSTRACT
The method for finding interactive multi- stage lattice fuzzy decision making problems in which all the parameters are represented by fuzzy numbers. In this interactive method a new algorithm is proposed to fuzzy lattice decision makings together with new representations of triangular fuzzy numbers. This paper will show the advantages of using proposed representations over the existing representations of fuzzy numbers and will present with great clarity the proposed method and illustrate numerical example.

Index terms: Fuzzy lattice, interactive method, triangular fuzzy numbers Convex set.

1. INTRODUCTION
In 1930's, the theory of the lattice order was proposed. Yaohuang Guo [3] establishes the lattice order decision making theory which starts a new direction of decision making. On the basis of these studies, interactive method of multi stage lattice fuzzy decision making is put forward to solve the problems using triangular fuzzy numbers. Multi - stage decision making problems usually arise when decisions are made in the sequential manner over time where earlier decisions may affect the feasibility and performance of later decisions. The multi stage decision making process can be separated into a number of sequential steps or stages, which is completed in one or more ways. The options for completing stages are known as decisions. The condition of the process at a given stage is known as state at that stage; each decision effect a transition from the current state to state associated with the next stage. The multi stage decision making process is finite if they are only a finite number of stages in the process and finite number of states associated with each each stage. The multi -stage decision process is deterministic if the outcome of each decision is known exactly.

In the lattice decision making, if the selected scheme can form limited lattice, the top factor will be the optimal scheme. If not, then take positive ideal solution (PIS) and negative ideal solution (NIS) as virtual schemes and regard them as top factors and bottom factors respectively. Delete the dominance scheme and construct a lattice. Choose the optimal solution or satisfying solution by comparing the closeness of scheme with positive ideal solution with that of negative ideal solution. The algorithm in this paper is firstly, weigh index value and establish positive and negative ideal solutions. Calculate the difference between every scheme and each ideal solution and make final choice. In this method, interactive is promising tool for dealing with multi stage decision making and lattice fuzzy problems under fuzziness.

2. PRELIMINARIES
In this section some basic definitions, existing representation of triangular fuzzy numbers, arithmetic operations between triangular fuzzy numbers are reviewed.

Definition 2.1: The characteristic function μ_A of a crisp set A C X assigns a value either 0 or 1 to each number in X. This function can be generalized to a function $\mu_{\hat{A}}$ such that the value assigned to the element of the universal set X fall within a specified image (i,e) $\mu_{\hat{A}} : X \to [0,1]$. The function $\mu_{\hat{A}}$ is called the membership function and the set $\hat{A} = \{ (x, \mu_{\hat{A}}(x)) / x \in X\}$ defined by $\mu_{\hat{A}}(x)$ for each $x \in X$ is called a fuzzy set.

Definition 2.2: A fuzzy set $\hat{A}$, defined on the universal set of real numbers R, is said to be a fuzzy number if its membership functions has the following characteristic;

(i) $\hat{A}$ is convex(i,e) $\mu_{\hat{A}}(\lambda x_1 + (1-\lambda) x_2) \geq \min \{ \mu_{\hat{A}}(x_1), \mu_{\hat{A}}(x_2) \}$ for all $x_1, x_2 \in R$,

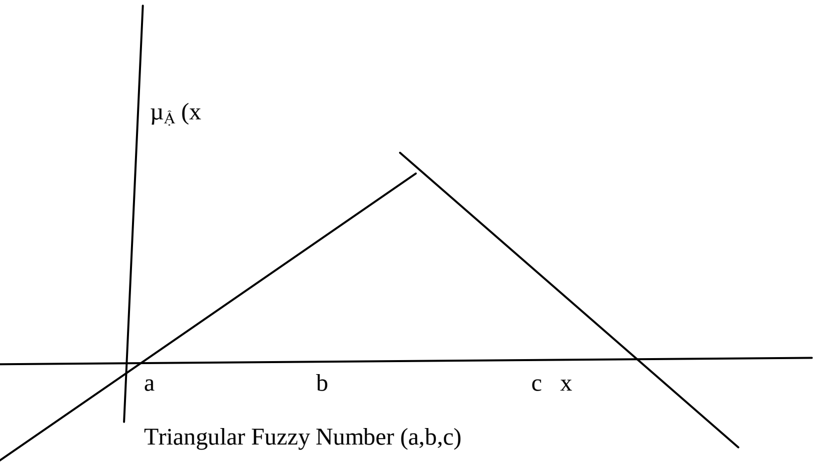
(ii) $\hat{A}$ is normal (i,e) There exists $x_0 \in R$ such that $\mu_{\hat{A}}(x_0) = 1$.

(iii) $\mu_{\hat{A}}(x)$ is piece wise continuous.

Definition 2.3:A fuzzy number $\hat{A}$ is called non- negative fuzzy number if $\mu_{\hat{A}}(x) = 0$.for all $x < 0$.

Definition 2.4: A fuzzy number $\hat{A} = (a,b,c)$ is said to a triangular fuzzy number if its membership function is given by

$$\mu_{\hat{A}}(x) = \begin{cases} 0 & \text{for } -\infty < x \leq a \\ x\text{-}a / b\text{-}a & \text{for } a \leq x < b \\ c\text{-}x/c\text{-}b & \text{for } b \leq x < c \\ 0 & \text{for } c \leq x < \end{cases}$$



Triangular Fuzzy Number (a,b,c)

A triangular fuzzy number $\hat{A}$ = (a,b,c) is said to be

(i) Zero triangular fuzzy number iff a =0 , b = 0, c = 0.

(ii) Non-Negative triangular fuzzy number iff a ≥ 0.

Definiton 2.5: Let $\hat{A}_1$ = (a_1,b_1,c_1) and $\hat{A}_2 = (a_2,b_2,c_2)$ be two triangular fuzzy numbers, then (i) $\hat{A}_1$ $\hat{A}_2$ = $(a_1 +a_2 , b_1 +b_2 , c_1 +c_2)$

(ii) $\hat{A}_1 \oplus \hat{A}_2 \otimes = (a^1,b^1,c^1)$ where $a^1 = \min \{a_1a_2, a_1c_2, c_1a_2,c_1c_2\}$

$b^1= b_1b_2$

$c^1 = \max \{ a_1a_2, a_1c_2, c_1a_2, c_1c_2 \}$

Definition 2.6: A lattice is a poset which has both Least Upper bound and Greatest Lower Bound.

Definition 2.7: Let µ be a fuzzy lattice ordered group of G and µ : X → G is called Fuzzy Lattice if
(FL1) µ (x +y) ≥ min {µ (x), µ (y)}
(FL2) µ (–x) ≥µ (x)
(FL3) µ (x V y) ≥min {µ (x), µ (y)}
(FL4) µ (x y) ≥ min { µ (x), µ (y)} for all x, y ε G.

Algorithms

F - Index matrix

F1 = normalized index matrix

R= decision matrix

M = fuzzy weighted index set

Sc = Scheme

d = distance.

D = difference between (+) ideal and (-) ideal solutions.

Step-1 : Form the normalized matrix using $F^1 = (u^1ij)^{mx4}$ unified method.

Step – 2; Construct weighted index decision matrix R = [rij] and $r^1ij = wj.x^1ij$ for all i , j.

Step-3: Choose $M^+ = (M_1, M_2, M_3)$ and $M^- = (m_1, m_2, m_3)$ as top factor and bottom factor.

Step- 4; Establish the difference between scheme Si (i = 1,2,3 …… n) and M^+

$Di^+ = \sqrt{\Sigma [d (rij , Mj)^2}$ for j = 1,2,3………. n

and $Di^- = \sqrt{\Sigma [d(rij. mj^1)^2}$ for j = 1,2,3 ………n

Step-5: Establish the comprehensive difference scheme S_i namely

$Di = q\ D_i^+ / D + (1-q) (1- D_i^- / D)$ Where i = 1,2,3 …..m , q [0,1].

Step-6: The decision maker chooses the optimal scheme according to the comprehensive difference D_i. The smaller of D_i is the better solution of the scheme.

3. NUMERICAL EXAMPLE

A company produce five products 1,2,3,4,5 whose positive and negative factor of the products are triangular fuzzy numbers as shown in Table- 1 and Table-2.

	Positive Factor
M1+	(90,100,105)
M2+	(70,85,90)
M3+	(5,10,15)
M4+	(10,15,20)
M5+	(25,35,45)

Table-1

	Negative Factor
M1-	(50,60,75)
M2-	(50,60,70)
M3-	(10,20,30)
M4-	(10,15,20)
M5-	(20,30,40)

Table-2

Step-1: Set the normalized matrix F_1^1 and F_2^1 using unified method

$$\begin{pmatrix} 90 & 100 & 105 \\ 75 & 85 & 90 \\ 5 & 10 & 15 \\ 10 & 15 & 20 \\ 20 & 35 & 45 \end{pmatrix}$$

and

$$\begin{pmatrix} 50 & 60 & 75 \\ 50 & 60 & 70 \\ 10 & 20 & 30 \\ 10 & 15 & 20 \\ 20 & 30 & 40 \end{pmatrix}$$

Step-2: Calculated the weighted matrix decision

$W^+ = M_1^+ - M_2^+ , M_2^+ - M_3^+ , M_3^+ - M_4^+ , M_4^+ - M_5^+$ and

$W^- = m_1^- - m_2^- , m_2^- - m_3^- , m_3^- - m_4^- , m_4^- - m_5^-$

Step-3 : Set the Top and bottom factor .

The positive ideal solution factor given by

(90-75) / 0.5 = 0.825, (75-70) / 0.2 = 0.390, (5-3) / 0.7 = 0.4323 ,(10-20) / 0.3 = -0.321,

(20-40) / 0.4 = -0.5421.

The negative ideal solution factor given by

(50-100) / .32 = -0.421, (50-85) /0 .4 = -0.451, (10-10) / 0.2 = 0 ,(10-15) / 0.3 = - 0.2421,

(20-35) / 0.4 = -0.4210.

Step-4: Set the difference scheme of D_i^+ and D_i^-

$D_1^+ = \sqrt{(90\text{-}50)^2 + (75\text{-}50)^2 + (5\text{-}10)^2 + (10\text{-}10)^2 + (20\text{-}20)^2} = 0.9754$

$D_2^+ = \sqrt{(100\text{-}60)^2 + (85\text{-}60)^2 + (10\text{-}20)^2 + (15\text{-}15)^2 + (35\text{-}30)^2} = 0.9254$

$D_3^+ = \sqrt{(105\text{-}75)^2 + (90\text{-}70)^2 + (15\text{-}30)^2 + (20\text{-}20)^2 + (45\text{-}40)2} = 0.8954$

and

$D_1^- = \sqrt{(75\text{-}105)^2 + (70\text{-}90)^2 + (30\text{-}15)^2 + (20\text{-}20)^2 + (40\text{-}45)^2} = 0.7412$

$D_2^- = \sqrt{(60\text{-}100)^2 + (60\text{-}85)^2 + (20\text{-}10)^2 + (15\text{-}15)^2 + (30\text{-}35)^2} = 0.6214$

$D_3^- = \sqrt{(50\text{-}90)^2 + (50\text{-}75)^2 + (10\text{-}5)^2 + (10\text{-}10)^2 + (20\text{-}20)^2} = \; = 0.7314$

Step-5: Establish the comprehensive difference scheme D_1^+, D_2^+, D_3^+, D_1^-, D_2^-, D_3^+

0.9754	0	0.9254
0	0.8954	0.7412
0.6214	0.7314	0

0.7412	0	0.6214
0.9754	0.7412	0
0	0.6214	0.7314

Step-6: Set the optimal solution of the comprehensive difference scheme D_i.

0.2	0.4	0.6
0.4	**0.3**	0.7
0.4	0.2	**0.1**

-0.1	-0.2	**-0.3**
-0.6	-0.5	-0.4
0.1	0.5	0.7

The optimal solution of the lattice decision maker is

$D_1^+ \rightarrow a_1$, $D_2^+ \rightarrow b_1$, $D_3^+ \rightarrow c_1$

$D_1^- \rightarrow c^1$,$D_2^- \rightarrow a^1$, $D_3^- \rightarrow a^1$

4. CONCLUSION

R.J. Li , established the concept of fuzzy multiple criteria decision making theory and application using triangular fuzzy numbers. In this paper, an interactive idea if lattice fuzzy multi stage decision n making problems is introduced using triangular fuzzy numbers under unified method.

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