Multi Agents System and Pareto Optimal Auctions

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ABSTRACT
In this work, we show that the results of the English auction multicriteria reverse based on the methods multi-criteria analysis are not optimal. We propose a model of multi-criteria auction sale which is intrinsically the Pareto optimum. We study the properties associated with to the multicriteria auction.

Keywords
Multi-agent system, MCA, Electronic Commerce, intelligent Agents.

1. INTRODUCTION
In a multi-agent system with agent's egocentric, one of the problems to be solved is how agents can reach a common understanding when they must coordinate their actions. Agents can be in competition for system resources or can be forced to ask other agents to perform actions on their behalf. Since the agents are egocentric, cooperation is far from guaranteed. Negotiation is the mechanism by which agents can reach a common agreement [2].

An approach to trading is considered by auction theory. The latter has generated much interest in the fields of economy and the Internet [5]. Auction protocols are implemented Dutch auctions, sealed bids in envelopes [9], the Vickrey auction [11] and the English auction. In many economic situations, ranging from banking to mobile operators, for example, products are no longer as unique characteristic of the price but other criteria, then the purchasing guidelines have established bid multicriteria which allows the taking into account not only price, but many other parameters [12] [10].

A reverse English auction is an iterative multi several stages, with a closing date where sales agents are competing to sell a product to a buyer agent. In each round of bidding, the buyer agent collects all proposals (one by Sales Agent), selects the best proposal as a proposal of reference for the next stage, the seller waits and makes the corresponding constraint for the ride Next.

One of the main components for multicriteria auction is the methods used to select the best offer therefore the use of multi criteria analysis is needed. Many methods are tools for decision support have been proposed to enable decision makers to make a "good" choice. The Specialists in multicriteria Support Decision agreed on three main [8]. The method of over-classification allows to compare the actions in pairs and to check whether, under certain predetermined conditions, one of the two outperforms the other actions or not. From all these comparisons, we then attempt a synthesis [6]. The methods based on utility theory produces a function that can classify all the actions of the best to worst, and other methods such as AHP (Analytical Hierarchy Process) and MACBETH (Measuring Attractiveness by a Categorical

Based Evaluation Technique) .

In this work, we opt for a negotiation mechanism using a reverse English auction multicriteria protocol. According to the classification adopted previously for multi-criteria analysis methods, we present the short comings of models of the auction based on the methods of over-ranking (ELECTRE I [1], PROMETHEE I [3]), and the theory of utility (Goal Programming [4], the harmonic mean), for optimal results in Pareto through an example of an auction where a buyer negotiates with three vendors (α, β, δ) purchase of a vehicle described by seven criteria.

Finally we propose a model of auction allows for optimal solutions in Pareto.

2. MULTICRITERIA MODEL
At first, the buyer must express these preferences; it can do it using a module that will be implemented by a class of preference relation.

The auction consists of several steps and every step the buyer agent selects the best proposal to be the proposal of reference, which implies the need for a multicriteria method for selecting the best offer by this agent. Any new proposal must beat the best proposal received in the previous round; we need a mechanism to ensure it [7].

In this section, we consider the basic concepts and notations:

- We have a set of P vendor noted S that compete to sell a product to a single buyer, denoted B.
- V is the set of solutions.
- Let $\xi = \{\xi_1,...,\xi_q\}$ a set of q criteria set by the buyer: $\xi_j : V \to E_j$ as E_j is an ordered set (these are public criteria for sellers).
- Is the dominance relation defined as follows:

$$\xi(a) \succcurlyeq \xi(b) \Leftrightarrow \xi_i(a) \geq \xi_i(b), \forall i = \{1,...,q\}$$
and $\exists k \in \{1,...,q\}, \xi_k(a) > \xi_k(b)$.

- Let k the maximum duration of the auction and $k_r, \forall r \in N$, the time of round r.
- $b_i^j(k_r)$ means the j[th] attribute r[th] the offer by the seller s_i if the index attribute is omitted, all attributes are considered.
- ε is not more or less great choice by the buyer.

3. AUCTION BASED ON UTILITY THEORY

3.1 Goal Programming

The best proposal is selected by the buyer agent on a function that defines the class of preference relations P_{GP} .

$$P_{GP} = \{\geqslant_{GP} \subset VxV, \forall b_i, b_j \in V,\ b_i \geqslant_{GP} b_j \Leftrightarrow d(b_i, C_*) < d(b_j, C_*)\}.$$ Where

$$d(b_i, C_*) = \left\{ \sum_{j=}^{g} \left| c_*^{J} - b_i^{j} \right|^{p} \right\}^{\frac{1}{p}}$$ and

$C_* = \{\max(score^j), \forall j \in \{1,...,q\}\}$: the ideal point in the space of criteria corresponding to the vector levels of aspiration.

Similarly any proposal the vendor is defined by a class of preference as follows:

$$P'_{GP} = \{\geqslant_{GP'} \subset VxV, \exists \varepsilon \in R^*, \forall b_i, b_j \in V,\ b_i \geqslant_{GP'} b_j \Leftrightarrow d(b_i, C_*) < d(b_j, C_*) - \varepsilon\}.$$

Experimentation: We present an example of an auction where a buyer negotiates with 3 sellers purchase a vehicle described by seven criteria (Tab 1).

The increment (ε) is set at 0.01. The weight mentioned above reflects the importance given to each setting.

Table 1.The user preferences

	brand			price		Air bague		ABS	
Poids	0.25			0.25		0.1		0.1	
Valeurs	α	β	δ	2000	6000	Oui	Non	Oui	Non
Scores	7	10	10	15	5	15	5	15	5

	dors				bagage		speed	
Poids	0.1				0.15		0.05	
Valeurs	2	3	4	5	200	1000	150	250
Scores	9	10	14	15	5	20	10	12

The auctions are conducted in four phases and end on an agreement with the seller for the proposal β_4 distances 16, 0.

Table 2.The evaluation of proposals by GP

p=1							
offer	price	Air bague	dors	ABS	bagage	speed	d
α_1	3098	N	3	N	320	185	39,7
α_2	3170	N	3	O	320	185	34,9
α_3	3250	O	5	O	320	185	20,1
α_4	3200	O	5	O	320	185	20,05
β_1	3073	N	5	N	380	170	30,9
β_2	3505	N	5	O	380	170	26,9
β_3	3545	O	5	O	380	170	17,0
β_4	3113	O	5	O	380	170	16.0
δ_1	3085	N	5	N	380	169	30,9
δ_2	3185	N	5	O	380	169	16,2
δ_3	3718	O	5	O	380	169	17,5
δ_4	3533	O	5	O	380	169	17,0

3.2 The harmonic mean

In our multicriteria auction, the proposals are compared using an aggregate function that defines the class of preference relations

$$P_{h(a)} = \{\geqslant_{w} \subset VxV, \forall b_i, b_j \in V,\ b_i \geqslant_{w} b_j, \forall j \in \{1,...,q\} \Leftrightarrow h(b_i) \geq h(b_j)\}.$$

Such as $h(b_i)\frac{1}{b_i}$.

Any proposal of the seller is defined by a class of preference as follows:

$$P'_{h(a)} = \{\geqslant_{w'} \subset VxV, \exists \varepsilon \in R^+, \forall b_i, b_j \in V,\ b_i \geqslant_{w'} b_j \Leftrightarrow h(b_i) > h(b_j) + \varepsilon\}.$$

Experiment: we evaluate the results of the previous example:

Table 3.Result of the harmonic mean

brand	Modèl	Harmonique
α	α_1	8.17
	α_2	8.49
	α_3	10.63
	α_4	11.38
β	β_1	9.16
	β_2	9.48
	β_3	11.56
	β_4	11.71
δ	δ_1	9.16
	δ_2	9.56
	δ_3	11.49
	δ_4	11.56

As shown in the table above the winner of this sale is the vendor β with the 4th proposal (β4).

Properties

- The method of goal programming selects the feasible solution that achieves the decision maker's objectives.

- Based on the solution of satisfaction, the technique of goal programming is very realistic.

- The problem of compensation in both models.

– Both models give satisfactory solutions rather than optimal.

4. AUCTION BASED ON THE METHODS OF OVER-CLASSIFICATION

4.1 ELECTRE I

The best proposal is selected by the buyer agent on a function that defines the class of preference relations $P_{ELECTRE1}$

$$P_{ELECTRE1} = \{ \succcurlyeq_{\text{ELECTR}} \subset VxV, \forall b_i, b_j \in V, \exists \hat{c}, \hat{d} \in [0,1],$$

$$b_i \succcurlyeq_{\text{ELECTRE1}} b_j \Leftrightarrow C(b_i, b_j) \geq \hat{c} \wedge D(b_i, b_j) \leq \hat{d} \}$$

where $C(b_i, b_j) = \dfrac{\sum k_{j_{\forall j: b_i^j \geq b_j^j}}}{k}$ Avec $k = \sum_{j=1}^{q} k_j$

and $D(b_i, b_j)$ is defined by :

$D(b_i, b_j) = 0, \forall j$ if $b_i^j \geq b_j^j$ else $D(b_i, b_j) = \dfrac{1}{\delta} \max[b_j^j - b_i^j]$:

With δ is the maximum difference between the same criteria to two offers.

Each new proposal must verify the following two conditions:

$C(b_i(k_{r+1}), b_f(k_r)) > C(b_i(k_r), b_f(k_r)) + \varepsilon$ and

$D(b_i(k_{r+1}), b_f(k_r)) > D(b_i(k_r), b_f(k_r)) - \varepsilon$

We contain the same example above, by multiplying the weight by 10, then we obtain the following results (Tab 4):

Table 4. Result of the Electre 1

Concordance round 1

	α1	β1	δ1
α_1	-	0.25	0.25
β_1	0.95	-	1
δ_1	0.75	0.7	-

Discordance round 1

	α1	β1	δ1
α1	-	0.5	0.5
β 2	0.03	-	0
δ 3	0.032	0.03	-

Concordance round 2

	α2	β2	δ2
α_1	-	0.5	0.5
β_2	0.7	-	0.75
δ_2	0.7	0.95	-

Discordance round 2

	α2	β2	δ2
α2	-	0.5	0.5
β2	0.083	-	0.083
δ2	0.032	0.002	-

Concordance round 3

	α_1	β_2	δ 3
α_3	-	0.6	0.6
β_3	0.7	-	1
δ_3	0.7	0.7	-

Discordance round 3

	α3	β 3	δ 3
α3	-	0.3	0.3
β 3	0.073	-	0
Δ3	0.032	0.043	-

Concordance round 4

	α4	α4	δ4
α_4	-	0.6	0.6
β_4	0.7	-	1
δ_4	0.7	0.7	-

Discordance round 4

	α4	β 4	δ 4
α4	-	0.3	0.3
β4	0.03	-	0
δ 4	0.003	0.083	-

The end result of this auction is $\beta_4 \succcurlyeq \delta_4$, $\delta_4 J \beta_4$ et $\alpha_4 J \delta_4$.

PROMETHEE I

The winning proposal in this method is defined as a class of preference relations $P_{promethee1}$:

$$P_{promethee1} = \{ \succcurlyeq_{\text{promethee1}} \subset VxV, \forall b_i, b_j \in V,$$

$$b_i \succcurlyeq_{\text{Promethee1}} b_j$$

$$\Leftrightarrow \begin{bmatrix} \left(\Phi^+(b_i) > \Phi^+(b_j)\right) \wedge \left(\Phi^-(b_i) < \Phi^-(b_j)\right) \\ \vee \\ \left(\Phi^+(b_i) = \Phi^+(b_j)\right) \wedge \left(\Phi^-(b_i) < \Phi^-(b_j)\right) \\ \vee \\ \left(\Phi^+(b_i) > \Phi^+(b_j)\right) \wedge \left(\Phi^-(b_i) = \Phi^-(b_j)\right) \end{bmatrix}$$

}

Where

$$\Phi^+(b_i) = \frac{1}{p-1} \sum \pi(b_i, b_t), \Phi^-(b_i) = \frac{1}{p-1} \sum \pi(b_t, b_i)$$

Such as $\pi(b_i, b_t) = \sum w_c . P_c(b_i, b_t)$ et

$P_c(b_i, b_t) = b_i^c - b_t^c, \forall t \in \{1,..., p\}$

Each new proposal $b_f(k_{r+1})$ must verify the following two conditions:

$$\left[\begin{array}{l} \left\{ \begin{array}{l} \Phi^+(b_i(k_{r+1})) > \Phi^+(b_f(k_r)) + \varepsilon \\ \wedge \\ \Phi^-(b_i(k_{r+1})) < \Phi^-(b_f(k_r)) - \varepsilon \end{array} \right\} \\ \vee \\ \left\{ \begin{array}{l} \Phi^+(b_i(k_{r+1})) = \Phi^+(b_f(k_r)) \\ \wedge \\ \Phi^-(b_i(k_{r+1})) < \Phi^-(b_f(k_r)) - \varepsilon \end{array} \right\} \\ \vee \\ \left\{ \begin{array}{l} \Phi^+(b_i(k_{r+1})) > \Phi^+(b_f(k_r)) + \varepsilon \\ \wedge \\ \Phi^-(b_i(k_{r+1})) = \Phi^-(b_f(k_r)) \end{array} \right\} \end{array} \right]$$

}

The results of this method are shown in Table 5.

Table5.Result of a method Promethee1

	1st round	2nd round	3rd round	4th round
	Π	π	Π	π
(α, β)	-14,19	-11,94	-7,19	0,41
(α, δ)	-14,10	-13,93	-6,10	-6,94
(β, δ)	0,08	-1,99	1,09	-7,36
(β, α)	14,19	11,94	7,19	-0,418
(δ, α)	14,10	13,93	6,10	6,94
(δ, β)	-7,58	1,99	-1,09	7,36

	1st_round	2nd_round	3th_round	4th_round
$\Phi^+(\alpha)$	-14,15	-12,93	-6,64	-3,26
$\Phi^-(\alpha)$	14,15	12,93	6,648	3,26
$\Phi^+(\beta)$	7,13	4,97	4,142	-3,89
$\Phi^-(\beta)$	-10,88	-4,97	-4,142	3,89
$\Phi^+(\delta)$	3,26	7,96	2,50	7,15
$\Phi^-(\delta)$	-7,01	-7,96	-2,50	-7,15

- Methods of classification are Partially compensatory
- Methods of classification are participate of the spirit of finding a solution "satisfactory", they speak often of compromise solution and not optimal.

5. THE PROPOSED MODEL

5.1 Description

The auction is initiated by the buyer agent who collects the preferences of the buyer. Then he opened the auction by sending all sellers these preferences, and the parameters of such sale (the length of a leg of a tour and the closing date of the auction...).

Each stage of the auction is divided into several rounds, during each step taken a reverse English auction united criteria of several towers, that is to say, the buying agent collects all proposals (one by sales agent) - regarding this criterion, only selects the best proposal as a proposal of reference for the next round, the seller waits and makes the corresponding constraint for the next round.

Each sales agent must send the proposal before the end of the round, otherwise it is discarded. The result at each stage is the best value achieved for the test during the negotiation stage (Pareto optimal). Before beginning the next step to negotiate on another criterion, sellers must put the agreement on the final value of the test trading at the previous step, if a seller does not accept this value can then be drawn from the auction, which is why it is necessary from the start classifying criteria of "less important» (i.e. which does not initially the sellers) and start the auction by him, "the most important " This classification can be USING multicriteria methods (type problems: β, γ).

The auction ends on the current best proposal, or when all sellers except one have either abandoned or when the closing date is reached.

The auction is synchronous, each sales agent must send the proposal before the end of the round, otherwise it is discarded. The values for each criterion for the buying agent and trader are converted into scores and at each stage the problem is to maximize the matching scores for the criterion values traded.

5.1.1 Strategy for the buyer agent

1. Classification criteria dealt
2. Scores are manipulated directly (conversion values for scores)

If the criterion is discreet, then their value is associated

Else

We use the following formula:

$$\frac{(v_i - \min_i).(T[\max_i] - T[mim_i])}{\max_i - \min_i} + T[\min_i]$$

v_i : Is the value to convert

$\min_i$: The minimum value for the criterion i

$\max_i$: The maximum value for the criterion i

$T[v_i]$: The score combines the value v_i .

Initially, $C = \{c_1,..., c_q\}$, and $b_{ref}^{i_0} = 0, \forall i \in \{1,..., q\}$.

3.1 The buyer agent repeats the following steps for each stage i:

- Put $\varepsilon^i = \{\varepsilon_i\}$
- Start_Timer $(T_{étape})$
- Wait (T_{Accord})
- For each Agreement C_j received, delete C_k for C as $k \neq j$ and put j in S^i
- The buyer agent repeats the following steps for each round r
    - Start_Timer (T_{Tour})
    - Receipt of proposals $b_j^{i_r}, \forall i \in S^i$
    - Test if $b_j^{i_r} \leq b_{ref}^{i_{r-1}}$ then $\forall j \in S^i, S_j$, was removed to put $S_i = S_i - S_j$ and $S^i = S^i - \{j\}$
    - Selected the winner $b_j^{i_r} \succcurlyeq b_k^{i_r} \Leftrightarrow b_j^{i_r} > b_k^{i_r}$.
    - Calculate $b_{final}^{i_r} = Max(b_j^{i_r}), \forall j \in S^i$.
    - Define the constraint for the next round $b_{final}^{i_r} = b_{final}^{i_r} + \varepsilon$
    - Send $b_{final}^{i_{r+1}}$.
    - Stop_timer (T_{tour})

5.1.2 Vendors Strategy

1. Receipt of buyer preferences and initial bid
2. The sales agent repeats the following steps for each step i :

- If the value $b_{final}^{i_{r-1}}$ is accepted then send the agreement C_i .
- The sales agent repeats the following steps for each round r .
    - Formulate and send the proposal $b_j^{i_r}$
    - Reception $b_{ref}^{i_r}$
    - If the value $b_{ref}^{i_r}$ is not accepted then abandoned

The result of this sale is an offer

$$b_{Enchère} = (Max(b_j^{i_r}), Max(b_j^{i+1_r}),..., Max(b_j^{q_r})),$$
$$\forall i \in \{1,...,q\} \text{ et } j \in \{1,..., n\}.$$

While it is impossible that the latter is dominated by any other offer $(b'_{Enchère})$. I.e. it can be improved with respect to a criterion without deteriorating for another.

- **Effectiveness**: An auction is efficient when any vendor, except perhaps the winning vendor, can provide a better proposal than the winning proposal by the preference relation of the buyer. Formally, the auction is efficient If $\exists b \in V / b_{Enchère} \Leftrightarrow$ if $\exists b \in V / b > b_{Enchère}$,

or $b_{Enchère} = (Max(b_j^{i_r}), Max(b_j^{i+1_r}),..., Max(b_j^{q_r})),$
$\forall i \in \{1,...,q\} \text{ et } j \in \{1,..., n\} \Rightarrow$ donc notre enchère logically effective. It should be noted that the efficiency can be

guaranteed only under the assumption that the auction closes naturally and not because the finite time.

- **Non-dominance winning proposals**: In the auction, the winning bid must not be always dominated.

Formally b_i s not dominated iff no b_k such that b_k b_i .

At each stage the winning entry is of the form $b^1_{final} = Max(b^{1_r}_j),...,b^i_{final}, null,..., null)$ therefore it is impossible the existence of b_k such that $b_k \geqslant b_i$.

The result of our auction is the supply $b_{Enchère} = (Max(b^{i_r}_j), Max(b^{i+1_r}_j),..., Max(b^{q_r}_j)),$
$\forall i \in \{1,...,q\}$ et $j \in \{1,...,n\}$.

So it is obvious that $b_{enchère}$ cannot be dominated (the dominance relation " is'>').

- **Fair competition of non-dominated proposals:** an auction met the fair competition of non-dominated proposals if any proposal for non-dominated is likely to be selected. The negotiation at each stage i for one and only selection criterion is proposed after the maximum value for the latter, then it is clear that a proposal b has a greater value on this criterion wins if it is given by a seller $\in s^I$.

- **The Pareto-optimality**: An auction is the Pareto optimal if the winning bid is better in any other proposals on all criteria. The winning bid is: $b_{Enchère} = (Max(b^{i_r}_j), Max(b^{i+1_r}_j),..., Max(b^{q_r}_j)),$
$\forall i \in \{1,...,q\}$ et $j \in \{1,...,n\}$ that maximizes all the values matching the criteria.

Experiment: We try to adapt the example described above with our model.The winner of this sale is the vendor β with an offer that bears the following values (assuming that the seller accepts these values):

criteria	brand	price	Air bague	doors	ABS	bagage	speed
order	7	6	1	2	3	4	5
values	β	3073	O	5	O	380	170

The seller α is excluded from the auction because he did not have a car that has a luggage capacity 380cc, and the seller δ is excluded because its top speed of 169km / H.

6. CONCLUSION

In this work, we presented a negotiation mechanism using a reverse English auction protocol based on methods of multicriteria decision support. Thus we have proposed a model.

The interest of the latter is to obtain an auction not only optimal, but Pareto, and no compensation between criteria.

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