On α-continuous Intuitionistic Fuzzy Multifunctions

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ABSTRACT
In 1999, Ozbakir and Coker [23] introduced the concept intuitionistic fuzzy multifunctions and studied their lower and upper intuitionistic fuzzy semi continuity from a topological space to an intuitionistic fuzzy topological space. The present paper introduces the concept of α-continuous intuitionistic fuzzy multifunctions. An Intuitionistic fuzzy multifunction F from a topological spaces $(X,\mathcal{T})$ to an intuitionistic fuzzy toplogical spaces (Y,Γ) is said to be Intuitionistic fuzzy α-continuous at a point $x_0 \in X$ if for any $\widetilde{G}_1, \widetilde{G}_2 \in IFO(Y)$ such that $F(x_0) \subset \widetilde{G}_1$ and $F(x_0) \cap \widetilde{G}_2$ there exists $U \in \alpha O(X)$ containing x_0 such that $F(u) \subset \widetilde{G}_1$ and $F(u) \cap \widetilde{G}_2, \forall u \in U$. F is called Intuitionistic fuzzy α-continuous if it has this property at each point of X. Several properties and characterizations of Intuitionistic fuzzy α-continuous

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1. INTRODUCTION
After the introduction of fuzzy sets by Zadeh [29] in 1965 and fuzzy topology by Chang [6] in 1967, several researches were conducted on the generalizations of the notions of fuzzy sets and fuzzy topology. The concept of intuitionistic fuzzy sets was introduced by Atanassov [2,3,4] as a generalization of fuzzy sets. In the last 27 years various concepts of fuzzy mathematics have been extended for intuitionistic fuzzy sets. In 1997 Coker [7] introduced the concept of intuitionistic fuzzy topological spaces as a generalization of fuzzy topological spaces. In 1999, Ozbakir and Coker [23] introduced the concept intuitionistic fuzzy multifunctions and studied their lower and upper intuitionistic fuzzy semi continuity from a topological space to an intuitionistic fuzzy topological space. In the present paper we introduce the concepts of intuitionistic fuzzy α-continuous multifunctions and obtain some of their characterizations and properties.

2. PRELIMINARIES
Throughout this paper $(X,\mathcal{T})$ and (Y, Γ) represents a topological space and an intuitionistic fuzzy topological space respectively. A subset A of a topological space $(X,\mathcal{T})$ is called Semi open [11] (resp. α-open[19]) if $A\subset Cl(Int(A))$ (resp.$A\subset Int\big(Cl(Int(A))\big)$. The complement of a semi open (resp. α-open) set is called semi closed (resp. α-closed). Every open (resp. closed) set is α-open (resp. α-closed) and every α-open (resp. α-closed) set is semi open (resp. semi closed) ,but the converses may not be true..The family of all α-open (resp. α-closed) subsets of topological space$(X,\mathcal{T})$ is denoted by $\alpha O(X)$ (resp. $\alpha C(X)$). The intersection of all α-closed (resp. semi closed) sets of X containing a set A of X is called the α-closure [14] (resp. semi closure) of A. It is denoted by $\alpha Cl(A)$ (resp. sCl(A)). The union of all α-open (resp. semi open) sub sets of A of X is called the α-interior [14] (resp. semi interior) of A .It is denoted by $\alpha Int(A)$ (resp. sInt(A)) . A subset A of X is α-closed (resp. semi closed) if and only if $A \supset Cl(Int(Cl(A)))$ $(resp. A \supset Int\big(Cl(A)\big)$. A subset N of a topological space$(X,\mathcal{T})$ is called a α-neighborhood [14] of a point x of X if there exists a α-open set O of X such that $x \in O \subset N$. A is a α -open in X if and only if it is a α -neighborhood of each of its points. A subset V of X is called a α -neighborhood of a subset A of X if there exists $U \in \alpha O(X)$ such that $A \subset U \subset V$. A mapping f from a topological space $(X,\mathcal{T})$ to another topological space $(X^*,\mathcal{T}^*)$ is said to be α-continuous [15, 16] if the inverse image of every open set of X* is α -open in X. Every continuous mapping is α-continuous but the converse may not be true [15]. A multifunction F from a topological space $(X,\mathcal{T})$ to another topological space $(X^*,\mathcal{T}^*)$ is said to be lower α-continuous [18] (resp. upper α-continuous[18]) at a point $x_0 \in X$ if for every α-neighborhood U of x_0 and for any open set W of X* such that $F(x_0) \cap W \neq \emptyset$ (resp. $F(x_0) \subset W$) there is a α-neighborhood U of x_0 such that $F(x) \cap W \neq \emptyset$ (resp. $F(x) \subset W$) for every $x \in U$.

Lemma 2.1[25]: Let A be a subset of a topological space $(X,\mathcal{T})$. Then:
(a) A is α-closed in X $\Leftrightarrow$sInt(Cl(A) ⊂ A;
(b) sInt(Cl(A)) = Cl(Int(Cl(A)));
(c) αCl(A) = A∪Cl(Int(Cl(A))).

Lemma 2.2[25]: Let A be a subset of a topological space $(X,\mathcal{T})$. Then the following conditions are equivalent :
(a) $A \in \alpha O(X)$
(a) $U \subset A \subset Int(Cl(U))$ for some open set U.
(b) $U \subset A \subset sCl(U)$ for some open set U.
(c) $A \subset sCl(Int(A))$.

Definition 2.1 [2, 3, 4]: Let Y be a nonempty fixed set. An intuitionistic fuzzy set $\tilde{A}$ in Y is an object having the form

$$\tilde{A} = \{< x, \mu_{\tilde{A}}(y), \upsilon_{\tilde{A}}(y) > y \in Y \}$$

where the functions $\mu_{\tilde{A}}$:Y→I and $\upsilon_{\tilde{A}}$:Y→I denotes the degree of membership (namely $\mu_{\tilde{A}}$(y)) and the degree of non membership (namely $\upsilon_{\tilde{A}}$(y)) of each element y∈Y to the set $\tilde{A}$ respectively, and $0 \le \mu_{\tilde{A}}(x) + \upsilon_{\tilde{A}}(x) \le 1$ for each y∈Y.

Definition 2.2 [2, 3, 4]: Let Y be a nonempty set and the intuitionistic fuzzy sets $\tilde{A}$ and $\tilde{B}$ be in the form

$\tilde{A} = \{< y, \mu_{\tilde{A}}(y), \upsilon_{\tilde{A}}(y) > : y \in Y\}$,
$\tilde{B} = \{< y, \mu_{\tilde{B}}(y), \upsilon_{\tilde{B}}(y) > : y \in Y\}$ and let
$\{\widetilde{A_\alpha} : \alpha \in \Lambda\}$ be an arbitrary family of intuitionistic fuzzy sets in Y. Then:
(a) $\tilde{A} \subseteq \tilde{B}$ if $\forall y \in Y$ [$\mu_{\tilde{A}}(y) \le \mu_{\tilde{B}}(y)$ and $\upsilon_{\tilde{A}}(y) \ge \upsilon_{\tilde{B}}(y)$];
(b) $\tilde{A} = \tilde{B}$ if $\tilde{A} \subseteq \tilde{B}$ and $\tilde{B} \subseteq \tilde{A}$;
(c) $\tilde{A}^C = \{ < y, \upsilon_{\tilde{A}}(y), \mu_{\tilde{A}}(y) > : y \in Y\}$;
(d) $\tilde{0}$ ={<y, 0,1>:y∈Y} and $\tilde{1}$ ={<y,1,0>:y∈Y}
(e) $\cap \widetilde{A_\alpha} = \{< y, \wedge \mu_{\tilde{A}}(y), \vee \upsilon_{\tilde{A}}(y) > : y \in Y\}$;
(f) $\cup \widetilde{A_\alpha} = \{< y, \vee \mu_{\tilde{A}}(y), \wedge \upsilon_{\tilde{A}}(y) >: y \in Y\}$;

Definition 2.3 [8] : Two Intuitionistic Fuzzy Sets $\tilde{A}$ and $\tilde{B}$ of Y are said to be quasi coincident ($\tilde{A}\, q \tilde{B}$ for short) if $\exists\, y \in Y$such that
$\mu_{\tilde{A}}(y) > \upsilon_{\tilde{B}}(y)$ or $\upsilon_{\tilde{A}}(y) < \mu_{\tilde{B}}(y)$.

Lemma 2.3[8]: For any two intuitionistic fuzzy sets $\tilde{A}$ and $\tilde{B}$ of Y, $\rceil(\tilde{A} q \tilde{B}) \Leftrightarrow \tilde{A} \subset \tilde{B}^c$.

Definition 2.4 [7]: An intuitionistic fuzzy topology on a non empty set Y is a family Γ of intuitionistic fuzzy sets in Y which satisfy the following axioms:
(O_1). $\tilde{0}, \tilde{1} \in \Gamma$,
(O_2). $\widetilde{A_1} \cap \widetilde{A_2} \in \Gamma$, for any $\widetilde{A_1}, \widetilde{A_2} \in \Gamma$,
(O_3). $\cup \widetilde{A_\alpha}$ for any family $\{\widetilde{A_\alpha} : \alpha \in \Lambda\} \in \Gamma$.
In this case the pair (Y, Γ) is called an intuitionistic fuzzy topological space and each intuitionistic fuzzy set in Γ, is known as an intuitionistic fuzzy open set in Y. The complement $\widetilde{A^C}$ of an intuitionistic fuzzy open set $\tilde{B}$ is called an intuitionistic fuzzy closed set in Y.

Definition 2.5 [7]: Let (Y, Γ) be an intuitionistic fuzzy topological space and $\tilde{A}$ be an intuitionistic fuzzy set in Y. Then the interior and closure of $\tilde{A}$ are defined by:
cl($\tilde{A}$) = ∩{$\tilde{K}$: $\tilde{K}$ is an intuitionistic fuzzy closed set in Y and $\tilde{A} \subseteq \tilde{K}$},
int($\tilde{A}$) = ∪{$\tilde{G}$: $\tilde{G}$ is an intuitionistic fuzzy open set in Y and $\tilde{G} \subseteq \tilde{G}$}.

Definition 2.6 [23]: Let X and Y are two non empty sets. A function F: X→Y is called intuitionistic fuzzy multifunction if F(x) is an intuitionistic fuzzy set in Y, ∀ x∈X.

Definition 2.7 [27]: Let F: X→Y is an intuitionistic fuzzy multifunction and A be a subset of X. Then F(A) = $\bigcup_{x \in A} F(x)$.

Definition 2.8 [23]: Let F :X→Y be an intuitionistic fuzzy multifunction. Then the upper inverse $F^+(\tilde{A})$ and lower inverse $F^-(\tilde{A})$ of an intuitionistic fuzzy set $\tilde{A}$ in Y are defined as follows:

$F^+(\tilde{A}) = \{x \in X : F(x) \subseteq \tilde{A}\}$

$F^-(\tilde{A}) = \{x \in X : F(x) q \tilde{A}\}$.

Definition 2.9 [23]: An Intuitionistic fuzzy multifunction F: $(X, \mathcal{T}) \to (Y, \Gamma)$ is said to be:

(a) Intuitionistic fuzzy upper α-continuous [28] (Intuitionistic fuzzy upper semi continuous[23]) at a point $x_0 \in X$ if for any intuitionistic fuzzy open set $\tilde{W} \subset Y$ such that $F(x_0) \subset \tilde{W}$ there exists an $U \in \alpha O(X)$ (resp. open set $U \subset X$) containing x_0 such that $F(U) \subset \tilde{W}$.
(b) Intuitionistic fuzzy lower α-continuous (resp. Intuitionistic fuzzy lower semi continuous) at a point $x_0 \in X$ if for any intuitionistic fuzzy open set $\tilde{W} \subset Y$ such that $F(x_0) q \tilde{W}$ there exists an $U \in \alpha O(X)$ (resp. open set $U \subset X$) containing x_0 such that $F(x) q \tilde{W}$, $\forall x \in \tilde{W}$.
(c) Intuitionistic fuzzy upper α-continuous (resp. intuitionistic fuzzy lower α-continuous Intuitionistic fuzzy upper semi-continuous, intuitionistic fuzzy lower semi-continuous) if it is intuitionistic fuzzy upper α-continuous (resp. intuitionistic fuzzy lower α-continuous intuitionistic fuzzy upper semi-continuous, intuitionistic fuzzy lower semi-continuous) at each point of X.

3. α-COONTINUOUS INTUITIONISTIC FUZZY MULTIFUNCTIONS

Definition 3.1: An Intuitionistic fuzzy multifunction F: $(X, \mathcal{T}) \to (Y, \Gamma)$ is said to be:
(a) Intuitionistic fuzzy α-continuous at a point $x_0 \in X$ if for any $\tilde{G}_1, \tilde{G}_2 \in IFO(Y)$ such that $F(x_0) \subset \tilde{G}_1$ and $F(x_0) \cap \tilde{G}_2$ there exists $U \in \alpha O(X)$ containing x_0 such that $F(u) \subset \tilde{G}_1$ and $F(u) \cap \tilde{G}_2, \forall u \in U$.
(b) Intuitionistic fuzzy α-continuous if it has this property at each point of X.

Theorem 3.1: If F: $(X, \mathcal{T}) \to (Y, \Gamma)$ is intuitionistic fuzzy α-continuous then F is intuitionistic fuzzy upper α-continuous and intuitionistic fuzzy lower α-continuous .

Proof: Obvious.

Theorem 3.2: Let F: $(X, \mathcal{T}) \to (Y, \Gamma)$ be an intuitionistic fuzzy multifunction, Then the following statements are equivalent:
(a) F is intuitionistic fuzzy α-continuous at a point $x \in X$;
(b) for any $\tilde{G}_1, \tilde{G}_2 \in IFO(Y)$ such that $F(x) \subset \tilde{G}_1$ and $F(x) q \tilde{G}_2$,there result the relation $x \in sCl(\,Int(F^+(\tilde{G}_1) \cap F^-(\tilde{G}_2)))$.
(c) for every $\tilde{G}_1, \tilde{G}_2 \in IFO(Y)$ such that $F(x) \subset \tilde{G}_1$ and $F(x) q \tilde{G}_2$, and for any semi-open set U of X containing x, there exists a non-empty open set $G_U \subset U$, such that $F(G_U) \subset \tilde{G}_1$ and $F(u) q \tilde{G}_2, \forall u \in G_U$.

Proof. **(a) ⟹ (b):** Let $\tilde{G}_1, \tilde{G}_2 \in IFO(Y)$ with $F(x) \subset \tilde{G}_1$ and $F(x) q \tilde{G}_2$,$\exists\, U \in \alpha$O(X) containing x such that $F(U) \subset \tilde{G}_1$ and $F(u) q \tilde{G}_2, \forall u \in U$. Thus, $x \in U \subset F^+(\tilde{G}_1)$ and $x \in U \subset F^-(\tilde{G}_2)$. Therefore $x \in U \subset F^+(\tilde{G}_1) \cap F^-(\tilde{G}_2)$. Since$U \in \alpha O(X)$. By Lemma 2.2 we have
$x \in U \subset sCl(IntU) \subset sCl(Int\left(F^+(\tilde{G}_1) \cap F^-(\tilde{G}_2)\right))$.
(b)⟹(c): Let $\tilde{G}_1, \tilde{G}_2 \in IFO(Y)$ with $F(x) \subset \tilde{G}_1$ and $F(x) q \tilde{G}_2$. Then $x \in sCl(\,Int(F^+(\tilde{G}_1) \cap F^-(\tilde{G}_2)))$. Let U be any semi-

open subset of X containing x. Then $U \cap Int\left(F^{+}(\tilde{G}_1) \cap F^{-}(\tilde{G}_2)\right) \neq \emptyset$. Put $G_U = Int(Int(F^{+}(\tilde{G}_1) \cap F^{-}(\tilde{G}_2)) \cap U)$, then $G_U \neq \phi$, $G_U \subset U$,
$G_U \subset Int(F^{+}(\tilde{G}_1)) \subset F^{+}(\tilde{G}_1)$ and
$G_U \subset Int(F^{-}(\tilde{G}_2)) \subset F^{-}(\tilde{G}_2)$. And thus $F(G_U) \subset \tilde{G}_1$ and $F(u)q\tilde{G}_2, \forall u \in G_U$.

(c) ⟹ (a): Let $\{U_x\}$ be the family of semi-open sets of X containing x. For any semi-open set U of X containing x and for every $\tilde{G}_1, \tilde{G}_2 \in IFO(Y)$ with $F(x) \subset \tilde{G}_1$ and $F(x)q\tilde{G}_2$, there exists a non-empty open set $G_U \subset U$ such that $F(G_U) \subset \tilde{G}_1$ and $F(u)q\tilde{G}_2, \forall u \in G_U$. Let $W = \cup\{G_U : U \in U_x\}$. Then W is open in X, $x \in sCl(W)$, $F(w) \subset \tilde{G}_1$ and $F(w)q\tilde{G}_2$, for every $w \in W$. Put $S = W \cup \{x\}$, then $W \subset S \subset sCl(W)$ thus $W \in \alpha O(X)$, $x \in S, F(S) \subset \tilde{G}_1$ and $F(t)q\tilde{G}_2, \forall\, t \in S$. Hence F is intuitionistic fuzzy α-continuous at x.

Definition 3.2: Let $\tilde{A}$ be an intuitionistic fuzzy set of an intuitionistic fuzzy topological space (Y, Γ). Then $\tilde{V}$ is said to be a neighbourhood of $\tilde{A}$ in Y if there exists an intuitionistic fuzzy open set $U\ of\ Y$ such that $\tilde{A} \subset \tilde{U} \subset \tilde{V}$.

Defination 3.3: Let (Y, Γ) be an intuitionistic fuzzy topological space, an intuitionistic fuzzy set $\tilde{V}$ is called a semi q-neighbourhood of an intuitionistic fuzzy set $\tilde{A}$ of Y if $\exists\ a\ \tilde{U} \in IFSO(Y)$ such that $\tilde{A}q\tilde{U} \subset \tilde{V}$.

Theorem 3.3: Let $F: (X, \mathcal{T}) \to (Y, \Gamma)$ be an intuitionistic fuzzy multifunction, Then the following statements are equivalent:

- **(a)** F is intuitionistic fuzzy α-continuous.
- **(b)** $F^{+}(\tilde{G}_1) \cap F^{-}(\tilde{G}_2) \in \alpha O(X)$, for every $\tilde{G}_1, \tilde{G}_2 \in IFO(Y)$
- **(c)** $F^{+}(\tilde{V}_2) \cup F^{-}(\tilde{V}_1) \in \alpha C(X)$, For any $\tilde{V}_1, \tilde{V}_2 \in IFC(Y)$.
- **(d)** $sInt(Cl(F^{-}(\tilde{B}_1) \cup F^{+}(\tilde{B}_2))) \subset F^{-}(Cl\tilde{B}_1) \cup F^{+}(Cl\tilde{B}_2)$, for any pair of intuitionistic fuzzy sets $\tilde{B}_1, \tilde{B}_2$ of Y.
- **(e)** $\alpha Cl(F^{-}(\tilde{B}_1) \cup F^{+}(\tilde{B}_2)) \subset F^{-}(Cl\tilde{B}_1) \cup F^{+}(Cl\tilde{B}_2)$, for any pair of intuitionistic fuzzy sets $\tilde{B}_1, \tilde{B}_2$ of Y.
- **(f)** $\alpha Int(F^{-}(\tilde{B}_1) \cap F^{+}(\tilde{B}_2))) \supset F^{-}\left(Int(\tilde{B}_1)\right) \cap F^{+}(Int(\tilde{B}_2))$, for any pair of intuitionistic fuzzy sets $\tilde{B}_1, \tilde{B}_2$ of Y.
- **(g)** For each point x of X for each neighbourhood $\tilde{V}_1$ of F(x) and for each q-neighbourhood $\tilde{V}_2$ of F(x), $F^{+}(\tilde{V}_1) \cap F^{-}(\tilde{V}_2)$ is a α-neighbourhood of x.

Proof: (a) ⟹(b). Let any $\tilde{G}_1, \tilde{G}_2 \in IFO(Y)$ and $x \in F^{+}(\tilde{G}_1) \cap F^{-}(\tilde{G}_2)$, thus $F(x) \subset \tilde{G}_1$ and $F(x)q\tilde{G}_2$, Since F being intuitionistic fuzzy α-continuous according to the theorem 3.2 (b).There follows that $x \in sCl(Int(F^{+}(\tilde{G}_1) \cap F^{-}(\tilde{G}_2)))$. And as x is chosen arbitrarily in $F^{+}(\tilde{G}_1) \cap F^{-}(\tilde{G}_2)$, we have $F^{+}(\tilde{G}_1) \cap F^{-}(\tilde{G}_2) \subset sCl(Int\left(F^{+}(\tilde{G}_1) \cap F^{-}(\tilde{G}_2)\right))$ and thus $F^{+}(\tilde{G}_1) \cap F^{-}(\tilde{G}_2) \in \alpha O(X)$ by Lemma 2.2.

(b) ⟹(c). It follows from Theorem 3.2 [27] (c) and (d).

(c) ⟹(d). Suppose that (c) holds and let $\tilde{B}_1, \tilde{B}_2$ be two intuitionistic fuzzy sets of Y . Then $Cl(\tilde{B}_1) \in IFC(Y)$, $Cl(\tilde{B}_2) \in IFC(Y)$ and thus by (c) $F^{-}(Cl(\tilde{B}_1)) \cup F^{+}(Cl(\tilde{B}_2)) \in \alpha C(X)$. Hence by Lemma2.1(a), $sInt[\ Cl(F^{-}(Cl(\tilde{B}_1)) \cup F^{+}\left(Cl(\tilde{B}_2)\right)] \subset F^{-}\left(Cl(\tilde{B}_1)\right) \cup F^{+}(Cl(\tilde{B}_2)$. Now $\tilde{B}_1 \subset Cl(\tilde{B}_1)$ and $\tilde{B}_2 \subset Cl(\tilde{B}_2)$By Theorem 3.2 [27] (e) and (f) $F^{+}(\tilde{B}_2) \subset F^{+}(Cl(\tilde{B}_2))$ and $F^{-}(\tilde{B}_1) \subset F^{-}(Cl(\tilde{B}_1))$. Consequently, $sInt(\ Cl(F^{-}(\tilde{B}_1)) \cup F^{+}(\tilde{B}_2)) \subset F^{-}\left(Cl(\tilde{B}_1)\right) \cup F^{+}(Cl(\tilde{B}_2)$.

(d) ⟹ (e). Suppose (d) hold. Since $\alpha Cl(A) = A \cup sInt(Cl(A))$ for each subset A of X, it followsthat, $\alpha Cl(F^{-}(\tilde{B}_1) \cup F^{+}(\tilde{B}_2)) = (F^{-}(\tilde{B}_1) \cup F^{+}(\tilde{B}_2)) \cup sInt(\ Cl(F^{-}(\tilde{B}_1) \cup F^{+}(\tilde{B}_2)))$
$\subset (F^{-}(\tilde{B}_1) \cup F^{+}(\tilde{B}_2)) \cup\ (F^{-}(Cl\tilde{B}_1)) \cup F^{+}(Cl\tilde{B}_2)) \subset F^{-}\left(Cl(\tilde{B}_1)\right) \cup F^{+}(Cl(\tilde{B}_2)$.

(e)⟹(f). $(\alpha Int\left(F^{-}(\tilde{B}_1) \cap F^{+}(\tilde{B}_2)\right))^{c} = \alpha Cl((F^{-}(\tilde{B}_1) \cap F^{+}(\tilde{B}_2))^{c})$
$= \alpha Cl((F^{-}(\tilde{B}_1)^{c}) \cup (F^{+}(\tilde{B}_2)^{c})$
$= \alpha Cl\ (F^{+}(\tilde{B}_1)^{c} \cup F^{-}(\tilde{B}_2)^{c})$
$\subset F^{+}\left(Cl(\tilde{B}_1)^{c}\right) \cup F^{-}\left(Cl(\tilde{B}_2)^{c}\right) = F^{+}\left((Int(\tilde{B}_1))^{c}\right) \cup F^{-}\left((Int(\tilde{B}_2))^{c}\right)$ $= (F^{-}(Int\tilde{B}_1))^{c} \cup (F^{+}(Int\tilde{B}_2))^{c} = (F^{-}(Int\tilde{B}_1)) \cap F^{+}(Int\tilde{B}_2))^{c}$.
And thus , $\alpha Int(F^{-}(\tilde{B}_1) \cap F^{+}(\tilde{B}_2)) \supset F^{-}(Int\tilde{B}_1) \cap F^{+}(Int\tilde{B}_2)$.

(a) ⟹(g). Let $x \in X$, $\tilde{V}_1$ is a neighbourhood of F(x) and $\tilde{V}_2$ is a q-neighbourhood of F(x). Then $\exists\ \tilde{U}_1, \tilde{U}_2 \in IFO(Y)$ such that $F(x) \subset \tilde{U}_1 \subset \tilde{V}_1$ and F(x) q$\tilde{U}_2 \subset \tilde{V}_2$. Therefore, $x \in F^{+}(\tilde{U}_1) \cap F^{-}(\tilde{U}_2)$. Therefore, by hypothesis
$x \in F^{+}(\tilde{U}_1) \cap F^{-}(\tilde{U}_2)$
$= F^{+}(Int(\tilde{U}_1)) \cap F^{-}\left(Int\ (\tilde{U}_2)\right)$
$\subset \alpha Int\ (F^{+}(\tilde{U}_1) \cap F^{-}(\tilde{U}_2))$
$\subset \alpha Int(F^{+}(\tilde{V}_1) \cap F^{-}(\tilde{V}_2))$
$\subset (F^{+}(\tilde{V}_1) \cap F^{-}(\tilde{V}_2))$.
It follows that $F^{+}(\tilde{V}_1) \cap F^{-}(\tilde{V}_2)$ is α-neighbourhood of x.

(g)⟹(a). Obvious**.**

Definition 3.4: An intuitionistic fuzzy multifunction $F: (X, \mathcal{T}) \to (Y, \Gamma)$ is called :

- **(a)** intuitionistic fuzzy strongly lower semi- continuous $F^{-}(\tilde{B})$ is a open set in X if for each intuitionistic fuzzy set $\tilde{B}\ of\ Y$.
- **(b)** intuitionistic fuzzy strongly upper semi-continuous if $F^{+}(\tilde{B})$ is a open set in X if for each intuitionistic fuzzy set $\tilde{B}\ of\ Y$.

Theorem 3.4: Let $F: (X, \mathcal{T}) \to (Y, \Gamma)$ be an intuitionistic fuzzy upper α-continuous and intuitionistic fuzzy strongly lower semi-continuous intuitionistic fuzzy multifunction then F is intuitionistic fuzzy α-continuous.

Proof: Let $\tilde{G}_1, \tilde{G}_2 \in IFO(Y)$ Now F being intuitionistic fuzzy upper α −continuous, and $\tilde{G}_1 \in IFO(Y)$, $F^{+}(\tilde{G}_1) \in \alpha O(X)$ by theorem 4.1[28]. Again F being intuitionistic fuzzy strongly lower semi-continuous,$F^{-}(\tilde{G}_2)$ is an open set in X. Hence $F^{+}(\tilde{G}_1) \cap F^{-}(\tilde{G}_2) \in \alpha O(X)$ and by Theorem 3.3, F is intuitionistic fuzzy α −continuous.

Theorem 3.5: Let $F:(X,\mathcal{T}) \rightarrow (Y,\Gamma)$ be an intuitionistic fuzzy lower α-continuous and intuitionistic fuzzy strongly upper semi-continuous intuitionistic fuzzy multifunction then F is intuitionistic fuzzy α-continuous.

Proof: Obvious.

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