The Split Domination in Arithmetic Graphs

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ABSTRACT
The paper concentrates on the theory of domination in graphs. The split domination in graphs was introduced by Kulli and Janakirm[5].In this paper; we have investigated some properties of the split domination number of an Arithmetic Graph and obtained several interesting results. The split domination of these arithmetic graphs have been studied as it enables us to construct graphs with a given split domination number in a very simple way. We have obtained an upper bound for the split domination number of the V_m graph as r+1,where m is a positive integer and $m = p_1^{a_1}.p_2^{a_2}\ldots\ldots p_r^{a_r}$ is the canonical representation, where $p_1, p_2, \ldots, p_r$ are distinct primes and $a_i' s > 1$.

General Terms
Split domination, graph theory

Keywords
Domination, Split domination set, Split domination number, Standard graphs, Arithmetic Graph.

1. INTRODUCTION
Graph theory is one of the most flourishing branches of modern mathematics and computer applications. The last 30 years have withnessed spectacular growth of Graph theory due to its wide applications to descrete optimization problems, combinatorial problems and classical algebraic problems.It has a very wide range of applications to many fields like engineering, physical, social and biological sciences, linguistics etc., The theory of domination has been the nucleus of research activity in graph theory in recent times. This is largely due to a variety of new parameters that can be developed from the basic definition of domination. The NP-completeness of the basic domination problems and its close relationship to other NP-completeness problems have contributed to the enormous growth of research activity in domination theory. It is clearly established from the exclusive coverage of the "Topics on domination in graph" in the 86th issue of the Journal of Discrete mathematics (1990), that the theory of domination is a very popular area for research activity in graph theory.

The rigorous study of dominating sets in graph theory began around 1960, even though the subject has historical roots dating back to 1862 when de Jaenisch studied the problems of determining the minimum number of queens which are necessary to cover or dominate a n x n chessboard.In 1958, Berge defined the concept of the domination number of a graph, calling this as "coefficient of External Stability". In 1962, Ore used the name 'dominating set' and 'domination number' for the same concept. In 1977 Cockayne and Hedetniemi made an interesting and extensive survey of the results know at that time about dominating sets in graphs. They have used the notation γ (G) for the domination number of a graph, which has become very popular since then.

The survey paper of Cockayane and Hedetniemi has generated lot of interest in the study of domination in graphs. In a span of about twenty years after the survey, more than 1,200 research papers have been published on this topic, and the number of papers continued to be on the increase. Since then a number of graph theorists Konig, Ore, Bauer, Harary, Lasker, Berge, Cockayne, Hedetniemi, Alavi, Allan, Chartrand, Kulli, Sampthkumar, Walikar, Armugam, Acharya, Neeralgi, Nagaraja Rao, Vangipuram many others have done very interesting and significant work in the domination numbers and the other related topics. Recent book on domination [3], has stimulated sufficient inspiration leading to the expansive growth of this field of study. It has also put some order into this huge collection of research papers, and organized the study of dominating sets in graphs into meaningful sub areas, placing the study of dominating sets in even broader mathematical and algorithmic contexts.

The split domination in graphs was introduced by Kulli & Janakiram [5]. They defined the split dominating set and the split domination number and obtained several interesting results regarding the split domination number of some standard graphs. They have also obtained relations of split domination number with the other parameters such as domination number, connected domination number, vertex covering number etc., Vasumathi & Vangipuram [8] and Vijayasaradhi & Vangipuram [9] obtained domination parameters of an arithmetic Graph and also they have obtained an elegant method for the construction of a arithmetic graph with the given domination parameter.

In most of the researches in Graph theory, the investigators are content with establishing the existence of a graph with a given graphical parameter. For example, given domination number as n does there exist a graph with this as the domination number? Similarly does there exist a graph with given bondage number or with given domatic number? These problems have been investigated successfully.

However in the matter of applications of these results to real life situations it becomes necessary to evolve the method of constructing such a graph with a given parameter. Construction of a graph with a given Graph theoretic parameter is generally difficult by the usual graph theoretic methods. In many applications of domination number, bondage number, or domatic number, it becomes necessary to construct a graph with as few vertices and/or edges as possible with a given domination number or bondage number or domatic number. It is in this context the usage of elementary number theoretic principles will help in the constructions of such graphs. In Vasumathi and Vangipuram [8], the construction of a graph with a given domination number has been given, using such a method. A similar method of construction using again elementary principles of number theory helped in the construction of a graph with a graceful degree sequence by Vijayasaradhi and Vangipuram [9].

Motivated by the study of domination and split domination we have investigated some properties of split domination number of Arithmetic graphs.

Important definitions:

1.1:**Dominating set:** A subset D of V is said to be a dominating set of G if every vertex in $V \setminus D$ is adjacent to a vertex in D.

1.2: Dominating number:The dominating number $\gamma(G)$ of G is the minimum cardinality of a dominating set.

1.3:Split dominating set :A dominating set D of graph G is called a split dominating set, if the induced sub graph $< V - D >$ is disconnected.

1.4: Split domination number: The split dominating number $\gamma_s(G)$ of G is the minimum cardinality of the split dominating set.

1.5:Arithmetic graph : The Arithmetic graph V_m is defined as a graph with its vertex set as the set of all divisors of m (excluding 1), where m is a natural number and $m = p_1^{a_1}.p_2^{a_2} \dots \dots p_r^{a_r}$ a canonical representation of m, where p_i 's are distinct primes and $a_i' s \geq 1$ and two distinct vertices a, b which are not of the same parity are adjacent in this graph if $(a,b) = p_i, for\ 1 \leq i \leq r$.

The vertices a and b are said to be of the same parity if both a and b are the powers of the same prime, for instance $a=p^2$, $b = p^5$.

2. SOME RESULTS ON SPLIT DOMINATION OF STANDARD GRAPHS

In this paper, we assume that the graph contains a split dominating set. Kulli and Janakiram have obtained several interesting relationships of $\gamma_s(G)$ with the other known parameters.The following are some of the results of Kulli and Janakiram[5].They have proved that

(i) $\gamma_s(G) \leq \alpha_o(G)$, where $\alpha_o(G)$ is a vertex covering number

(ii) $\gamma(G) \leq \gamma_s(G)$

(iii) $k(G) \leq \gamma_s(G)$

(iv) $\gamma_s(G) \leq$ P. $\Delta(G)/\Delta(G) + 1$, where $\Delta(G)$ is the maximum degree of G.

(v) $\gamma_s(G) \leq \delta(G)$; If Diam (G) = 2, where $\delta(G)$ is the minimumdegree of G.

They have obtained the split domination number of some standard graphs as follows:

(vi) $\gamma_s(C_P) = [p/3]$, where $[x]$ is the lease + ve integer not less than x and C_P is a cycle with $p \geq 4$ vertices

(vii) $\gamma_s(W_P) = 3$,Where W_P is a wheel with $p \geq 5$ vertices

(viii) $\gamma_s(K_{m,n}) = m$, where $2 \leq m \leq n$ and $K_{m,n}$ is a complete bipartite graph.

3. SPLIT DOMINATION – ARITHMETIC GRAPHS

In this paper, we have developed a method of construction of graph with a given number as the split domination number of the graph. For this purpose we make use of an arithmetic graph V_m with its vertex set as the set of all devisors of m (except 1) and defining the adjancey property of the arithmetic graph suitably.

The split domination of these arithmetic graphs have been studied as it enables us to construct graphs with a given split domination number in a very simple way.

We have obtained that the split domination number of the $V_m$graph is $r + 1$, where m is a positive integer and $m = p_1^{a_1}.p_2^{a_2} \dots \dots p_r^{a_r}$ is the canonical representation, where $p_1, p_2, \dots, p_r$ are distinct primes and a_i's > 1.

Theorem 3.1: If $m = p_1^{a_1}.p_2^{a_2} \dots \dots p_r^{a_r}$, where a_i's > 1, for i = 1,2,…… r; then $\gamma_s(V_m) \leq r + 1$, where r is the number of distinct prime factors of m.

Proof: Let $m = p_1^{a_1}.p_2^{a_2} \dots \dots p_r^{a_r}$where $a_i > 1$, for $1 \leq i \leq r$ and $p_1, p_2, \dots \dots, p_r$ are distinct primes.

The set of vertices

$D = \{p_1, p_2, \dots \dots, p_r, \ p_1.p_2 \dots \dots p_r\}$,is a split dominating set.

For , If v is any vertex in $V - D$, then v is of the form

$p_1^{b_1}.p_2^{b_2} \dots \dots p_r^{b_r}$, where $0 \leq b_i \leq a_i$ not all b_i's are '0' at the same time.

Then the vertex $p_1^{b_1}.p_2^{b_2} \dots \dots p_r^{b_r}$ is adjacent with p_i in$D$.

Thus D is dominating set ofV_m.

Further this is also a split dominating set.

For, the vertex $p_1^{a_1}.p_2^{a_2} \dots \dots p_r^{a_r}$ is not adjacent with any vertex in$< V - D >$.

Hence D is a split dominating set.

Further this is also a minimal split dominating set.

For, If we remove any vertex v from$D$, then v is of form eitherp_i, for $1 \leq i \leq r$ or $p_1.p_2 \dots \dots p_r$.

If v is of the formp_i, say for the sake of definiteness, $v = p_i$

If we remove any vertex p_i, then the vertex $p_1^{a_1}.p_2^{a_2} \ldots \ldots p_r^{a_r}$ is adjacent with p_i .
So $D - \{v\}$ is not a split dominating set.
On the other hand if we remove $p_1.p_2 \ldots \ldots p_r$ from D , the vertices $p_1^2, p_1^3, \ldots\ldots p_1^{a_1}, p_2^2, p_2^3, \ldots\ldots p_2^{a_2}$ etc., are not adjacent with any vertex in $D - \{v\}$.
This means that $D - \{p_1.p_2 \ldots \ldots p_r\}$ is not a dominating set. Hence D is a minimal split dominating set.
Also in view of the definition of the arithmetic graph and the form of the set of vertices being devisors of m and in view of the definition of the adjacency of any two vertices in V_m, it follows that D is a split dominating set of minimum cardinality.
Hence $\gamma_s(V_m) \leq r + 1$, where r is the core of 'm'.

3.1 Construction of a graph with the given split domination number

With the help of the above theorem we will now construct a graph with the given split domination number.

These constructions are quite useful in the applications of domination theory in real life situations.

If we are required to construct a graph with a given split domination number 't', we proceed as follows:

Choose $m = p_1^{a_1}.p_2^{a_2} \ldots \ldots p_r^{a_r}$, where p_i's are distinct primes and a_i' $s > 1$.
Consider an arithmetic graph V_m .
By Theorem 3.2, the split domination number of V_m is t and the split dominating set is
$D = \{p_1, p_2, \ldots \ldots, p_{t-1}, p_1.p_2 \ldots \ldots p_{t-1}\}$.

4. ILLUSTRATIONS

The construction of a graph with a given split domination number as 3:

(i) Given $t = 3$; we have $t - 1 = 2$, choose any two primes p_1, p_2 and let $m = p_1^2 p_2^2$

The vertices of Vm are the devisors of m (except 1) :

$$V = \{p_1, p_2, p_1^2, p_2^2, p_1p_2, p_1p_2^2, p_1^2p_2, p_1^2p_2^2\}$$

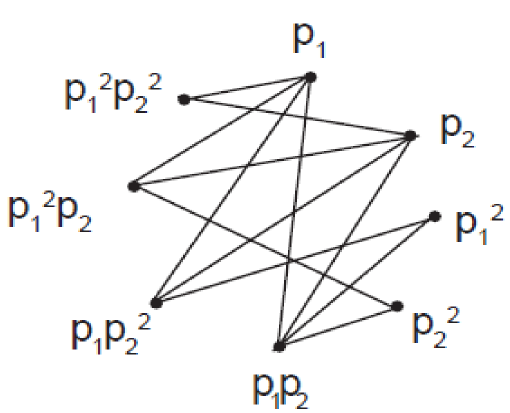

Fig. 1

The V_m graph with $m = p_1^2 p_2^2$

$D = \{p_1, p_2, p_1\,p_2\}$ is the minimum split dominating set, since $< V - D >$ is disconnected.

$$\gamma_s(V_m) = 3$$

(ii) Given $t = 3$; we have $t - 1 = 2$, choose any two primes p_1, p_2 and let $m = p_1^2 p_2^3$

The vertices of Vm are the devisors of m (except 1) :

$$V = \{p_1, p_2, p_1^2, p_2^2, p_2^3, p_1p_2, p_1p_2^2, p_1p_2^3, p_1^2p_2, p_1^2p_2^2, p_1^2p_2^3\}$$

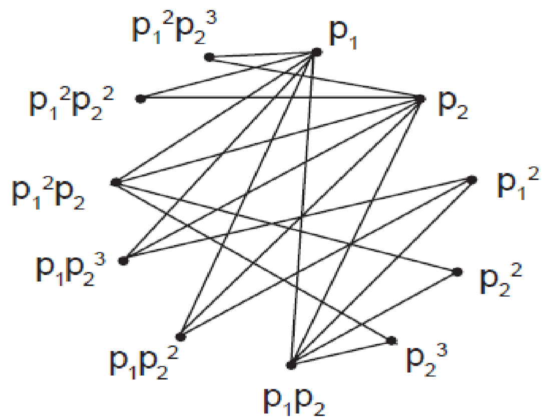

Fig. 2

The V_m graph with $m = p_1^2 p_2^3$

$D = \{p_1, p_2, p_1\,p_2\}$ is the minimum split dominating set, since $< V - D >$ is disconnected

$$\gamma_s(V_m) = 3$$

5. CONCLUSION

The tools of number theory enable us to develop a simple method of constructing a graph with a given cardinality of the spilt dominating set with amazing ease. It is also amazing to observe how such a graph with a given domination number can be enlarged to include more vertices and edges in a methodical, simple manner without affecting the domination number. We can apply this to many applications such as to eradicate pests in Agriculture, to control viruses which produces diseases in an epidemic form, to maintain confidential in transferring the information, especially very useful for Defense sector. To some extent this may be due to the ever growing importance of computer science and its connection with graph theory.

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