Design of Fuzzy Subtractive Clustering Model using Particle Swarm Optimization for the Permeability Prediction of the Reservoir

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ABSTRACT
Permeability is the key parameter of the reservoir and has a significant impact on petroleum fields operations and reservoir management. In most reservoirs, permeability measurements are rare and therefore permeability must be measured in the laboratory from reservoir core samples or evaluated from well test data. However, core analysis and well test data are usually only available from a few wells in a field. Unfortunately, coring every well in large fields is very expensive and uneconomical. This paper proposes an intelligent technique using a Takagi-Sugeno-Kang (TSK) fuzzy modeling approach based on subtractive clustering and particle swarm optimization (PSO) to predict reservoir permeability from well logs data. Subtractive clustering technique (SCT) is employed to identify fuzzy inference system. The radius of influence of cluster center (r_a) in the SCT is selected by PSO. This intelligent technique is applied to predict permeability of Mansuri Bangestan reservoir located in Ahwaz, Iran utilizing available geophysical well log data. The performance of the technique is recorded in terms of MSE and R^2 value. The results showed that the proposed technique was well performed in predicting the reservoir permeability.

Keywords
TSK Fuzzy Modeling, Subtractive Clustering, Particle Swarm Optimization, Permeability, Log Data

1. INTRODUCTION
Permeability is one of the most important rock parameters in reservoir engineering that affects fluids flow in reservoir. In most reservoirs, permeability measurements are rare and Permeability is determined from rock sample or well testing data. Core analysis and well test data are expensive and time consuming.

In recent years, fuzzy systems, which is based on fuzzy logic [1], have attracted the growing attention and interest in different subjects Because of the following two useful properties and capabilities: capability of approximating any complex nonlinear system and model determination through the input-output data (learning process). Fuzzy system is adaptive and relies on input-output data rather than on a classical method, so the resulting scheme is valuable, efficient and capable of reflecting changes in the reservoir permeability behavior. Takagi–Sugeno–Kang (TSK) ([2], [3]) fuzzy system is a more general class of fuzzy systems which is used in this paper. In this system, the consequent part is a crisp function.

One of the important tasks to design a fuzzy system is how to determine the number of rules (structure identification). There are two approaches to generate initial fuzzy rules: manually and automatically. The manual approach forces designers to spend troubled time on tuning fuzzy rules. In many cases the expert's knowledge is not easily available and in some of them, this knowledge is faulty, contains uncertainty, so in this situation, the manual approach becomes more difficult to generate suitable rules. The basic idea behind of the automatically approaches is to estimate fuzzy rules through learning process from input-output sample data. An automatic data-driven based method for generating the initial fuzzy rules is Chiu's subtractive clustering technique (SCT) [4], which is an extension of the grid-based mountain clustering method [5]. The main idea of the SCT is to obtain useful information by grouping data from a large dataset that represent a system behavior. Each cluster center obtained by this technique represents a rule.

Although the SCT is fast, robust and accurate, the user-specified parameter r_a (the radius of influence of cluster center) in this method, strongly affects the number of rules generated. A large r_a generally results in fewer rules, while a small r_a can produce immoderate number of clusters. Determination of r_a to obtain optimum number of fuzzy rule with minimum error in output of the model is a very important problem. Search-based intelligent algorithms such as Genetic Algorithm (GA) [6], simulated annealing [7], ant colony optimization (ACO) [8], can be used for this determination. Recently, a new Evolutionary Algorithm has been proposed by Eberhart and Kennedy [9], which has inspired by social behavior in the nature, called Particle Swarm Optimization (PSO). PSO has a simple structure and easy implementation in practice. In this work, we propose for choosing the best radius of influence of cluster center.

The rest of the paper is organized as follows. In section 2, we briefly describe the subtractive clustering based TSK fuzzy modeling method. The PSO method is explained in Section 3.

Section 4 describes the reservoir. In section 5, the results are presented and discussed. Finally, Section 6 concludes the paper.

2. TSK FUZZY MODELING BASED ON SUBTRACTIVE CLUSTERING

Fuzzy models are effective techniques for the modeling of nonlinear, uncertain and complex systems, where classical methods are difficult to apply because of lack of exact knowledge and accurate numerical values. Among various fuzzy models, the model introduced by Takagi, Sugeno and Kang (TSK fuzzy system) ([2],[3]) is more suitable for sample-data based fuzzy modeling, because it needs less rules, each rule's consequence with linear function can describe the input-output mapping in a large range, and the fuzzy implication used in the model is also simple.

The TSK fuzzy system is a systematic approach to generating fuzzy rules from a given input-output data set. This model consists of rules with fuzzy sets in the antecedents and crisp function (generally is a polynomial in the input variables) in the consequent part. The kth rule of the TSK model can be expressed as:

$$\begin{aligned}&\text{IF } x_1 \text{ is } A_{1k} \text{ and } x_2 \text{ is } A_{2k} \text{ and } \ldots \text{ and } x_n \text{ is } A_{nk}\\ &\text{THEN } y^k = p_o^k + p_1^k x_1 + p_2^k x_2 + \ldots + p_n^k x_n\end{aligned} \quad (1)$$

where x_j ($j \in [1, n]$, n is the number of inputs) is jth input, y^k is the consequent of the kth rule, A_{jk} and p_j^k is the MF and regression parameter in the kth rule, respectively. Construction of the TSK model includes two steps: structure identification and parameters estimation [2]. Structure identification involves an initial rule generation, which is usually done by fuzzy clustering. Parameters estimation of each cluster that includes consequent parameter estimation is usually done with a least-squares method [4].

To extract rules from data, we chose the subtractive clustering method by Chiu [5]. The Subtractive clustering is one-pass algorithm for estimating the number of clusters and initial location of cluster centers, and extracts the TSK fuzzy rules through the training data. This method operates by finding the point with the highest number of neighbors as center for a cluster based on the density of surrounding data points [10]. The subtractive clustering method is described as follows:

Consider a collection of m data points $\{x_1, x_2, ..., x_m\}$ in an N-dimensional space. Without loss of generality, the data points are assumed normalized. In this algorithm, all data point can be considered as a potential cluster center. Then, based on the density of surrounding data points, the potential value for each data point is calculated as follows:

$$p_i = \sum_{j=1}^{m} e^{-\alpha \left\| x_i - x_j \right\|^2} \quad , \quad \alpha = \frac{4}{r_a^2} \quad (2)$$

where $\|\cdot\|$ denotes the Euclidean distance, and r_a is a positive constant called cluster radius. After the potential of each data point has been calculated, the data point with the highest potential is selected as the first cluster center. Let x_1^* be the center of the first cluster and p_1^* its potential value. The potential of each data point x_i^* is revised as follows:

$$p_i \Leftarrow p_i - p_1^* e^{-\beta \left\| x_i - x_1^* \right\|^2} \quad , \quad \beta = \frac{4}{r_b^2} \quad , \quad r_b = \eta r_a \quad (3)$$

where η is a positive constant greater than 1 and is called the squash factor. When the potentials of all data points have been revised by (2), the data point with the highest remaining potential is selected as the second cluster center. In general, after the kth cluster center has been obtained, the potential of each data point is revised as follows:

$$p_i \Leftarrow p_i - p_k^* e^{-\beta \left\| x_i - x_k^* \right\|^2} \quad (4)$$

where x_k^* is the center of the kth cluster and p_k^* is its potential value.

The process of acquiring new cluster center and revising potential repeats in relation to squash factor together with the accept ratio, reject ratio and influence range. The accept ratio sets the potential, as a fraction of the potential of the first cluster center, above which another data point will be accepted as a cluster center. But reject ratio sets the potential, as a fraction of the potential of the first cluster center, below which a data point will be rejected as a cluster center.

By the end of clustering, a sufficient number of cluster centers and cluster sigma is generated. The initial number of rules and antecedent membership functions are determined by this information and then fuzzy inference system of TSK model is identified.

The parameter r_a strongly affects the number of clusters that will be generated. A large value of r_a generally results in fewer clusters that lead to a coarse model, while, a small value of r_a can produce an excessive number of rules that may result in an over defined system. In this work, particle swarm optimization (PSO) is used to suggest the best r_a for TSK model based on SCT.

3. PARTICLE SWARM OPTTIMIZATION

The particle swarm optimization algorithm inspired by the behavior of the social organisms such as flock of birds. Similar to other population-based algorithms, such as genetic algorithms, the PSO algorithm is initialized with a population of random solutions, called particles. These particles moves over the search space with an adaptable velocity, and record the best position it has discovered in the search space. Each particle can adjust its velocity vector, based on its own flying experience and the flying experience of the other particles in the search space.

Suppose that the dimension for a searching space is D, the total number of particles is N. The position and the velocity of the

i th particle can be represented as vector $X_i = (x_{i1}, x_{i2}, ..., x_{iD})$ and $V_i = (v_{i1}, v_{i2}, ..., v_{iD})$, respectively. The best previous position of the i th particle is $P_i = (p_{i1}, p_{i2}, ..., p_{iD})$, and the best previous of the swarm is $P_g = (p_{g1}, p_{g2}, ..., p_{gD})$. Then, the velocity of the particle and its new position will be determined according to the following two equations:

$$V_{id}(t+1) = V_{id}(t) + c_1 r_1 [P_{id} - X_{id}(t)] + c_2 r_2 [P_{gd} - X_{id}(t)] \quad (5)$$

$$X_{id}(t+1) = X_{id}(t) + V_{id}(t+1) \quad (6)$$

where c_1 and c_2 are the individual and social learning rates, respectively, and r_1 and r_2 are random numbers in the range 0 and 1 with uniformly distribution.

It is found that usually the particles velocities build up too fast and they may converge to a suboptimal solution. Shi and Eberhart [11] introduced the concept of inertia weight to the original version of PSO, in order to reduce the velocity. The velocity of the particle, with the inertia term expressed as follows:

$$V_{id}(t+1) = \theta V_{id}(t) + c_1 r_1 [P_{id} - X_{id}(t)] + c_2 r_2 [P_{gd} - X_{id}(t)] \quad (7)$$

where θ is the inertia weight, decreases linearly with the iteration number as follows:

$$\theta_i = \theta_{\max} - \left(\frac{\theta_{\max} - \theta_{\min}}{i_{\max}}\right) i \quad (8)$$

where $\theta_{\min}$ and $\theta_{\max}$ are the initial and final values of the inertia weight, respectively, i is the current iteration number and $i_{\max}$ is the maximum number of iterations used in PSO. The values of $\theta_{\max} = 0.9$ and $\theta_{\min} = 0.4$ are the proper value through empirical studies [11].

4. RESERVOIR DESCRIPTION

The under study field is located at 40 km away from south of Ahwaz city. This field dimension at WOC is 30 km in length and 3.5 km in width. Mansuri field has two reservoirs: Asmari and Bangestan. More than 46 wells have been drilled in Bangestan reservoir and all the wells have logging data. Only six wells in this reservoir have core data, wells: 1, 4, 14, 25, 44 and 54. The core data and logging data of all wells that core analysis data were available were used for this study.

5. RESULTS AND DISCUSSION

In this study, first-order TSK fuzzy approach based on subtractive clustering was used to predict permeability of the reservoir. The process of model building using subtractive clustering was carried out by making of clusters in the data space and translation of these clusters into TSK rules. The first-order TSK fuzzy model is defined as follows:

R_1 : IF x_1 is A_{11} and x_2 is A_{21} and x_3 is A_{31} and x_4 is A_{41} and x_5 is A_{51}

THEN $y^1 = p_0^1 + p_1^1 x_1 + p_2^1 x_2 + p_3^1 x_3 + p_4^1 x_4 + p_5^1 x_5$

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R_k : IF x_1 is A_{1k} and x_2 is A_{2k} and x_3 is A_{3k} and x_4 is A_{4k} and x_5 is A_{5k}

THEN $y^k = p_0^k + p_1^k x_1 + p_2^k x_2 + p_3^k x_3 + p_4^k x_4 + p_5^k x_5$

where x_1, x_2, x_3, x_4, x_5 are CT, DT, NPHI, RHOB and GR, y^k is the consequent of the rule k, and $p_0^k, p_1^k, p_2^k, p_3^k, p_4^k, R^2$ are the regression parameters identified by using the LSE algorithm. The squash factor, accept ratio and reject ratio, parameters of the SCT be 1.25, 0.5 and 0.15, respectively. The parameter r_a (cluster radius) strongly affects the number of clusters that will be generated. PSO is used to choose best r_a and the Mean Square Error (MSE) used as a cost function in this algorithm. In these simulations, the initial and final values of the inertia as $\theta_{\min}$ and $\theta_{\max}$ are 0.4 and 0.9 weight, respectively; the acceleration constants, c_1 and c_2 are 2, r_1 and r_2 are two random numbers in the range of [0, 1]. The population size and number of generation is 40 and 100, respectively. All data in this work normalized to the interval [-1, 1] and split into two parts: one part used to train the model and another used to test the model. The number of training and testing data is 700 and 300, respectively.

The best cluster radius obtained by proposed method is [0.38 0.87 0.79 0.11 0.86 0.89]. The rule base of the models built by using subtractive clustering method and particle swarm optimization with minimum modeling error is shown in Table 1. Each row in the table represents a rule. The results of these experiments are in good agreement with those predicted using the proposed model as shown in Figure 1 to 4. Figures 1 and 2 show the comparison between proposed model output with the actual measurements at training and validation phase, respectively. Figures 3 and 4 show the scatter plot of the model at training and validation phase, respectively. Table 2 gives the MSE and R^2 values of the model of the training and validation phase.

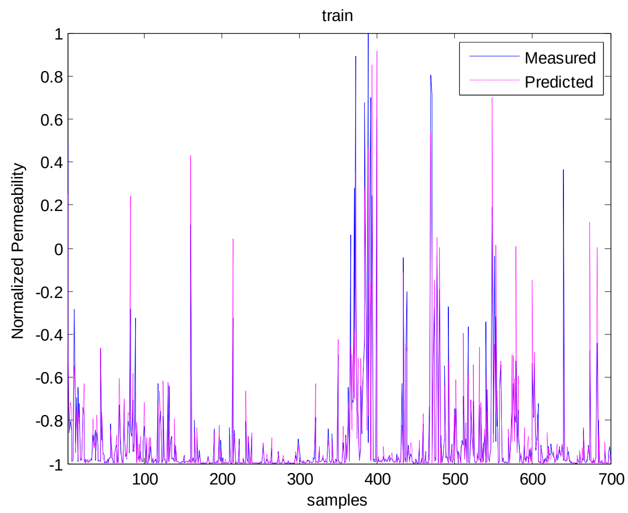

Fig 1: Comparison between measured and predicted permeability, Training phase

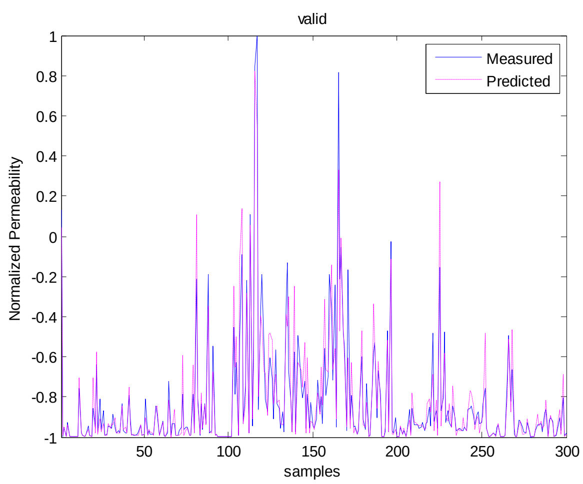

Fig 2: Comparison between measured and predicted permeability, validation phase

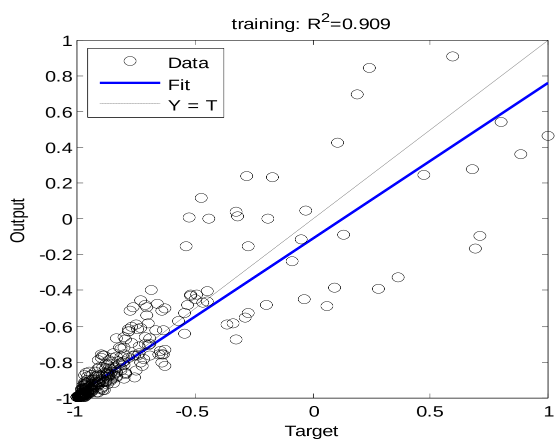

Fig 3: R^2 Training phase

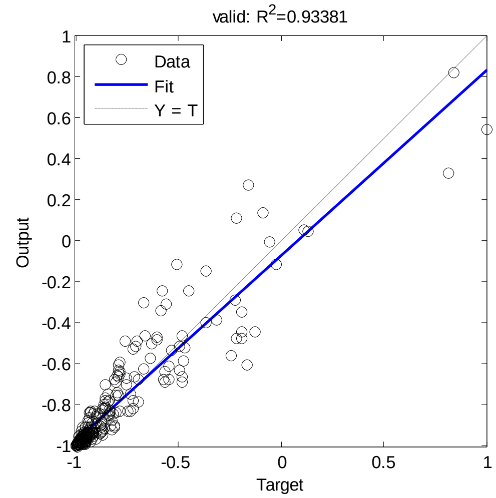

Fig 4: R^2 Validation phase

6. CONCLUSION

This paper has presented a framework for the construction of a TSK fuzzy approach based on subtractive clustering technique with particle swarm optimization to predict permeability of the reservoir. This approach has been tested with the dataset of Mansuri Bangestan reservoir in Ahwaz, Iran. In this paper, a method proposed to use particle swarm optimization to choose the value of radius of influence (r_a). The experimental results show that by choosing the value of r_a in this way gives us the good approximation.

As evidenced from the results obtained it can be concluded that Clustering and fuzzy logic together provide a simple yet powerful method, which can be applied to many other petroleum industries.

7. REFERENCES

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Table 1: A rule base of TSK model

R_k	c_1^k	c_2^k	c_3^k	c_4^k	c_5^k	p_0^k	p_1^k	p_2^k	p_3^k	p_4^k	p_5^k
R_1	-0.531	-0.2712	0.037	0.4246	-0.2625	0.1441	0.0449	-0.0538	0.0443	0.1136	-0.858
R_2	-0.1531	0.0335	0.3852	0.6532	0.083	-0.0681	-0.1777	0.2608	0.4877	0.1628	-1.2946
R_3	-0.8173	-0.7386	-0.2807	-0.0576	-0.5802	-0.0022	0.04	-0.0319	0.0504	0.0105	-0.9688
R_4	-0.6786	-0.5359	-0.0976	0.2099	-0.3971	0.0324	-0.028	0.0337	0.0974	-0.0576	-1.0183
R_5	-0.9681	-0.8094	-0.4674	-0.3926	-0.6648	0.0267	-0.0061	0.0009	0.0154	0.0025	-0.9675
R_6	0.3408	0.4374	0.6357	0.7774	0.4075	-0.0246	0.4456	0.5071	-0.0679	-0.455	-0.888
R_7	-0.9346	-0.7999	-0.3318	-0.2251	-0.5857	0.009	0.0016	0.0094	0.0414	-0.003	-0.975
R_8	-0.9984	-0.9047	-0.8876	-0.8233	-0.8249	0.0071	-0.0008	0.003	0.0011	0.0014	-0.9882
R_9	0.8274	1	0.8643	0.9067	0.8208	-0.1061	-0.3012	3.0695	-4.0802	1.643	0.2403

Table 2: Performance of the model

	Training	**Validation**
MSE	0.0118	0.01
R^2	0.909	0.9338