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Edge – Odd Gracefulness of Few Fan Graph Merging a Finite Number of Circuits and a Star

K. Thirusangu
Associate Professor of Mathematics,
SIVET College,
Gowrivakkam, Chennai

P. Nedumaran
Assistant Professor of Mathematics,
Gurunanak College,
Velacherry, Chennai

A. Solairaju
Associate Professor of Mathematics,
Jamal Mohamed College,
Tiruchirappalli, Tamilnadu,
India

ABSTRACT
The Friendship graphs $F(nC_3 * S_k)$, $F(nC_5 * S_k)$ and $F(2nC_3 * S_k)$ are all even vertex graceful where n is a positive integer.

Keywords
Friendship graphs, Star, Circuits

1. INTRODUCTION
A.Solairaju, and A.Sasikala [2008] got gracefulness of a spanning tree of the graph of product of P_m and C_n, A.Solairaju and K.Chitra [2009] obtained edge-odd graceful labeling of some graphs related to paths. A.Solairaju, and C. Vimala [2008] gracefulness of a spanning tree of the graph of Cartesian product of S_m and S_n,

A.Solairaju and P.Muruganantham [2009] proved that ladder $P_2 \times P_n$ is even-edge graceful (even vertex graceful). They found [2010] the connected graphs $P_n \circ nC_3$ and $P_n \circ nC_7$ are both even vertex graceful, where n is any positive integer. They also obtained [2010] that the connected graph $P_n \Delta nC_4$ is even vertex graceful, where n is any even positive integer.

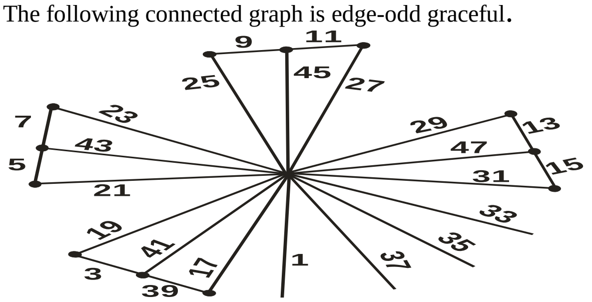
Section I - Preliminaries and definitions:

The following definitions are now given:

Definition 1.1: Let G = (V,E) be a simple graph with p vertices and q edges. A map f :V(G) → {0,1,2,…,q} is called a graceful labeling if f is one – to – one; The edges receive all the labels (numbers) from 1 to q where the label of an edge is the absolute value of the difference between the vertex labels at its ends. A graph having a graceful labeling is called a graceful graph.

Definition 1.2
A graph is odd-edge graceful if there exists an injective map f : E (G) → {1,3,5, …, 2q} so that the induced map f+: V(G) → {0, 1, 2, 3 ,…, 2k-2} defined by f+ (x) = ∑ f(xy) (mod 2k) where k = max { p, q } makes all distinct.

Example 1.3
The following connected graph is edge-odd graceful.



Definition 1.4
A friendship graph or a fan graph F(nC3 * Sk) is defined as the following connected graph containing n copies of circuits of each length 3 with some arbitrary labeling of edges in

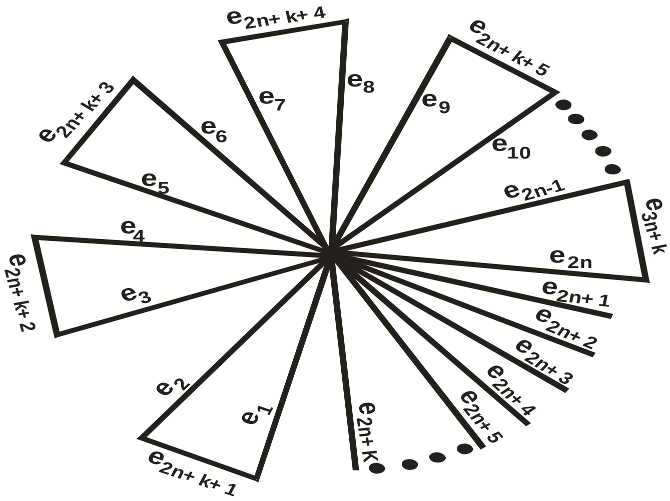


Figure 1: Friendship graph $F(nC_3)$ with some arbitrary labelings for edges

2. NEW CLASSES OF EDGE-ODD GRACEFUL GRAPHS
The discussion is started with the following theorem:

Theorem 2.1
The friendship graph F(nC3 * Sk) is edge-odd graceful where n ≡ 0 (mod 3)

Proof
The graph F(nC3 * Sk) has vertex set {V0, V1, V2, V3, V4, …, V2n-1, V2n, V2n+1, V2n+2, …, V2n+k}. It has edge set {ei = V0Vi: i varies from 1 to n} ∪ { e2n+k+i = ViVi+1: i varies from 1, 3, 5, …, 2n-1} ∪ {e2n+i = V0Vi: i varies from 2n+1, 2n+2,…, 2n+k}.

Define f: E(G) → {1,3,5,…2q-1} ,by f(ei) =2i-1 (i =1 to 3n+k ; I ≠2n+k, i ≠ 2n+k+1)

f(e2n+k) = (4n + 2k – 1) ; f(e2n+k+1) = (4n + 2k + 1) (if k ≥ n)

f(e2n+k) = (4n + 2k + 1) ; f(e2n+k+1) = (4n + 2k – 1) (if k < n).

Then the induced map f+ (u) = ∑ f(uv) (mod 2q) where the sum runs over all edges uv through v. Now, f and f+ both satisfy edge-odd graceful labeling. Thus the connected graph F(nC3 * Sk) is an edge-odd graceful.

Example 2.2: The friendship graph F(6C3 * S2) is edge-odd graceful. The graph has p = 15 vertices, q = 20 edges. The edge-odd graceful labelings are mentioned below in figure 2:

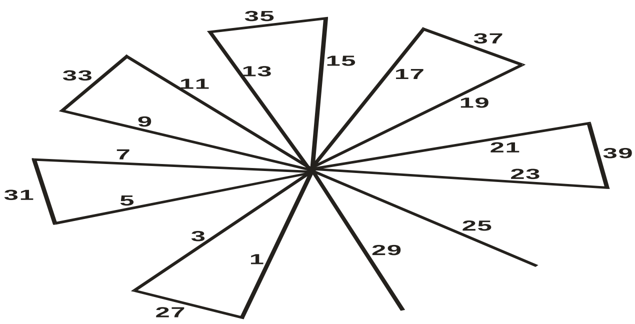


Figure 2: Edge-odd graceful of the friendship graph $F(6C_3 * S_2)$

Theorem 2.2 The friendship graph F(nC3 * Sk) is edge-odd graceful where n≡1(mod 3)

Proof: The graph F(nC3 * Sk) has vertex set {V0, V1, V2, V3, V4, …, V2n-1, V2n, V2n+1, V2n+2, …, V2n+k}. It has edge set {ei = V0Vn+k+i: i varies from 1 to 2n} ∪ { ei = ViVi+1: i varies from 1, 3, 5, …, 2n-1} ∪ {e2n+i = V0Vi: i varies from 2n+1, 2n+2,…, 2n + k}. The other arbitrary labelings od edges for the graph F(nC3 * Sk) are as follows:

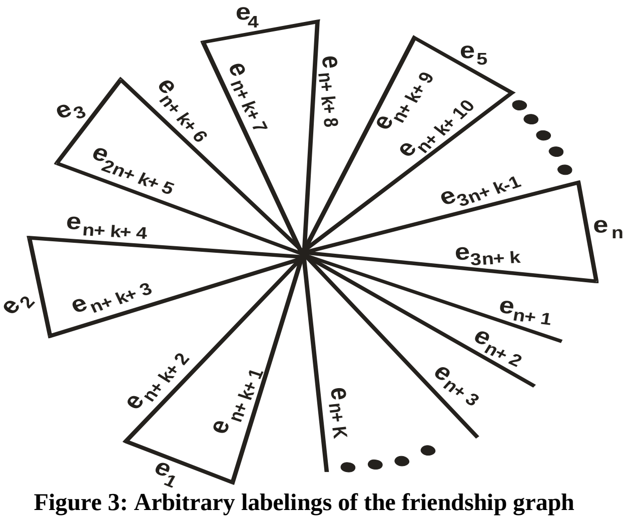


Figure 3: Arbitrary labelings of the friendship graph $F(nC_3 * S_k)$

To get the required edge-odd graceful labelings, define f: E(G)→{1,3,5,…2q-1} ,by f(ei) = (2i-1) , i = 1 to (3n + k).

Then the induced map f+ (u) = ∑ f(uv) (mod 2q) where the sum runs over all edges uv through v. Now, f and f+ both satisfy edge-odd graceful labeling. Thus the connected graph F(nC3 * Sk) is an edge-odd graceful.

Example 2.4: The friendship graph F(7C3 * S4) is edge-odd graceful

The graph has p = 19 vertices, q = 25 edges. The edge-odd graceful labelings are mentioned below:

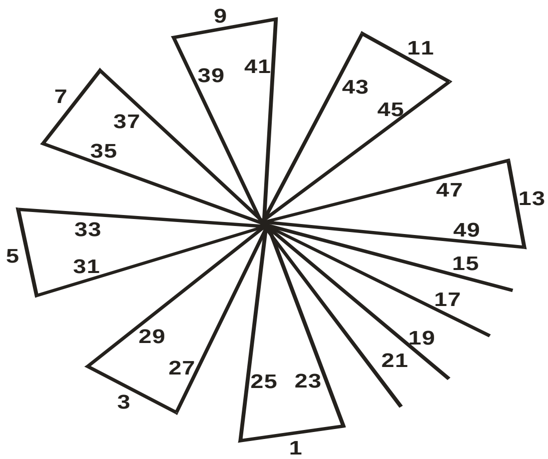


Figure 4: Edge-odd graceful of the friendship graph $F(7C_3 * S_4)$

Theorem 2.5: The friendship graph $F(nC_3 * S_k)$ is edge-odd graceful where n ≡ 2 (mod 3) Proof: The graph $F(nC_3 * S_k)$ has vertex set $\{V_0, V_1, V_2, V_3, V_4, \ldots, V_{2n-1}, V_{2n}, V_{2n+1}, V_{2n+2}, \ldots, V_{2n+k}\}$. It has edge set $\{e_{1+3(i-1)/2} = V_0V_i$: i varies from 1,3,5,…, to 2n-1} ∪ $\{e_{3i\ /\ /2} = V_0V_i$: i varies from 2,4,6,…, to 2n} ∪ $\{ e_{2+3(i-1)/2} = V_iV_{i+1}$: i varies from 1, 3, 5, …, 2n-1} ∪ $\{e_{2n+i} = V_0V_i$: i varies from 2n + 1, 2n + 2,…, 2n + k}.

The third arbitrary labelings of the edges for the graph $F(nC_3 * S_k)$ are as follows:

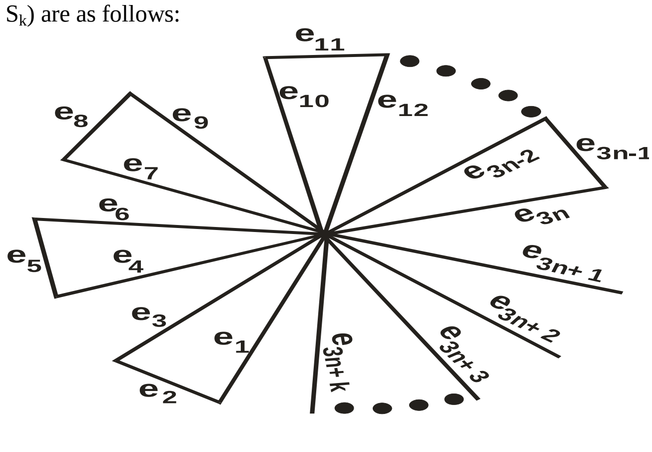


Figure 5: An arbitrary labelings of the friendship graph $F(nC_3 * S_k)$

To get edge-odd graceful labelings in this cases, Define f: E(G)→{1,3,5,…2q-1} ,by $f(e_i) = (2i-1)$, i = 1 to 3n+k , i ≠ 3n-2, i ≠ 3n-1; $f(e_{3n-2}) = 6n-3$, $f(e_{3n-1}) = 6n-5$ if k < n; $f(e_{3n-2}) = 6n-5$, $f(e_{3n-1}) = 6n-3$ if k ≥ n.

Then the induced map $f^+ (u) = \sum f(uv)$ (mod 2q) where the sum runs over all edges uv through v. Now, f and f^+ both satisfy edge-odd graceful labeling. Thus the connected graph $F(nC_3 * S_k)$ is an edge-odd graceful.

Example 2.5: The friendship graph $F(5C_3 * S_4)$ is edge-odd graceful

The graph has p = 14 vertices, q = 19 edges. The edge-odd graceful labelings are mentioned below:

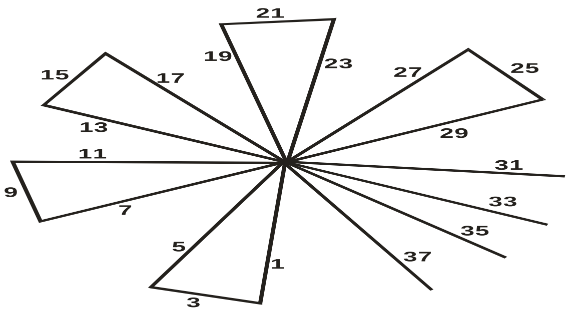


Figure 6: Edge-odd graceful of the friendship graph F($5C_3 * S_4$)

3. DIFFERENT TYPES OF FRIENDSHIP GRAPHS HAVING EDGE-ODD GRACEFUL LABELINGS

The following is now to be verified:

Theorem 3.1: The friendship graph F($nC_5 * S_k$) is edge-odd graceful where n is any positive integer.

Proof: The graph has vertex set {V_0, V_1, V_2, V_3, V_4, V_5, V_6, V_7, V_8, …, V_{2n-3}, V_{2n-2}, V_{4n-1}, V_{4n},…, V_{4n+1}, V_{4n+2}, …, V_{4n+K}}. It has edge set {$e_I = V_iV_{i+1}$: i varies from {1,2,3, 5,6,7, …, 4n-3, 4n-2, 4n-1} ∪ {V_4V_0, V_8V_0, $V_{12}V_0$, …, $V_{4n}V_0$} ∪ { V_0V_1, V_4V_o, V_0V_5, V_8V_0, V_0V_9, $V_{12}V_0$, …, V_0V_{4n-3}, $V_{4n}V_0$} ∪ { V_0V_i : i varies from 1 to k}. The arbitrary labelings of the edges for the given graph F$nC_5 * S_k$) are as follows:

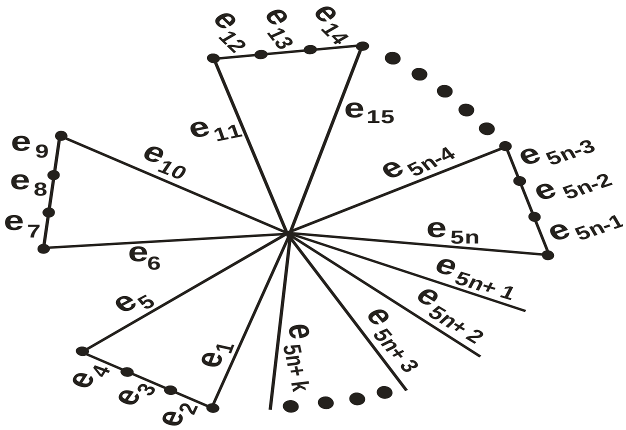


Figure 7: An arbitrary labelings of the friendship graph F($nC_5 * S_4$)

Case (i) : n is odd

Subcase (a): k is even Define f: E(G)→{1,3,5,…2q-1} , by f (e_i) = (2i-1), i = 1 to 5n + k.

Subcase (b): k is odd Define f: E(G)→{1,3,5,…2q-1} by f (e_i) =2i-1 , i = 1 to $5\left(\frac{n+1}{2}\right)$; 5n+1,5n+2,…,5n+k.

f (e_i) = f($e_{5\left(\frac{n+1}{2}\right)}$) + 2 + 4 (i- 1 - $\frac{5(n+1)}{2}$), i = $\frac{5(n+1)}{2}$ + 1,…, $\frac{15(n+1)}{4}$

f (e_i) = f($e_{5\left(\frac{n+1}{2}\right)}$+4) + 4(i- 1 - $\frac{15(n+1)}{4}$) ; i = $\frac{15(n+1)}{4}$ +1,…, (5n).

Case (ii) n is even

Subcase (c): Either k is odd or k is even with k ≥ n: Define f (e_i) = 2i-1 ,i = 1 to 5n+k.

Subcase (d); k is even with k < n. Define f(e_i) = 2i + 1; i = 1 to 5n / 2;

f(e_i) =f($e_{5n/2}$) + 2 + 4 (i- $1 - \frac{15n}{8}$) ; i = $\frac{5n}{2} + 1$,…, $\frac{15n}{8}$; f (e_i) =f(($e_{5n/2}$) + 4) + 4 (i- $1 - \frac{15n}{8}$) ; i = $\frac{15n}{8} + 1$,…,5n; f (e_i) = 2i-1 , i = 5n+1,…,5n(k-1).; f (e_{5n+k}) = 1.

In all cases, the induced map f^+ (u) = ∑ f(uv) (mod 2q) where the sum runs over all edges uv through v. Now, f and f^+ both satisfy edge-odd graceful labeling. Thus the connected graph F($nC_5 * S_k$) is an edge-odd graceful.

Example 3.2: The friendship graph F($5C_5 * S_4$) is edge-odd graceful.

The graph has p = 25 vertices, q = 29 edges. The edge-odd graceful labelings are mentioned below:

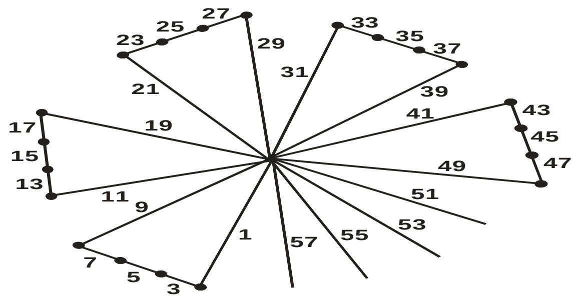


Figure 8: Edge-odd graceful of the friendship graph F($5C_5 * S_4$)

Example 3.3: The friendship graph F($4C_5 * S_3$) is edge-odd graceful.

The graph has p = 21 vertices, q = 24 edges. The edge-odd graceful labelings are mentioned below:

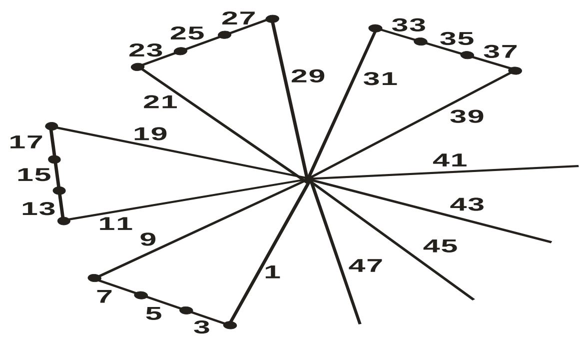


Figure 9: Edge-odd graceful of the friendship graph F($4C_5 * S_3$)

Theorem 3.5 The friendship graph F($2nC_3 * S_k$) is edge-odd graceful where n is any positive integer.

Proof: The graph has vertex set {V_0, V_1, V_2, V_3, V_4, V_5, V_6,…, V_{3n-2}, V_{3n-1}, V_{3n}, V_1, V_2, .., V_k}. It has edge set {V_0V_i: i varies from 1 to 3n+k } ∪ {V_iV_{i+1} : i varies from 1 to 3n and i is not a multiple of 3}. The arbitrary labelings of the edges or the given graph $F(2nC_3 * S_k)$ are as follows:

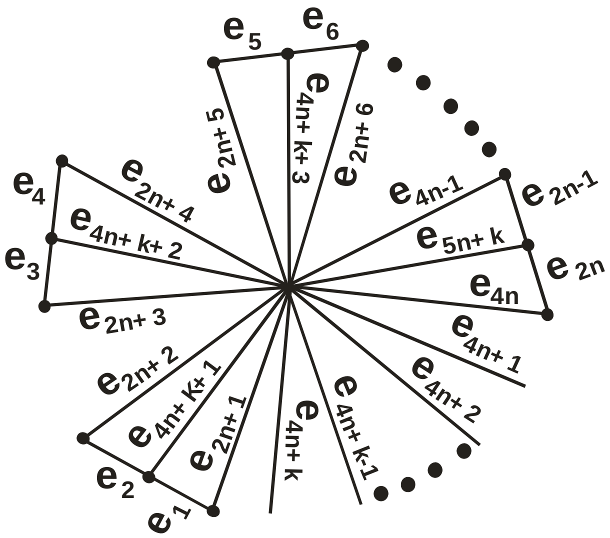


Figure 10: An arbitrary labelings of the friendship graph $F(2nC_3 * S_k)$

Define $f(e_i) = 2i-1$, where $i = 1$ to $5n+k$; $i \neq 1$, and $i \neq 4n+k$

Case (i): n is odd ; k is even < n: Define $f(e_{4n+k}) = 1$; $f(e_1) = 8n+2k-1$

Case (ii): All other cases: Define $f(e_{4n+k}) = 8n+2k-1$; $f(e_1) = 1$

Then the induced map $f^+(u) = \sum f(uv) \pmod{2q}$ where the sum runs over all edges uv through v. Now, f and f^+ both satisfy edge-odd graceful labeling. Thus the connected graph $F(2nC_3 * S_k)$ is an edge-odd graceful.

Example 3.6: The friendship graph $F(2.2C_3 * S_4)$ is edge-odd graceful.

The graph has p = 17 vertices, and q = 24 edges. The edge-odd graceful labelings are mentioned below:

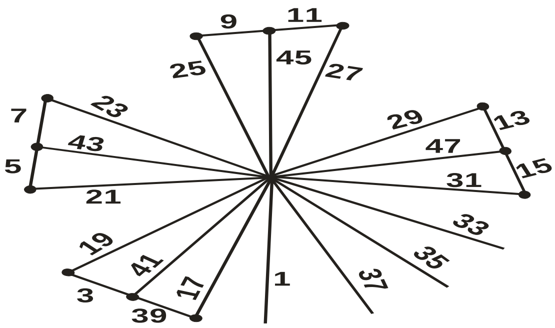


Figure 11: Edge-odd graceful of the friendship graph $F(2.2C_3 * S_4)$

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