Quantum Inspired GA based Neural Control of Inverted Pendulum

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ABSTRACT
This paper deals with comparison of artificial neural network and quantum inspired evolutionary neural network control of an inverted pendulum. First, a properly tuned PID controller was utilized to stabilize the inverted pendulum to generate the training data. Secondly, a feed-forward neural network was trained on the basis of these data. Thirdly, a quantum genetic algorithm optimized neural network was developed. If a disturbance occurs in the system, the controllers counteract this disturbance and balance inverted pendulum. All these three schemes are tested and compared. The results establish that the quantum genetic algorithm neural controller has the best control action.

Keywords
Quantum GA ANN, Inverted Pendulum Control, Adaptive Control, Nonlinear system control, Neural Control

1. INTRODUCTION
The inverted pendulum control problem is the same as trying to balance a stick on your fingertip. The inverted pendulum is related to rocket or missile guidance, where thrust is actuated at the bottom of a tall vehicle. The understanding of a similar problem is built in the technology of Segway, a self-balancing transportation device. The largest implemented uses are on huge lifting cranes on shipyards. Through continuous research new control methods have been developed and then applied to high-tech industries such as aeronautical engineering and robotics. In addition to educational purposes, an inverted pendulum is also an excellent problem for researchers to test new control stretagies, as it is a highly unstable, non linear, under-actuated system.

In robot systems, the relation is pertinent with inverted pendulum systems [1]. In biomechanics, the pendulum is used to model bipedal dynamic walking [2]. The pendulums are also used in the study of wheeled motion [3] and balancing mechanisms [4]. A self balancing human transportation vehicle has been used as a pedagogical tool for teaching the concepts of feedback control to engineering students [5].

There are many soft computing techniques were used in the past. Adaptive fuzzy logic control addresses the problem of controlling classes of unknown nonlinear systems using both indirect adaptive schemes [6-8], and direct adaptive schemes [9, 10]. Quantization of the state space of the inverted pendulum was plasticized with which single layer neural networks could learn to balance the pendulum [11]. Anderson extended the work of Barto et al. by applying a form of the popular error back-propagation method to two-layered neural networks that learn to balance the pendulum given the actual state variables of the inverted pendulum as input [12]. Many researchers have applied the neural network theory to control the inverted pendulum systems [13-17]. Researchers used a combination of neural network and GA [18], neural network [19] and fuzzy theory [20].

For attenuating external disturbances or/and system uncertainties, $H\infty$ schemes for fuzzy systems are discussed based on kinds of sufficient conditions of the quadratic stabilization [21-24].

This paper is divided mainly in four sections. First section deals with the mathematical modeling of the system, followed by second section which explains the proposed controller. Third section portrays the comparison of proposed controller with simple ANN and conventional controller and last section concludes the work done.

2. DEVELOPMENT OF INVERTED PENDULUM MODEL
Consider the Single Inverted Pendulum (SIP) system shown in Figure-1. It comprises of an inverted pendulum, a cart, a tram-road and control arrangement. The cart is confined to linear motion along a track and a rod attached to the cart by a hinge joint. The cart is equipped with a motor that accepts a voltage and supplies torque as a function of the voltage applied. The torque developed in turn moves the cart by a rack and pinion gear attached to the track and motor. The control object is the cart that can freely move right-and-left on the tram road, and the objective is to make the pole angle of inverted pendulum follow the desired output asymptotically.

The equations of motion may be derived using Lagrange's equations of motion. Lagrangian $L = T - V$, where T be the kinetic energy of the system and V be the potential energy.

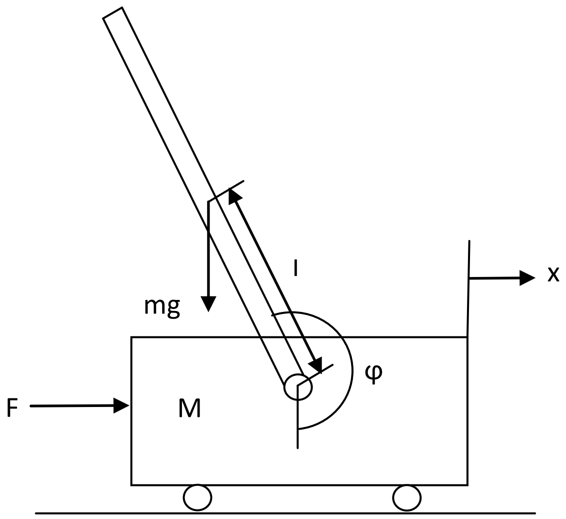

Fig 1: Single Inverted Pendulum System

Let;

M = Mass of the cart

m = Mass of inverted pendulum

l = length between pivot and the centre of mass of the pendulum

ϕ = Angle of inverted pendulum from vertical

F = force applied to the cart

g = acceleration due of gravity

x = displacement of cart from reference position

B = viscous damping coefficient as seen at the motor pinion

b = viscous damping coefficient as seen at the pendulum axis

Analysing the physical model of inverted pendulum and cart, we can obtain the mathematical model of the inverted pendulum-cart system as follows;

$$(M+m)\ddot{x} + B\dot{x} + ml\ddot{\phi}\cos\phi - ml\dot{\phi}^2 \sin\phi = F$$

$$(I+ml^2)\ddot{\phi} + b\dot{\phi} + mgl\sin\phi = -ml\ddot{x}\cos\phi \qquad \text{......(1)}$$

While the SIP is running, the inverted pendulum is required to be stabilized at vertically upward position i.e.- $\phi = \pi$ radians. It may be assumed that pendulum stays within a small vicinity of this equilibrium position. Let θ be a small angle by which pendulum deviates from its equilibrium position. i.e.- $\phi = (\pi + \theta)$. Therefore, approximation processing can be made as;

$$\cos\phi = \cos(\pi+\theta) \approx -1,$$

$$\sin\phi = \sin(\pi+\theta) \approx -\theta,$$

$$\left(\frac{d\phi}{dt}\right)^2 \approx 0$$

By substituting the above given approximations and u representing the input force F, the expressions of equation (1) can be simplified as follows:

$$(M+m)\ddot{x} + B\dot{x} - ml\ddot{\theta} = u$$

$$(I+ml^2)\ddot{\theta} + b\dot{\theta} - mgl\sin\phi = ml\ddot{x} \qquad \text{......(2)}$$

Solving the equation (2), we get the final state space equations of the system as follows:

$$\ddot{x} = \left[\frac{m^2gl^2}{I(M+m)+Mml^2}\right]\theta - \left[\frac{B(I+ml^2)}{I(M+m)+Mml^2}\right]\dot{x}$$
$$-\left[\frac{bml}{I(M+m)+Mml^2}\right]\dot{\theta} + \left[\frac{(I+ml^2)}{I(M+m)+Mml^2}\right]u$$

and

$$\ddot{\theta} = \left[\frac{(M+m)mgl}{I(M+m)+Mml^2}\right]\theta - \left[\frac{Bml}{I(M+m)+Mml^2}\right]\dot{x}$$
$$-\left[\frac{b(M+m)}{I(M+m)+Mml^2}\right]\dot{\theta} + \left[\frac{ml}{I(M+m)+Mml^2}\right]u \qquad \text{.......(3)}$$

Finally, the system state-space model will be expressed by following matrix equation:

$$\begin{bmatrix}\dot{x}\\ \dot{\theta}\\ \ddot{x}\\ \ddot{\theta}\end{bmatrix} = \begin{bmatrix}0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\\ 0 & \frac{m^2gl^2}{D} & \frac{-B(I+ml^2)}{D} & \frac{-bml}{D}\\ 0 & \frac{(M+m)mgl}{D} & \frac{Bml}{D} & \frac{b(M+m)}{D}\end{bmatrix}\begin{bmatrix}x\\ \theta\\ \dot{x}\\ \dot{\theta}\end{bmatrix} + \begin{bmatrix}0\\ \frac{(I+ml^2)}{D}\\ 0\\ \frac{ml}{D}\end{bmatrix}u$$

..............(4)

where; $D = I(M+m) + Mml^2$

3. CONTROL STRETEGY FOR BALANCING OF INVERTED PENDULUM

3.1 Conventional PID Controller

PID control is an important ingredient of a distributed control system. The controllers are also embedded in many special-purpose control systems. PID control is often combined with logic, sequential functions, selectors and simple function blocks to build the complicated automation systems used for energy production, transportation and manufacturing. Many sophisticated control strategies, such as model predictive control are also organized hierarchically.

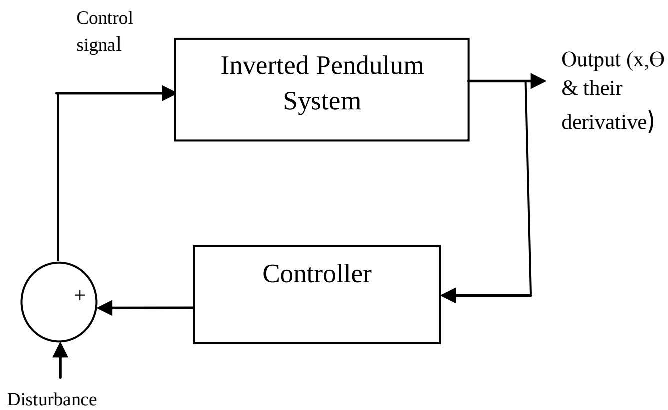

Fig. 2: Control scheme for Inverted Pendulum System

The proportional term in the controller generally helps in establishing system stability and improving the transient response while the derivative term is often used when it is necessary to further improve the closed loop response. Conceptually the effect of the derivative term is to feed information on the rate of change of the measured variable into the controller action. The PID controller is tuned for different disturbances to achieve better performance.

3.2 Artificial Neural Network (ANN) Controller

During the controlling of pendulum by conventional fixed parametric PID controller, the performance is deteriorates as the operating conditions or disturbances are changed. Hence, ANN controller is used [25]. The training data of ANN is obtained from the pre-tuned PID controller for a given disturbance.

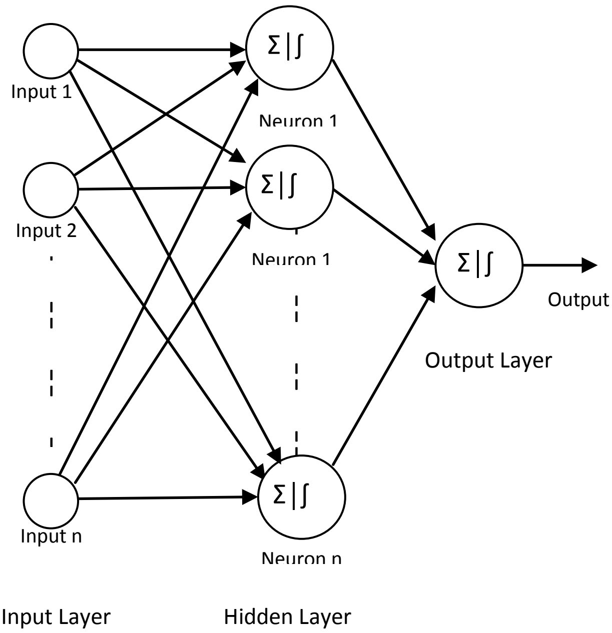

Fig. 3: Structure of Artificial Neural Network

The three layer structure of the ANN is used to control SIP as depicted in figure-3. It is consisting of one input layer having four input variables (viz- cart displacement, pendulum angle from vertical, cart speed and pendulum's angular speed), one hidden layer with five neurons and one output layer having one output variable (viz- controller output voltage). The tansigmoid function is used in hidden layer neurons. The training function 'trainlm' was used for training the ANN. The ANN training pattern is consisting of input vector and controller output.

Input vector = $X(t\text{-}\tau)$=[x dx/dt θ $d\theta/dt$]

Output = $u(t)$

Training Pattern = [$X(t\text{-}\tau)$ $u(t)$] ---------------(5)

After training, the trained ANN was used to control inverted pendulum system.

3.3 Quantum Inspired GA-ANN Controller

After the implementation of classical ANN controller, we optimised the weights of ANN using quantum inspired evolutionary algorithm.

The smallest unit of a digital information is bit, which takes one of the two values {0,1}. The corresponding unit of quantum information stored in a two state quantum computer is called quantum bit or qubit [26-28]. A qubit may be in a state '0' or '1', or in any superposition of the two. The state of a qubit can be represented as;

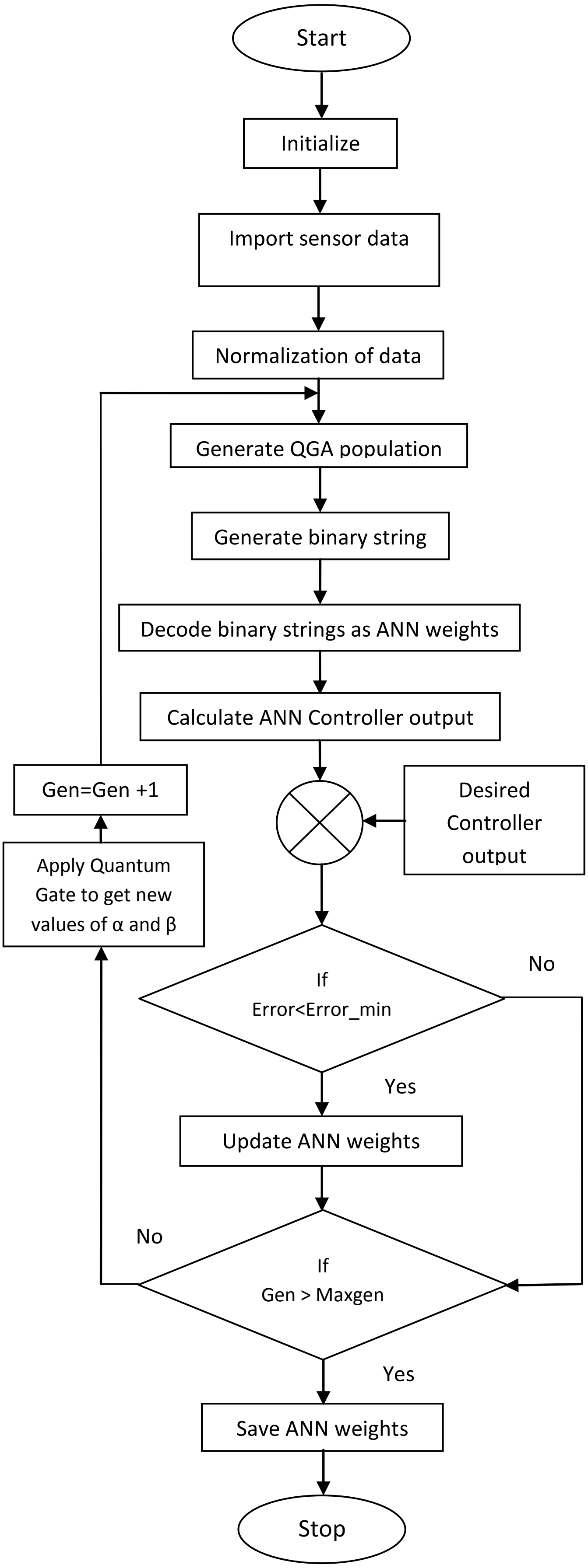

Fig.4: Flowchart for Quantum GA Optimization of ANN weights

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle \quad ....(6)$$

Where α and β are complex numbers that specify the probability amplitudes of the corresponding states. A qubit is a state in a two dimensional Hilbert space that can take any value of the form of equation-(6). $|\alpha|2$ give the probability that the qubit will be found in '0' state and $|\beta|2$ give the probability that the qubit will be found in '1' state. Normalization of the state to unity guarantees;

$$|\alpha|2 + |\beta|2 = 1 \quad ....(7)$$

If there is a system of n qubits, the system can represent 2n states at a time. However, in the act of observing a quantum state, it collapses to a single state [29].

The quantum state can be used to generate a binary string according to state value. His binay state is decoded to get decimal values of ANN weights between the permissible upper and lower limits. The ANN with these weights is tested for the system input variables and ANN output is obtained. This output is compared with desired output and error is calculated.

Now, for next generation of population, the state of a qubit can be changed by operation with a quantum rotational gate. A quantum gate is a reversible gate and can be represented as a unitary operator, U acting on a qubit basis state satisfying U†U = UU†, where U† is the Hermitian adjoint of U [30]. The changed values of quantum state are then used to generate a new binary string and hence, new weights. The ANN is tested again to get new output and error is again calculated.

This process is used repeatedly for certain number of generations and weights are updated for each reduced value of error. The whole algorithm of Quantum GA optimization is illustrated in a flowchart in figure-4.

While the classical computing techniques are dealing in binary, the quantum computing processes the data at different points between binary levels. Classical techniques using two states (0 and 1) confine the calculations to two binary levels, whereas using qubits in quantum computing technique results in higher resolution and more continuous levels for computing. It results to the availability of solutions which lie between binary levels and were unavailable in classical computing techniques.

4. RESULTS

All these developed controllers were implemented to the inverted pendulum model, tested for different disturbances and results were obtained.

The typical disturbances used for testing the controller performance were pulse wave, ramp wave and an irregular manual voltage variation using the slider. For the different disturbances, the performance characteristics of the controllers are drawn in figures 5-8. These figures (5-8) illustrates the behaviour of developed controllers for different disturbances. Quantum inspired techniques are proved to be faster and more effective for complex problems.

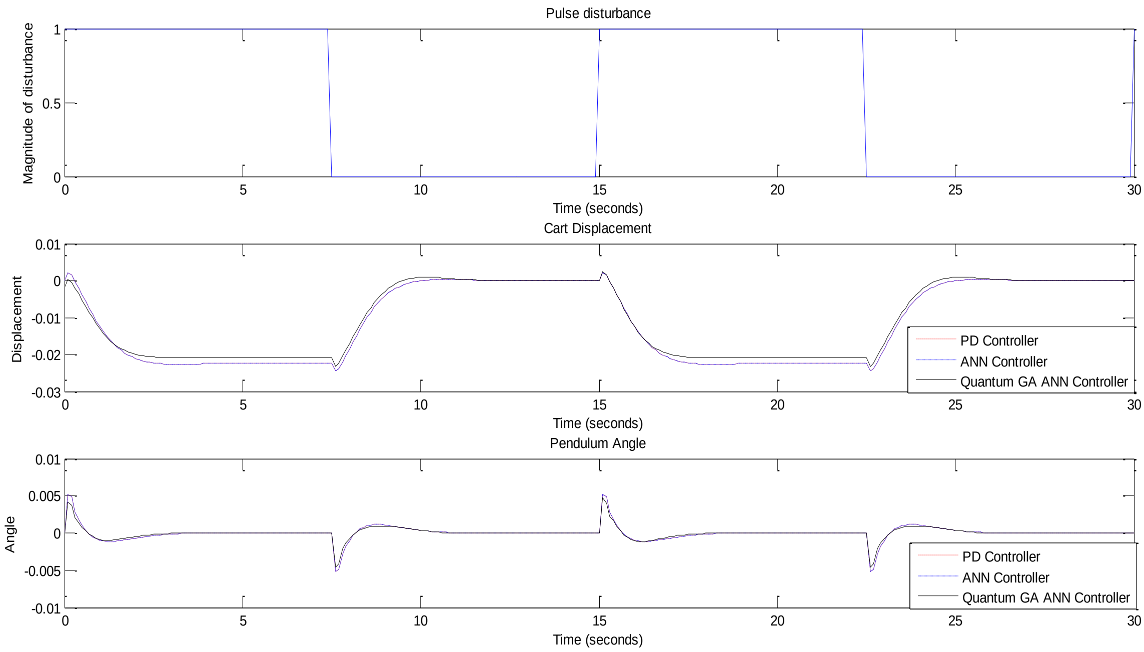

Figure 5: Behaviour of different controllers for pulse disturbance

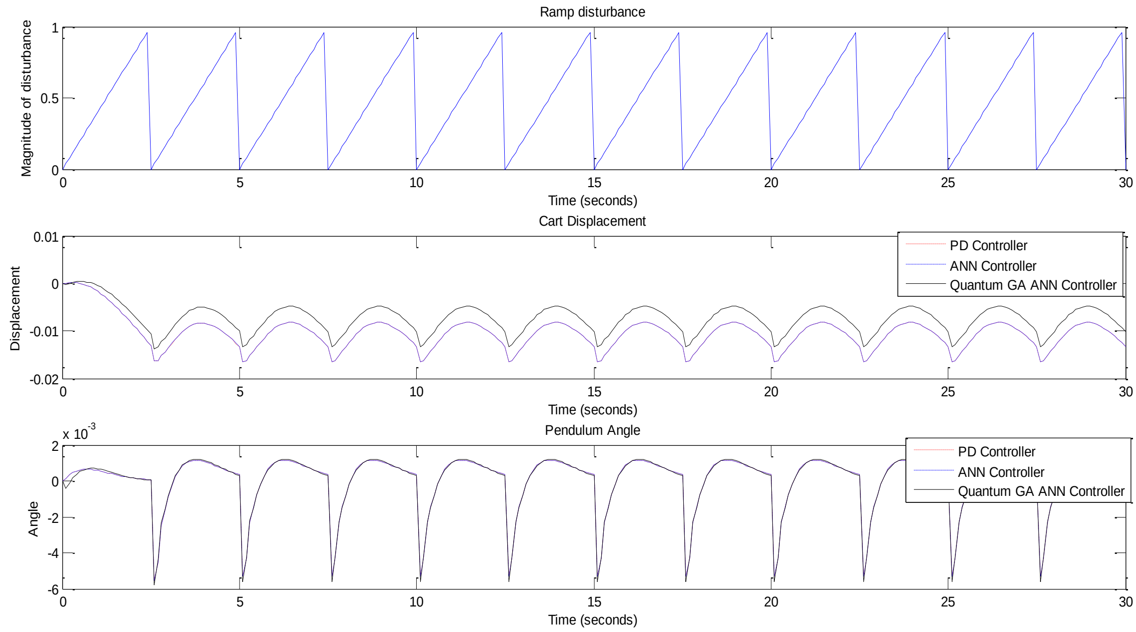

Figure 6: Behaviour of different controllers for ramp disturbance

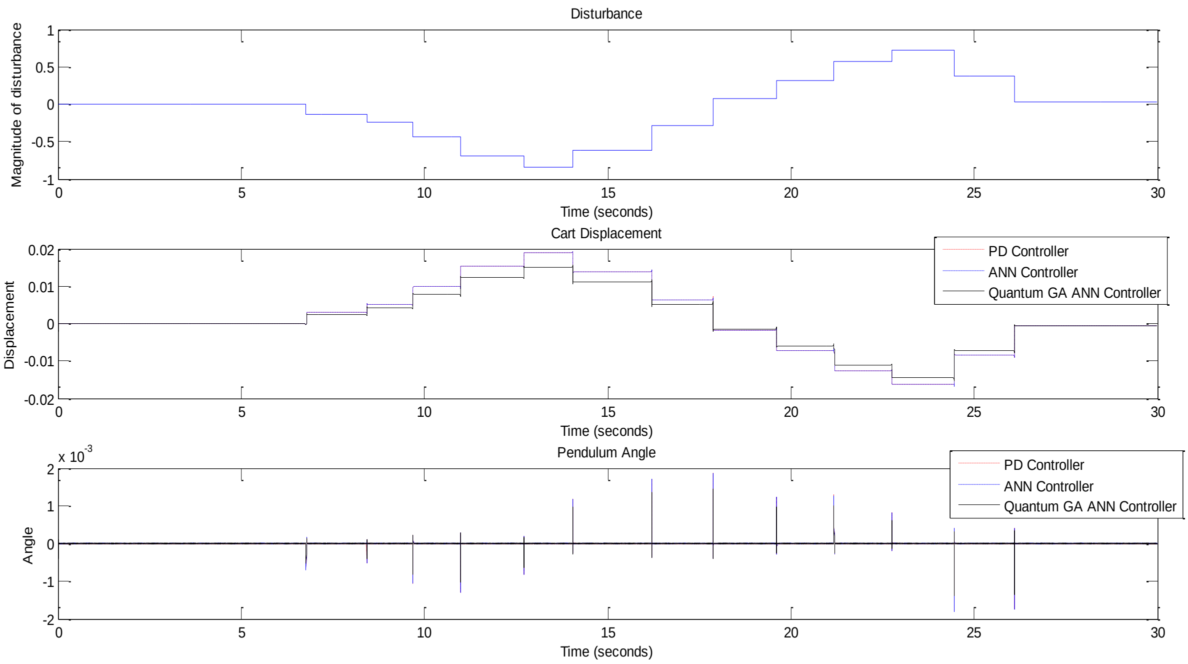

Fig. 7: Behaviour of different controllers for random disturbance using slider gain tool

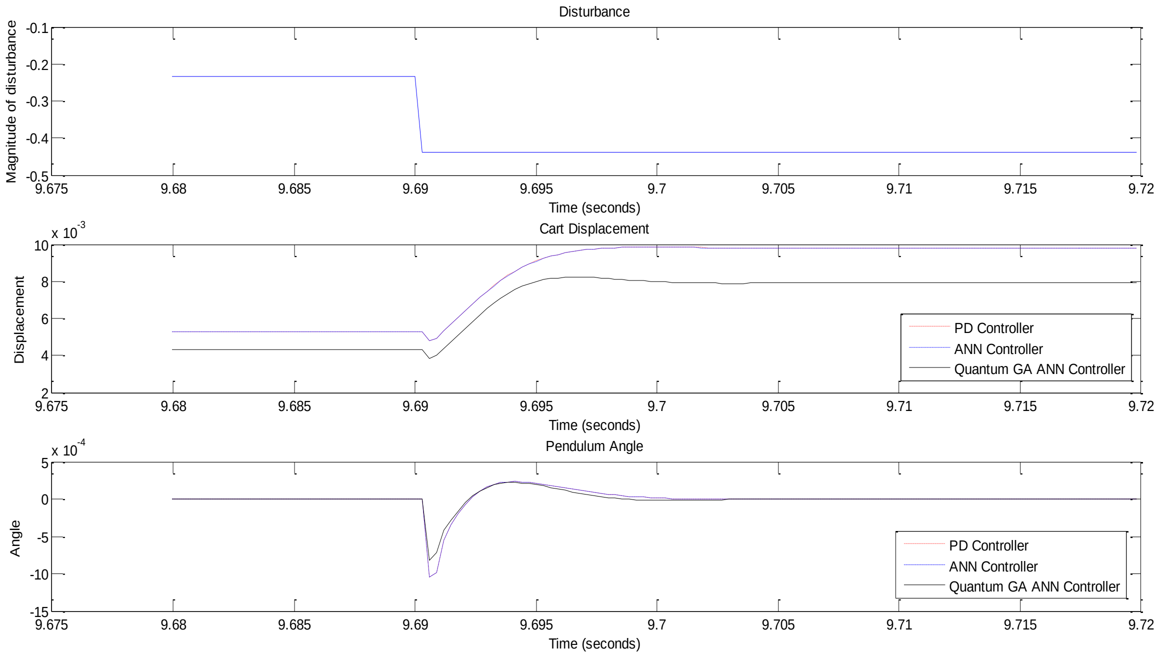

Fig. 8: Zoomed in view of controller performance for slider gain disturbance (given in figure-7)

5. CONCLUSIONS

In this paper, the inverted pendulum system is modelled and controlled using conventional PID controller and ANN controller. The ANN controller is further improved by training it using Quantum Inspired GA. Classical binary GA technique uses two states (0 and 1) and confines the calculations to two binary levels, whereas QGA technique uses qubits, which results in higher resolution and more continuous levels for computing. The performance of proposed QGA-ANN controller has been compared with conventional PD controller and ANN. The results shows that QGA-ANN controller is superior that the conventional controller and simple ANN.

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