System of Linear Fractional Integro-Differential Equations by using Adomian Decomposition Method

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ABSTRACT
In this paper, Adomian decomposition method is applied to solve system linear fractional integro-differential equations. The fractional derivative is considered in the Caputo sense. Special attentions are given to study the convergence of the proposed method. Finally, some numerical examples are provided to show that this method is computationally efficient.

Keywords
System linear fractional integro-differential equations, Adomian decomposition method, Caputo fractional derivative, Riemann-Liouville

1. INTRODUCTION
The Adomian decomposition method was first proposed by Adomian and used to solve a wide class of linear and integral differential equations. The method is very powerful in finding the solutions for various physical problems. Some important problems in science and engineering can usually be reduced to a system of integral and fractional integro-differential equations. Integro-differential equations has attracted much attention and solving this equation has been one of the interesting tasks for mathematicians. In this paper we try to introduce a solution of system of linear fractional integro-differential equations in the following form:

$$\begin{cases} D^{\alpha}u_1(x) = g_1(x) + \int_a^b k_1(x,t,u_1(t),u_2(t),...,u_p(t))dt, \\ D^{\alpha}u_2(x) = g_2(x) + \int_a^b k_2(x,t,u_1(t),u_2(t),...,u_p(t))dt, \\ \quad\quad . \\ \quad\quad . \\ D^{\alpha}u_p(x) = g_p(x) + \int_a^b k_p(x,t,u_1(t),u_2(t),...,u_p(t))dt, \end{cases} \quad (1.1)$$

with initial conditions:

$$u_i^{(j)}(x_0) = u_{ij},\ i = 1,2,...,p,\ j = 0,1,...,n_i - 1. \quad (1.2)$$

Adomian decomposition offers certain advantages over routine numerical methods. Numerical methods use discretization which gives rise to rounding off errors causing loss of accuracy, and requires large computer power and time. Adomian decomposition method is better since it does not involve discretization of the variables hence is free from rounding off errors and does not require large computer memory or time. There are only a few techniques for the solution of fractional integro-differential equations, since it is relatively a new subject in mathematics. These methods are: Adomian decomposition method ([2], [6], [7], [9], [10]), Iterative Decomposition Method [8], the collocation method [3] and fractional differential trams form method ([1],[11]). In this study presented, fractional differentiations and integration are understood in Remann - Liouville sense.

The paper has been organized as follows. Section 2 gives notations and basic definitions. Section 3 consists of main results of the paper, in which Adomian decomposition of the system of fractional integro-differential equations has been developed. Some illustrative examples are given in Section 4 followed by the discussion and conclusions presented in Section 5.

2. BASIC DEFINITIONS
In this section we introduce some basic definitions and properties of fractional calculus.

Definition 2.1 A real function $f(x),\ x > 0,$ is said to be in the space $C_{\alpha},\ \alpha \in R,$ if there exists a real number $p > \alpha,$ such that $f(x) = x^p f_1(x),$ where $f_1(x) \in C[0,\infty).$

Definition 2.2 A real function $f(x),\ x>0,$ is said to be in the space $C_\alpha^k,\ k\in N,$ if $f^k\in C_\alpha.$

Definition 2.3 I^α denotes the fractional integral operator of order α in the sense of Riemann-Liouville, defined by

$$I^\alpha f(x)=\begin{cases}\dfrac{1}{\Gamma(\alpha)}\displaystyle\int_0^x \frac{f(t)}{(x-t)^{1-\alpha}}dt, & \alpha>0,\\ f(x), & \alpha=0,\end{cases} \quad (2.1)$$

Definition 2.4 Let $f\in C_{-1}^m,\ \mathrm{m}\in N.$ Then the Caputo fractional derivative of $f(x),$ defined by

$$D^\alpha f(x)=\begin{cases}\dfrac{1}{\Gamma(m-\alpha)}\displaystyle\int_0^x \frac{f^{(m)}(t)}{(x-t)^{\alpha-m+1}}dt, & 0\le m-1<\alpha\le m,\\ \dfrac{d^m f(x)}{dx^m}, & \alpha=m\in N,\end{cases} \quad (2.2)$$

Or

$$D^\alpha f(x)=I^{m-\alpha}\frac{d^m}{dx^m}f(x),\ m=1,\ 2,\ \cdots . \quad (2.3)$$

Hence, we have the following properties:

$$\left.\begin{array}{l}(1) I^\alpha I^\beta f(x)=I^{\alpha+\beta}f(x)=I^\beta I^\alpha f(x).\\ (2) I^\alpha x^\gamma=\dfrac{\Gamma(\gamma+1)}{\Gamma(\alpha+\gamma+1)}x^{\alpha+\gamma}.\\ (3) D^\alpha\left[I^\alpha f(x)\right]=f(x).\\ (4) I^\alpha\left[D^\alpha f(x)\right]=f(x)-\displaystyle\sum_{k=0}^{m-1}f^k(0)\frac{x^k}{k!},\quad 0\le m-1<\alpha\le m\in N.\end{array}\right\} \quad (2.4)$$

3. ADOMIAN DECOMPOSITION METHOD FOR SOLVING THE SYSTEM

Consider equations (1.1) with initial conditions (1.2) where D^α is the operator defined as (2.4). Operating with J^α on both sides of the equation (1.1) as follows:

$$\begin{cases}u_1(x)=\sum_{k=0}^{m-1}\ u_1^{(k)}(0^+)\ \frac{x^k}{k!}+J^\alpha g_1(x)+J^\alpha\int_a^b k_1(x,t,u_1(t),u_2(t),\dots,u_p(t))dt,\\ u_2(x)=\sum_{k=0}^{m-1}\ u_2^{(k)}(0^+)\ \frac{x^k}{k!}+J^\alpha g_2(x)+J^\alpha\int_a^b k_2(x,t,u_1(t),u_2(t),\dots,u_p(t))dt,\\ \quad\vdots\\ u_p(x)=\sum_{k=0}^{m-1}\ u_p^{(k)}(0^+)\ \frac{x^k}{k!}+J^\alpha g_p(x)+J^\alpha\int_a^b k_p(x,t,u_1(t),u_2(t),\dots,u_p(t))dt,\end{cases} \quad (3.1)$$

In which

$$\begin{cases}f_1(x)=\sum_{k=0}^{m-1}\ u_1^{(k)}(0^+)\ \frac{x^k}{k!}+J^\alpha g_1(x),\\ f_2(x)=\sum_{k=0}^{m-1}\ u_2^{(k)}(0^+)\ \frac{x^k}{k!}+J^\alpha g_2(x),\\ \quad\vdots\\ f_p(x)=\sum_{k=0}^{m-1}\ u_p^{(k)}(0^+)\ \frac{x^k}{k!}+J^\alpha g_p(x),\end{cases} \quad (3.2)$$

by substituting from (3.2) into (3.1), we get

$$\begin{cases}u_1(x)=f_1(x)+J^\alpha\int_a^b k_1(x,t,u_1(t),u_2(t),\dots,u_p(t))dt,\\ u_2(x)=f_2(x)+J^\alpha\int_a^b k_2(x,t,u_1(t),u_2(t),\dots,u_p(t))dt,\\ \quad\vdots\\ u_p(x)=f_p(x)+J^\alpha\int_a^b k_p(x,t,u_1(t),u_2(t),\dots,u_p(t))dt.\end{cases} \quad (3.3)$$

Adomian decomposition method decomposed the solution of ui(x) as the following:

$$\begin{cases} u_1(x) = \sum_{i=0}^{\infty} u_{1i}(x), \\ u_2(x) = \sum_{i=0}^{\infty} u_{2i}(x), \\ \quad \vdots \\ u_p(x) = \sum_{i=0}^{\infty} u_{pi}(x), \end{cases} \qquad (3.4)$$

by substituting from (3.4) into (3.3), we get

$$\begin{cases} \sum_{i=0}^{\infty} u_{1i}(x) = f_1(x) + J^{\alpha} \int_a^b k_1(\sum_{i=0}^{\infty}(x,t,u_{1i}(t),u_{2i}(t),...,u_{pi}(t)))dt, \\ \sum_{i=0}^{\infty} u_{2i}(x) = f_2(x) + J^{\alpha} \int_a^b k_2(\sum_{i=0}^{\infty}(x,t,u_{1i}(t),u_{2i}(t),...,u_{pi}(t)))dt, \\ \quad \vdots \\ \sum_{i=0}^{\infty} u_{pi}(x) = f_p(x) + J^{\alpha} \int_a^b k_p(\sum_{i=0}^{\infty}(x,t,u_{1i}(t),u_{2i}(t),...,u_{pi}(t)))dt, \end{cases} \qquad (3.5)$$

We set

$$\begin{cases} u_{10}(x) = f_1(x) = \sum_{k=0}^{m-1} u_{10}^{(k)}(0^+) \frac{x^k}{k!} + J^{\alpha} g_{10}(x), \\ u_{20}(x) = f_2(x) = \sum_{k=0}^{m-1} u_{20}^{(k)}(0^+) \frac{x^k}{k!} + J^{\alpha} g_{20}(x), \\ \quad \vdots \\ u_{p0}(x) = f_p(x) = \sum_{k=0}^{m-1} u_{p0}^{(k)}(0^+) \frac{x^k}{k!} + J^{\alpha} g_{p0}(x), \end{cases} \qquad (3.6)$$

And

$$\begin{cases} u_{11}(x) = J^{\alpha} \int_a^b k_1(x,t,u_{10}(t),u_{2o}(t),\ldots,u_{p0}(t))dt, \\ u_{21}(x) = J^{\alpha} \int_a^b k_2(x,t,u_{10}(t),u_{20}(t),\ldots,u_{p0}(t))dt, \\ \quad \vdots \\ u_{p1}(x) = J^{\alpha} \int_a^b c_p(x,t,u_{10}(t),u_{20}(t),\ldots,u_{p0}(t))dt, \end{cases} \qquad (3.7)$$

And also we take

$$\begin{cases} u_{1,s+1}(x) = J^{\alpha} \int_a^b k_1(x,t,u_{1,s}(t),u_{2,s}(t),\ldots,u_{p,s}(t))dt, \\ u_{2,s+1}(x) = J^{\alpha} \int_a^b k_2(x,t,u_{1,s}(t),u_{2,s}(t),\ldots,u_{p,s}(t))dt, \\ \quad \vdots \\ u_{p,s+1}(x) = J^{\alpha} \int_a^b c_p(x,t,u_{1,s}(t),u_{2,s}(t),\ldots,u_{p,s}(t))dt, \end{cases} \qquad (3.8)$$

or generally we have recursive relations as follows:

$$\begin{cases} u_{i0}(x) = \sum_{k=0}^{m-1} u_{i0}^{(k)}(0^+) \frac{x^k}{k!} + J^{\alpha} g_{i0}(x), \\ u_{i,s+1}(x) = J^{\alpha} \int_a^b k_i(x,t,u_{1,s}(t),u_{2,s}(t),\ldots,u_{p,s}(t))dt. \end{cases}$$

$i = 1,2,3,...,p \qquad s = 1,2,3,.....$.

4. NUMERICAL RESULTS

In this section, we have applied Adomian decomposition method for solving system of linear fractional Integro-differential equations with known exact solution. All the results are calculated by using the symbolic computation software Maple 16.

Example 4.1

Consider the following system of fractional integro-differential equation:

$$D^{\frac{2}{3}}y_1(x) = \frac{3}{2}\frac{x^{\frac{1}{3}}\ \sqrt{3}\ \Gamma(\frac{2}{3})}{\pi} - \frac{1}{6}x + \int_0^1 2x\ t(y_1(t)+y_2(t))dt, \quad (4.1)$$

$$D^{\frac{2}{3}}y_2(x) = \frac{9}{4}\frac{x^{\frac{4}{3}}\ \sqrt{3}\ \Gamma(\frac{2}{3})}{\pi} + \frac{5}{6}x^3 + \int_0^1 x^3(y_1(t)-y_2(t))dt, \quad (4.2)$$

subject to $y_1(0) = -1,\ y_2(0) = 0$ with the exact solution $y_1(x) = x - 1,\ y_2(x) = x^2.$

The adomian decomposition method suggests that by applying the inverse operator $D^{-\alpha} = J^{\alpha}$ which is the inverse of to both sides of (4.1), (4.2) and by using equation (2.4) we have

$$y_1(x) = \Sigma_{k=0}^{m-1} y^{(k)}(0^+)\frac{x^k}{k!} + J^{\frac{2}{3}}(\frac{3}{2}\frac{x^{\frac{1}{3}}\ \sqrt{3}\ \Gamma(\frac{2}{3})}{\pi}) - J^{\frac{2}{3}}(\frac{1}{6}x) + J^{\frac{2}{3}}\int_0^1 2x\ t(y_1(t)+y_2(t))dt, \quad (4.3)$$

$$y_2(x) = \Sigma_{k=0}^{m-1} y^{(k)}(0^+)\frac{x^k}{k!} + J^{\frac{2}{3}}(\frac{9}{4}\frac{x^{\frac{4}{3}}\ \sqrt{3}\ \Gamma(\frac{2}{3})}{\pi}) + J^{\frac{2}{3}}(\frac{5}{6}x^3) + J^{\frac{2}{3}}\int_0^1 x^3(y_1(t)-y_2(t))dt, \quad (4.4)$$

By assuming

$$f_1(x) = -1 + J^{\frac{2}{3}}(\frac{3}{2}\frac{x^{\frac{1}{3}}\ \sqrt{3}\ \Gamma(\frac{2}{3})}{\pi}) - J^{\frac{2}{3}}(\frac{1}{6}x), \qquad f_2(x) = J^{\frac{2}{3}}(\frac{9}{4}\frac{x^{\frac{4}{3}}\ \sqrt{3}\ \Gamma(\frac{2}{3})}{\pi}) + J^{\frac{2}{3}}(\frac{5}{6}x^3),$$

And with starting of the initial approximation

$y_{10}(x) = f_1(x)$ *and* $y_{20}(x) = f_2(x)$, we have

$$y_{10}(x) = x - 1 - 0.1107732168x^{\frac{5}{3}},$$
$$y_{20}(x) = x^2 + 0.3398723697x^{\frac{11}{3}},$$

$$y_{11}(x) = J^{\frac{2}{3}}\lambda\int_0^1 2x\ t(y_{10}(t)+y_{20}(t))dt,$$
$$y_{11}(x) = 0.1503413208x^{\frac{5}{3}},$$

$$y_{21}(x) = J^{\frac{2}{3}}\lambda\int_0^1 x^3(y_{10}(t)-y_{20}(t))dt,$$
$$y_{21}(x) = -0.3865177110x^{\frac{11}{3}},$$

$$y_{12}(x) = J^{\frac{2}{3}}\lambda\int_0^1 2x\ t(y_{11}(t)+y_{21}(t))dt,$$
$$y_{12}(x) = -0.0361654561 5x^{\frac{5}{3}},$$

$$y_{22}(x) = J^{\frac{2}{3}}\lambda\int_0^1 x^3(y_{11}(t)-y_{21}(t))dt,$$
$$y_{22}(x) = 0.05677359351x^{\frac{11}{3}},$$

$$y_{13}(x) = J^{\frac{2}{3}}\lambda\int_0^1 2x\ t(y_{12}(t)+y_{22}(t))dt,$$
$$y_{13}(x) = 0.0002067868597x^{\frac{5}{3}},$$

$$y_{23}(x) = J^{\frac{2}{3}}\lambda\int_0^1 x^3(y_{12}(t)-y_{22}(t))dt,$$
$$y_{23}(x) = -0.01049300858x^{\frac{11}{3}},$$

hance $\sum_{i=0}^{\infty} y_{1i}(x) = y_{10} + y_{11} + y_{12} + y_{13} + \dots$.

And $\sum_{i=0}^{\infty} y_{2i}(x) = y_{20} + y_{21} + y_{22} + y_{23} + \dots$.

The approximation solution for $\alpha = 2/3$ is

$$y_1(x) = x - 1 + 0.0036094348x^{\frac{5}{3}}.$$

$$y_1(x) = x^2 - 0.0003647564x^{\frac{11}{3}}.$$

Table 1, figure 1 and figure 2 show the comparison between approximate and exact for Adomian decomposition method.

Table 1.

x	Exact y_1	Exact y_2	Approx y_1	Approx y_2	Error y_1	Error y_2
0	-1	0	-1	0	0	0
0.1	-0.9	0.01	-0.8999222371	0.009999921416	0.0000777629	7.8584×10^{-8}
0.2	-0.8	0.04	-0.7997531181	0.03999900204	0.0002468819	9.9796×10^{-7}
0.3	-0.7	0.09	-0.6995147399	0.08999558652	0.0004852601	0.00000441348
0.4	-0.6	0.16	-0.5992161989	0.1599873267	0.0007838011	0.0000126733
0.5	-0.5	0.25	-0.4988630993	0.2499712772	0.0011369007	0.0000287228
0.6	-0.4	0.36	-0.3984593951	0.3599439523	0.0015406049	0.0000560477
0.7	-0.3	0.49	-0.2980080920	0.4899013655	0.0019919080	0.0000986345
0.8	-0.2	0.64	-0.1975115867	0.6398390592	0.0024884133	0.0001609408
0.9	-.1	0.81	-0.09697185465	0.8097521292	0.0030281453	0.0002478708
1.0	0	1.00	0.0036094348	0.9996352436	0.003609435	0.0003647564

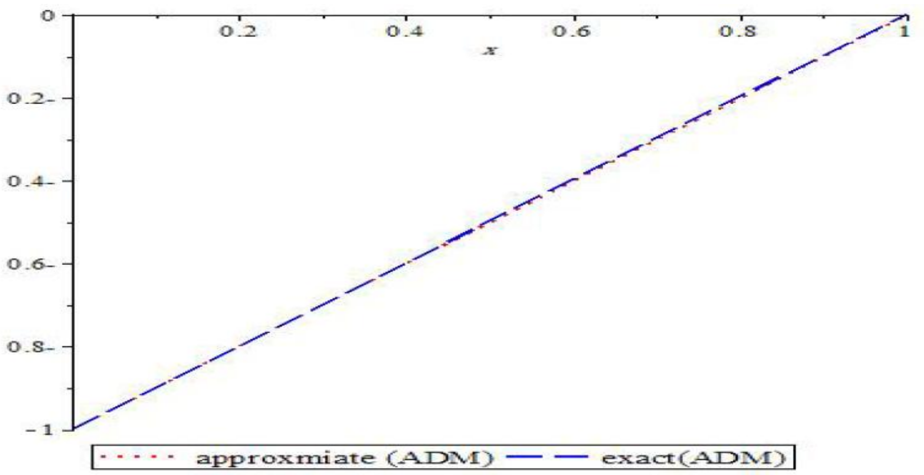

Figure 1: Numerical results of Example 4.1.

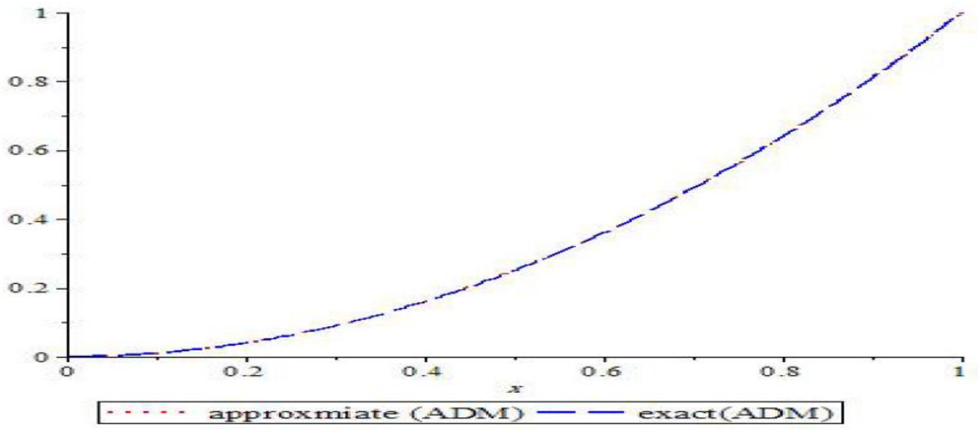

Figure 2: Numerical results of Example 4.1.

Example 4.2

Consider the following system of fractional integro-differential equation:

$$D^{\frac{3}{4}}y_1(x) = \frac{2}{15}\frac{x^{\frac{1}{4}}\sqrt{2}\,\Gamma(\frac{3}{4})(-15+32x^2)}{\pi} - \frac{1}{20} - \frac{1}{12}x + \int_0^1 (x+t)(y_1(t)+y_2(t))dt, \quad (4.7)$$

$$D^{\frac{3}{4}}y_2(x) = \frac{2}{5}\frac{x^{\frac{1}{4}}\sqrt{2}\,\Gamma(\frac{3}{4})(8x-5)}{\pi} - \frac{1}{15}\sqrt{x} + \int_0^1 \sqrt{x}\,t^2(y_1(t)-y_2(t))dt, \quad (4.8)$$

subject to $y_1(0)=0,\ y_2(0)=0$ with the exact solution $y_1(x) = x - x^3,\ y_2(x) = x^2 - x.$

The adomian decomposition method suggests that by applying the inverse operator $D^{-\alpha} = J^{\alpha}$ which is the inverse of to both sides of (4.7), (4.8) and by using equation (2.4) we have

$$y_1(x) = \Sigma_{k=0}^{m-1} y^{(k)}(0^+)\frac{x^k}{k!} + J^{\frac{3}{4}}(\frac{2}{15}\frac{x^{\frac{1}{4}}\sqrt{2}\,\Gamma(\frac{3}{4})(-15+32x^2)}{\pi}) - J^{\frac{3}{4}}(\frac{1}{20}+\frac{1}{12}x) + J^{\frac{3}{4}}\int_0^1 (x+t)(y_1(t)+y_2(t))dt, \quad (4.9)$$

$$y_2(x) = \Sigma_{k=0}^{m-1} y^{(k)}(0^+)\frac{x^k}{k!} + J^{\frac{3}{4}}(\frac{2}{5}\frac{x^{\frac{1}{4}}\sqrt{2}\,\Gamma(\frac{3}{4})(8x-5)}{\pi}) - J^{\frac{3}{4}}(\frac{1}{15}\sqrt{x}) + J^{\frac{3}{4}}\int_0^1 \sqrt{x}\,t^2(y_1(t)-y_2(t))dt, \quad (4.10)$$

Substituting from equation (3.4) into equations (4.9) and (4.10) we get:

$$\sum_{i=0}^{\infty} y_{1i}(x) = 0 + J^{\frac{3}{4}}(\frac{2}{15}\frac{x^{\frac{1}{4}}\sqrt{2}\,\Gamma(\frac{3}{4})(-15+32x^2)}{\pi}) - J^{\frac{3}{4}}(\frac{1}{20}+\frac{1}{12}x) + J^{\frac{3}{4}}\int_0^1 (x+t)(\sum_{i=0}^{\infty}(y_{1i}(t)+y_{2i}(t)))dt, \quad (4.11)$$

$$\sum_{i=0}^{\infty} y_{2i}(x) = 0 + J^{\frac{3}{4}}(\frac{2}{5}\frac{x^{\frac{1}{4}}\sqrt{2}\,\Gamma(\frac{3}{4})(8x-5)}{\pi}) - J^{\frac{3}{4}}(\frac{1}{15}\sqrt{x}) + J^{\frac{3}{4}}\int_0^1 \sqrt{x}\,t^2(\sum_{i=0}^{\infty}(y_{1i}(t)-y_{2i}(t)))dt, \quad (4.12)$$

By assuming

$$f_1(x) = J^{\frac{3}{4}}(\frac{2}{15}\frac{x^{\frac{1}{4}}\sqrt{2}\,\Gamma(\frac{3}{4})(-15+32x^2)}{\pi}) - J^{\frac{3}{4}}(\frac{1}{20}+\frac{1}{12}x), \qquad f_2(x) = J^{\frac{3}{4}}(\frac{2}{5}\frac{x^{\frac{1}{4}}\sqrt{2}\,\Gamma(\frac{3}{4})(8x-5)}{\pi}) - J^{\frac{3}{4}}(\frac{1}{15}\sqrt{x}),$$

and with starting of the initial approximation

$y_{10}(x) = f_1(x)$ *and* $y_{20}(x) = f_2(x)$, we have

$$y_{10}(x) = x - x^3 - 0.05440326263x^{3/4} - 0.05181263108x^{7/4},$$
$$y_{20}(x) = x^2 - x - 0.1042923805x^{5/4},$$

$y_{11}(x) = J^{\frac{3}{4}} \lambda \int_0^1 (x+t)(y_{10}(t) + y_{20}(t))dt,$
$y_{11}(x) = -0.01707138413x^{3/4} - 0.008050048391x^{7/4},$

$y_{21}(x) = J^{\frac{3}{4}} \lambda \int_0^1 \sqrt{x}\, t^2(y_{10}(t) - y_{20}(t))dt,$
$y_{21}(x) = 0.1036071206x^{5/4},$

$y_{12}(x) = J^{\frac{3}{4}} \lambda \int_0^1 (x+t)(y_{11}(t) + y_{21}(t))dt,$
$y_{12}(x) = 0.02559636338x^{3/4} + 0.02074489180x^{7/4},$

$y_{22}(x) = J^{\frac{3}{4}} \lambda \int_0^1 \sqrt{x}\, t^2(y_{11}(t) - y_{21}(t))dt,$
$y_{22}(x) = -0.02395486120x^{5/4},$

$y_{13}(x) = J^{\frac{3}{4}} \lambda \int_0^1 (x+t)(y_{12}(t) + y_{22}(t))dt,$
$y_{13}(x) = 0.008126773648x^{3/4} + 0.007164745169x^{7/4},$

$y_{23}(x) = J^{\frac{3}{4}} \lambda \int_0^1 \sqrt{x}\, t^2(y_{12}(t) - y_{22}(t))dt,$
$y_{23}(x) = 0.01316389936x^{5/4},$

$y_{14}(x) = 0.009701428840x^{3/4} + 0.008144855120x^{7/4},$
$y_{24}(x) = 0.0004522005183x^{5/4},$

$y_{15}(x) = 0.006353095813x^{3/4} + 0.005413228542x^{7/4},$
$y_{25}(x) = 0.003281575848x^{5/4},$

$y_{16}(x) = 0.005182955215x^{3/4} + 0.004387865838x^{7/4},$
$y_{26}(x) = 0.001612608250x^{5/4},$

$y_{17}(x) = 0.003863715028x^{3/4} + 0.003279112229x^{7/4},$
$y_{27}(x) = 0.001506851813x^{5/4},$

$y_{18}(x) = 0.002984632538x^{3/4} + 0.002530500003x^{7/4},$
$y_{28}(x) = 0.001068561524x^{5/4},$

hance $\sum_{i=0}^{\infty} y_{1i}(x) = y_{10} + y_{11} + y_{12} + y_{13} + \dots$.

And $\sum_{i=0}^{\infty} y_{2i}(x) = y_{20} + y_{21} + y_{22} + y_{23} + \dots$.

The approximation solution for $\alpha = 3/4$ is
$y_1(x) = x - x^3 - 0.00966568229x^{3/4} - 0.008197480771x^{7/4}.$

$y_1(x) = x^2 - x - 0.0035544239x^{5/4}.$

Table 2, figure 3 and figure 4 show the comparison between approximate and exact for adomain decomposition method.

Table 2.

x	$Exact\ y_1$	$Exact\ y_2$	$Approx\ y_1$	$Approx\ y_2$	$Error\ y_1$	$Error\ y_2$
0	0	0	0	0	0	0
0.1	0.099	-0.09	0.09713539751	-0.09019987994	0.00186460249	0.0001998799
0.2	0.192	-0.16	0.1886189631	-0.1604753973	0.0033810369	0.0004753973
0.3	0.273	-0.21	0.2680850392	-0.2107891704	0.0049149608	0.0007891704
0.4	0.336	-0.24	0.3294891734	-0.2411306917	0.0065108266	0.0011306917
0.5	0.375	−0.25	0.3668156253	-0.2514944512	0.0081843747	0.0014944512
0.6	0.384	-0.24	0.3740575097	-0.2418769741	0.0099424903	0.0018769741
0.7	0.357	-0.21	0.3452116030	-0.2122758402	0.0117883970	0.0022758402
0.8	0.288	-0.16	0.2742764486	-0.1626892533	0.0137235514	0.0026892533
0.9	0.171	-0.09	0.1552515251	-0.09311581998	0.0157484749	0.0031158200
1.0	0	0	-0.01786316306	-0.0035544239	0.017863163	0.003554424

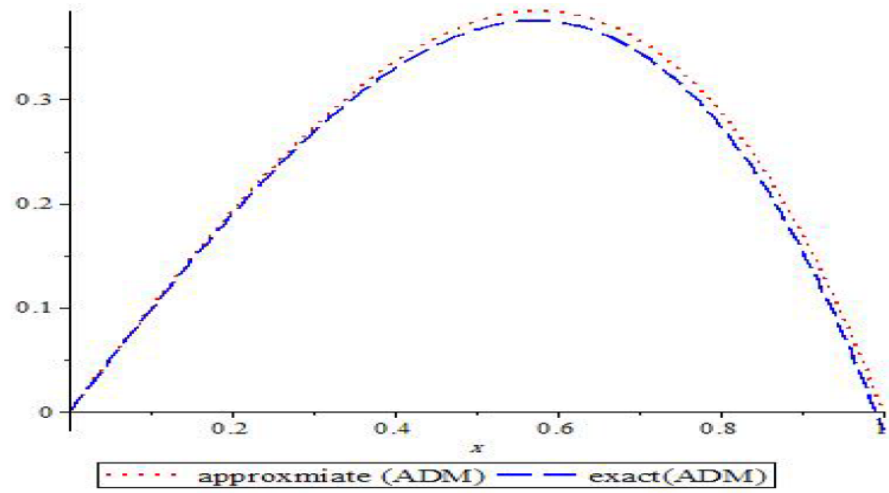

Figure 3: Numerical results of Example 4.2.

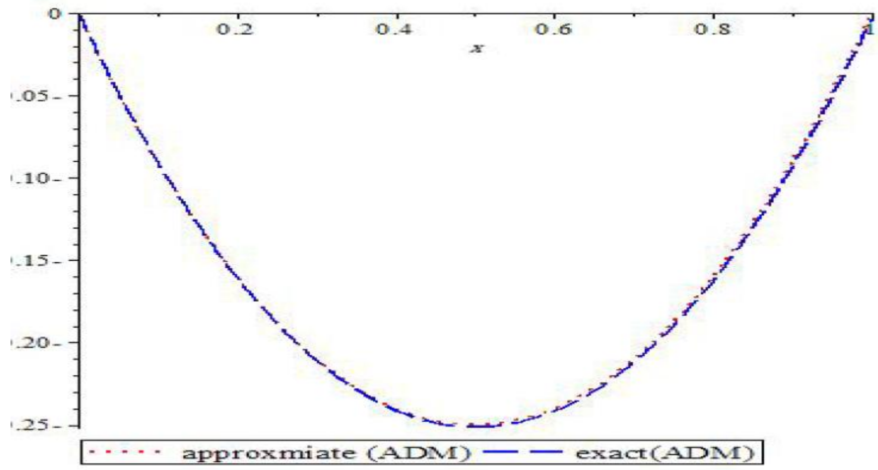

Figure 4: Numerical results of Example 4.2.

Example 4.3

Consider the following system of fractional integro-differential equation:

$$D^{\frac{4}{5}}y_1(x) = \frac{25}{33}\frac{x^{\frac{6}{5}}\ \sin(\frac{1}{5}\pi)\ \Gamma(\frac{4}{5})(15x-11)}{\pi} + \frac{83}{80}x + \int_0^1 2x\ t(y_1(t)-y_2(t))dt, \quad (4.13)$$

$$D^{\frac{4}{5}}y_2(x) = \frac{125}{8}\frac{x^{\frac{6}{5}}\ \sin(\frac{1}{5}\pi)\ \Gamma(\frac{4}{5})}{\pi} - \frac{67}{160} - \frac{13}{24} + \int_0^1 (x+t)(y_1(t)+y_2(t))dt, \quad (4.14)$$

subject to $y_1(0)=0,\ y_2(0)=0$ with the exact solution $y_1(x)=x^3-x^2,\ y_2(x)=\frac{15}{8}x^2$.

The adomian decomposition method suggests that by applying the inverse operator $D^{-\alpha} = J^{\alpha}$ which is the inverse of to both sides of (4.13), (4.14) and by using equation (2.4) we have

$$y_1(x) = \Sigma_{k=0}^{m-1} y^{(k)}(0^+)\frac{x^k}{k!} + (J^{\frac{4}{5}}\frac{25}{33}\frac{x^{\frac{6}{5}}\ \sin(\frac{1}{5}\pi)\ \Gamma(\frac{4}{5})(15x-11)}{\pi}) + J^{\frac{4}{5}}(\frac{83}{80}x) + J^{\frac{4}{5}}\int_0^1 2x\ t(y_1(t)-y_2(t))dt, \quad (4.15)$$

$$y_2(x) = \Sigma_{k=0}^{m-1} y^{(k)}(0^+)\frac{x^k}{k!} + J^{\frac{4}{5}}(\frac{125}{8}\frac{x^{\frac{6}{5}}\ \sin(\frac{1}{5}\pi)\ \Gamma(\frac{4}{5})}{\pi}) - J^{\frac{4}{5}}(\frac{67}{160} + \frac{13}{24}) + J^{\frac{4}{5}}\int_0^1 (x+t)(y_1(t)+y_2(t))dt, \quad (4.16)$$

Substituting from equation (4.2) into equations (4.15) and (4.16) we get:

$$\sum_{i=0}^{\infty} y_{1i}(x) = 0 + (J^{\frac{4}{5}}\frac{25}{33}\frac{x^{\frac{6}{5}}\ \sin(\frac{1}{5}\pi)\ \Gamma(\frac{4}{5})(15x-11)}{\pi}) + J^{\frac{4}{5}}(\frac{83}{80}x) + J^{\frac{4}{5}}\int_0^1 2x\ t(y_1(t)-y_2(t))dt, \quad (4.17)$$

$$\sum_{i=0}^{\infty} y_{2i}(x) = 0 + J^{\frac{4}{5}}(\frac{125}{8}\frac{x^{\frac{6}{5}}\ \sin(\frac{1}{5}\pi)\ \Gamma(\frac{4}{5})}{\pi}) - J^{\frac{4}{5}}(\frac{67}{160}+\frac{13}{24}) + J^{\frac{4}{5}}\int_0^1 (x+t)(y_1(t)+y_2(t))dt, \quad (4.18)$$

By assuming

$$f_1(x) = (J^{\frac{4}{5}}\frac{25}{33}\frac{x^{\frac{6}{5}}\ \sin(\frac{1}{5}\pi)\ \Gamma(\frac{4}{5})(15x-11)}{\pi}) + J^{\frac{4}{5}}(\frac{83}{80}x), \qquad f_2(x) = J^{\frac{4}{5}}(\frac{125}{8}\frac{x^{\frac{6}{5}}\ \sin(\frac{1}{5}\pi)\ \Gamma(\frac{4}{5})}{\pi}) - J^{\frac{4}{5}}(\frac{67}{160}+\frac{13}{24}),$$

and with starting of the initial approximation

$y_{10}(x) = f_1(x)$ and $y_{20}(x) = f_2(x)$, we have

$$y_{10}(x) = x^3 - x^2 + 0.6188521925x^{9/5},$$
$$y_{20}(x) = \tfrac{15}{8}x^2 - 0.4495998459x^{\frac{4}{5}} - 0.3230955222x^{\frac{9}{5}},$$

$$y_{11}(x) = J^{\frac{4}{5}}\lambda\int_0^1 2x\ t(y_{10}(t)-y_{20}(t))dt,$$
$$y_{11}(x) = -0.1315815747x^{\frac{9}{5}},$$

$$y_{21}(x) = J^{\frac{4}{5}}\lambda\int_0^1 (x+t)(y_{10}(t)+y_{20}(t))dt,$$
$$y_{21}(x) = 0.3607635646x^{\frac{4}{5}} + 0.2371121629x^{\frac{9}{5}},$$

$$y_{12}(x) = J^{\frac{4}{5}}\lambda\int_0^1 2x\ t(y_{11}(t)-y_{21}(t))dt,$$
$$y_{12}(x) = -0.2694542668x^{\frac{9}{5}},$$

$$y_{22}(x) = J^{\frac{4}{5}}\lambda\int_0^1 (x+t)(y_{11}(t)+y_{21}(t))dt,$$
$$y_{22}(x) = 0.1420310210x^{\frac{9}{5}} + 0.1681533890x^{\frac{4}{5}},$$

$$y_{13}(x) = J^{\frac{4}{5}}\lambda\int_0^1 2x\ t(y_{12}(t)-y_{22}(t))dt,$$
$$y_{13}(x) = -0.2008247049x^{\frac{9}{5}},$$

$$y_{23}(x) = J^{\frac{4}{5}}\lambda\int_0^1 (x+t)(y_{12}(t)+y_{22}(t))dt,$$
$$y_{23}(x) = 0.02857769795x^{\frac{9}{5}} + 0.02847628389x^{\frac{4}{5}},$$

$$y_{14}(x) = -0.08415096017x^{\frac{9}{5}},$$
$$y_{24}(x) = -0.02725731083x^{\frac{9}{5}} - 0.03774819730x^{\frac{4}{5}},$$

$$y_{15}(x) = -0.001778135567x^{\frac{9}{5}},$$

$$y_{25}(x) = -0.03624230409x^{\frac{9}{5}} - 0.04595255620x^{\frac{4}{5}},$$

$$y_{16}(x) = 0.03039819298x^{\frac{9}{5}},$$
$$y_{26}(x) = -0.02332725394x^{\frac{9}{5}} - 0.02836318056x^{\frac{4}{5}},$$

$$y_{17}(x) = 0.02895092896x^{\frac{9}{5}},$$
$$y_{27}(x) = 0.007892669174x^{\frac{9}{5}} + 0.008878116810x^{\frac{4}{5}},$$

hance $\sum_{i=0}^{\infty} y_{1i}(x) = y_{10} + y_{11} + y_{12} + y_{13} + \ldots$.

And $\sum_{i=0}^{\infty} y_{2i}(x) = y_{20} + y_{21} + y_{22} + y_{23} + \ldots$.

The approximation solution for $\alpha = 4/5$ is

$$y_1(x) = x^3 - x^2 - 0.0124167254x^{\frac{9}{5}}.$$

$$y_1(x) = \frac{15}{8}x^2 + 0.0046075743x^{\frac{4}{5}} + 0.0056911601x^{\frac{9}{5}}.$$

Table 3, figure 5 and figure 6 show the comparison between approximate and exact for Adomian decomposition method.

Table 3.

x	*Exact* y_1	*Exact* y_2	*Approx* y_1	*Approx* y_2	*Error* y_1	*Error* y_2
0	0	0	0	0	0	0
0.1	-0.009	0.01875000000	-0.009196791836	0.01957045012	0.00019679184	0.00082045012
0.2	-0.032	0.07500000000	-0.03268526897	0.07658553189	0.00068526897	0.00158553189
0.3	-0.063	0.1687500000	-0.06442175687	0.1711602662	0.00142175687	0.0024102662
0.4	-0.096	0.3000000000	-0.09838624516	0.3033074346	0.0023862452	0.0033074346
0.5	-0.125	0.4687500000	-0.1285657680	0.4730307131	0.0035657680	0.0042807131
0.6	-0.144	0.6750000000	-0.1489508450	0.6803311180	0.0049508450	0.0053311180
0.7	-0.147	0.9187500000	-0.1535340665	0.9252086490	0.0065340665	0.0064586490
0.8	-0.128	1.200000000	-0.1363093883	1.207662867	0.0083093883	0.007662867
0.9	-0.081	1.518750000	-0.09127172997	1.527693135	0.0102717300	0.008943135
1.0	0	1.875000000	-0.0124167254	1.885298734	0.012416725	0.010298734

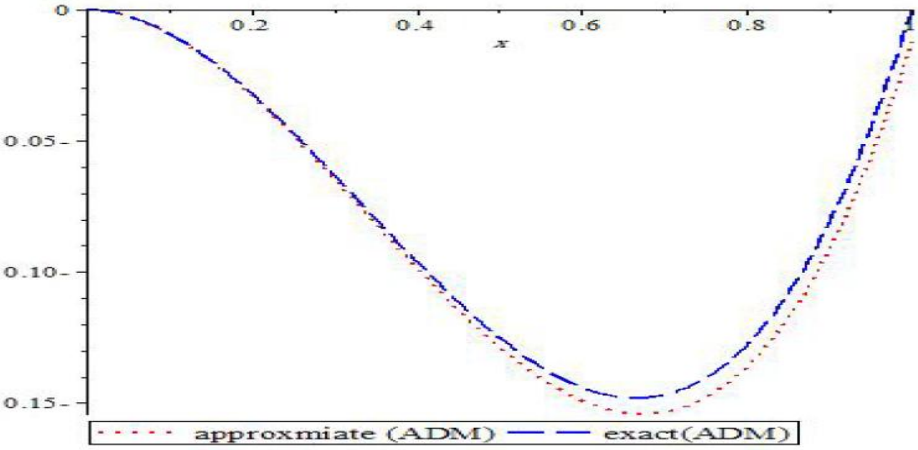

Figure 5: Numerical results of Example 4.3.

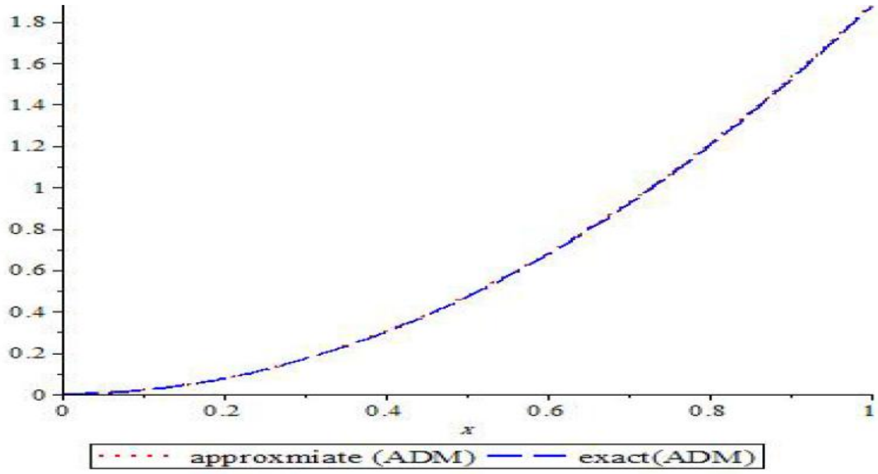

Figure 6: Numerical results of Example 4.3.

5. CONCLUSION

In this article, Adomian decomposition method has been successfully applied to find the solution of a system of linear Fredholm fractional integro-differential equations are presented in Table 1, 2 and 3, for differential result of x to show the stability of the method. The approximate solution obtained by Adomian decomposition method is compared with exact solution.

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