Analysis of Total Average Cost for $M^X_{(m,N)}$/M/1/BD/MV with Fuzzy Parameters using Robust Ranking Technique

K.Julia Rose Mary
Asso.Prof of Mathematics
Nirmala College for Women (Auto)
Coimbatore –18

G. Majula Christina
M.Phil Research Scholar
Nirmala College for Women (Auto)
Coimbatore - 18

ABSTRACT
This paper proposes the procedure to find out the total average cost in terms of crisp values for $M^X_{(m,N)}$/M/1/BD/MV with fuzzy parameters. In which arrival rate, service rate, batch size, setup, vacation, breakdown, repair rates, and the start up ,build up, holding, setup, dormant, breakdown costs and cost for busy and vacation periods are all considered as trapezoidal fuzzy numbers. As ranking technique is a systematic procedure and plays a vital role in decision making under fuzzy environment, Robust ranking technique is applied for the $M^X_{(m,N)}$ /M/1/BD/MV model with fuzzy parameters. Numerical example is also presented to elucidate the validity of the proposed system.

Keywords
Bi-Level Threshold policy, buildup period, Dormant period, Breakdown, Multiple vacations, Fuzzy sets (Normal, convex), Trapezoidal fuzzy number, Membership function, and Fuzzy ranking.

1. INTRODUCTION
Queuing system with bi level control (m,N) policy have been extensively studied in the literature by numerous authors namely Lee and park(1997),Lee et al(2004), Ke.J.C(2004),and Choudhary(2006). Past work regarding queues may be divided into two categories i)the case of controlling the service ii)the case of controlling the arrivals. The control policy of service has been investigated by many authors Yadin and Naor(1963), Lee et al(1994), Lee & srinivasan(1989) and Leet et al (1995).

Regarding the control policy of arrivals many authors Rue and Roshen shine(1981), Neuts(1984) and Stidham(1985) deal with the policy where not all arriving batches are allowed to join the system at all times. Particularly in the over saturated queue where the arrivals are occurring faster than the departures. For any queueing system cost analysis constitute a very important aspect of its investigation and they find out the total average cost per unit time for the system.

Within the context of classical queueing theory the inter arrival time and service times are necessary to follow certain probability. However in many real life applications the statistical information may be derived subjectively(i.e)the arrival and service mode are more correctly described by linguistic terms such as fast, slow (or) moderate, rather than by probability distributions. Hence fuzzy queues are much more realistic than the regularly used crisp queues. Fuzzy queuing models have been describe by such researchers like Li and Lee(1989), Nagi and Lee(1992), Kao et al(1993) proposed a general approach for queuing systems in a fuzzy environment based on Zadeh's extension principle. Moreover Julia Rose Mary & Shanmuhapriya(2014) have discussed $FM^X_{(m,N)}$ /G_{SOS}/1 with fuzzy breakdowns and fuzzy multiple vacation. They derived the membership function of total average cost using Zadeh extension principle.

Ranking technique has been discussed by researchers like Choobinesh & Li (1993), R.R Yager, S.P Chen(2005), A.Nagoor Khani and V.Ashok Kumar(2009) have analyzed bulk arrival fuzzy queues with fuzzy outputs. Some fuzzy numbers are not directly comparable. In order to express a crisp preference of alternatives we need a method for constructing a Crisp total ordering from fuzzy number. Further Kao applied α-cut approach to reduce a fuzzy queue into a family of crisp queue. In Literature we have many methods for converting fuzzy into crisp, for which Robust ranking techniques is the most successful and very convenient method for converting fuzzy to crisp values. B. Palpandi & G.Geedharami have analyzed evaluations of performance measures of bulk arrival queue with fuzzy using Robust ranking technique.

Also Julia Rose Mary & Angel Jenitta(2014) have studied the cost analysis for Bi level threshold policy and single vacations of an unreliable server with fuzzy parameters using Robust ranking technique successively. Thus in this paper we apply Robust Ranking Technique which helps to provide system characteristics of interest in terms of crisp values for bulk arrival queue with fuzzified Poisson arrival rate, service rate, breakdown rate, and multiple vacation.

2. MATHEMATICAL FORMULATION
Consider bi-level threshold policy of FM^x/FM/1/BD Multiple vacation queue. Customers arrive in batches in accordance with time homogeneous Poisson process with arrival rate $\overline{\lambda}$, the service time $\overline{\mu}$ Not all arriving batches are allowed to join the system state which falls into one of the categories namely (build up, setup, dormant and vacation) or busy or breakdown period.$r_i(0\leq r_i\leq 1)$ denotes the probability that an arriving batch is allowed to join the system while the server is idle and r_i (i=2,3) $(0\leq r_i\leq 1)$ denotes the probability with which an arriving batch joins the system during the server's busy and breakdown periods respectively. A cycle starts whenever the system empties the server leaves for a vacation of random length $\bar{\upsilon}$. whenever the system empties following an exponential distribution of parameter $\overline{\eta}$. After returning from the vacation, if the server finds **m** or more customers in the system, then he immediately starts the setup operation of random length D. Otherwise the server takes the repeated number of vacations. Until he finds **m** or more customers accumulate in the system. It is assumed that the sequence of vacations $\{v_1, v2\ldots..\}$are independent and identically distributed and are denoted by v. At the end of the setup period if the queue length is greater than or equal to N, then

the server begins to serve the customers, one at a time. Otherwise server remains idle in the system waiting for the queue length to reach at least N, to start the service. The server is subject to breakdown at anytime while working, with Poisson rate α. whenever the system fails the server is sent immediately for repair facility where the repair item is an independent and identically distributed random variable B_r.

In this queuing model,by considering the arrival rate($\overline{\lambda}$),service rate($\overline{\mu}$),setup rate($\overline{\gamma}$),vacation rate($\overline{\eta}$),repair rate($\overline{\beta}$),breakdown rate($\overline{\alpha}$) as fuzzy numbers which are approximately known. Then we represent

$\overline{\lambda} = \{p, \theta_{\overline{\lambda}}(p)/p \epsilon S(\overline{\lambda})\}$, $\overline{\mu} = \{q, \theta_{\overline{\mu}}(q)/q \epsilon S(\overline{\mu})\}$, $\overline{\eta} = \{t, \theta_{\overline{\eta}}(t)/t \epsilon S(\overline{\eta})\}$, $\overline{\gamma} = \{f, \theta_{\overline{\gamma}}(f)/f \epsilon S(\overline{\gamma})\}$, $\overline{\beta} = \{e, \theta_{\overline{\beta}}(e)/e \epsilon S(\overline{\beta})\}$, and $\overline{\alpha} = \{a, \theta_{\overline{\alpha}}(a)/a \epsilon S(\overline{\alpha})\}$

Here $\theta_a(b)$ and S(a) denote the membership function and support of a where a=$\overline{\lambda}, \overline{\mu}, \overline{\eta}, \overline{\gamma}, \overline{\beta}, \overline{\alpha}$ are fuzzy numbers and b=p,q,t,f,e,a are the crisp values corresponding to arrival rate, service rate,setup cost,vacation rate,repair rate and breakdown rate respectively.

We assume that the system characteristic is the average cost and is denoted by T_c^s(m,n).Here in this model start up cost(C_y),build up cost(C_{build}),holding Cost(Ch),setup cost(Cset),dormant cost(Cdor),cost Cost(C_h),setup cost(C_{set}),dormant cost(C_{dor}), cost for busy(C_{busy}),vacation period(C_v),breakdown cost(C_{br}),are convex fuzzy sets which are approximately known. Then we represent these convex fuzzy sets as,

$\overline{C_y} = \{y, \theta_{\overline{C_y}}(y)/y \in S(\overline{C_y})\}, \overline{C_{build}} = \{b, \theta_{\overline{C_{build}}}(b)/b \in S(\overline{C_{build}})\}$,

$\overline{C_h} = \{h, \theta_{\overline{C_h}}(h)/h \in S(\overline{C_h})\}, \overline{C_{set}} = \{s, \theta_{\overline{C_{set}}}(s)/s \in S(\overline{C_{set}})\}$,

$\overline{C_{dor}} = \{d, \theta_{\overline{C_{dor}}}(d)/d \in S(\overline{C_{dor}})\}, \overline{C_{busy}} = \{u, \theta_{\overline{C_{busy}}}(u)/u \in S(\overline{C_{busy}})\}$,

$\overline{C_v} = \{v, \theta_{\overline{C_v}}(v)/v \in S(\overline{C_v})\}, \overline{C_{br}} = \{r, \theta_{\overline{C_{br}}}(r)/r \in S(\overline{C_{br}})\}$

Here $\theta_a(b)$ and S(a) denote the membership function and support of a where $a = \overline{C_y}, \overline{C_{build}}, \overline{C_h}, \overline{C_{set}}, \overline{C_{dor}}, \overline{C_{busy}}, \overline{C_v}, \overline{C_{br}}$ are convex fuzzy sets and b=y,b,h,s,d,u,v,r are the crisp values corresponding to startup,buildup, holding cost, setup, dormant, busy, vacation, breakdown cost respectively.

On the basis of the concept of α -cuts, we develop a mathematical programming approach for deriving the α -cuts of $\overline{C_y}, \overline{C_{build}}, \overline{C_h}, \overline{C_{set}}, \overline{C_{dor}}, \overline{C_{busy}}, \overline{C_v}, \overline{C_{br}}$ as crisp intervals which are given by,

$C_y(\alpha) = \{y \in Y/\theta_{\overline{C_y}}(y) \ge \alpha\}, C_{build}(\alpha) = \{b \in B/\theta_{\overline{C_{build}}}(b) \ge \alpha\}$,

$C_h(\alpha) = \{h \in H/\theta_{\overline{C_h}}(h) \ge \alpha\}, \overline{C_{set}}(\alpha) = \{s \in S/\theta_{\overline{C_{set}}}(s) \ge \alpha\}$,

$\overline{C_{dor}}(\alpha) = \{d \in D/\theta_{\overline{C_{dor}}}(d) \ge \alpha\}, \overline{C_{busy}}(\alpha) = \{u \in U/\theta_{\overline{C_{busy}}}(u) \ge \alpha\}$,

$\overline{C_v} = \{v \in V/\theta_{\overline{C_v}}(v) \ge \alpha\}, \overline{C_{br}}(\alpha) = \{r \in R/\theta_{\overline{C_{br}}}(r) \ge \alpha\}$

where 0<α≤1 Hence a fuzzy queue can be reduced to a family of crisp queues with different α -cuts $\{C_y(\alpha)/0 < \alpha \le 1\}, \{C_{build}(\alpha)/0 < \alpha \le 1\}, \{C_h(\alpha)/0 < \alpha \le 1\}, \{C_{set}(\alpha)/0 < \alpha \le 1\}, \{C_{dor}(\alpha)/0 < \alpha \le 1\}, \{C_v(\alpha)/0 < \alpha \le 1\}, \{C_{br}(\alpha)/0 < \alpha \le 1\}$ .These sets represents sets of movable boundaries and they form nested structure zimmermann(2001) for expressing the relationship between the crisp sets and fuzzy sets . Let the confidence interval of the fuzzy sets $C_y(\alpha), C_{build}(\alpha), C_h(\alpha), C_{set}(\alpha), C_{dor}(\alpha), C_{busy}(\alpha), C_v(\alpha), C_{br}(\alpha)$ be

$[l_{C_{y(\alpha)}}, u_{C_{y(\alpha)}}], [l_{C_{build(\alpha)}}, u_{C_{build(\alpha)}}], [l_{C_{h(\alpha)}}, u_{C_{h(\alpha)}}], [l_{C_{set(\alpha)}}, u_{C_{set(\alpha)}}], [l_{C_{dor(\alpha)}}, u_{C_{dor(\alpha)}}], [l_{C_{busy(\alpha)}}, u_{C_{busy(\alpha)}}]$,

$[l_{C_{v(\alpha)}}, u_{C_{v(\alpha)}}], [l_{C_{br(\alpha)}}, u_{C_{br(\alpha)}}]$

By using the concept of α-cut the $FM^x_{(m,n)}$/FM/1 with fuzzy breakdown and fuzzy multiple vacation can be reduced to $M^x_{(m,N)}$/M/1/BD/MV. The expected total average total cost of crisp queuing system with bulk arrival is given by

$T_c^R(m,N) = hL_1 + (h(\frac{a}{e}) + u)P_{busy} + \frac{1}{E(f) + E(t)\sum_{n=0}^{m-1}\frac{e_n}{1-a_0} + \sum_{n=m}^{N-1}\frac{\phi_n^e}{P_1}} + R(y + E(f)s) + h\ P_1\ E(X)\ \frac{E(f^2)}{2} + [h(\ P_1\ E(X)(\ \frac{E(t^2)}{2} + E(f)E(t)\ ) - RE(t)V]\sum_{n=0}^{m-1}\frac{e_n}{1-a_0} + hE(t)\sum_{n=0}^{m-1}\frac{ne_n}{1-a_0} + Rd\sum_{n=m}^{N-1}\frac{\phi_n^R}{P_1} + h\sum_{n=m}^{N-1}\frac{n\phi_n^R}{P_1}$

3. ROBUST RANKING TECHNIQUE-ALGORITHM

To find the characteristics of system interest in terms of crisp value we defuzzyify the numbers into crisp ones by a fuzzy number ranking method. Robust ranking technique Nagarajan & Solai raju (2010) which satisfies compensation, linearity and additive properties results which are consistent with human intuition. By giving a convex fuzzy number a, the Robust Ranking Index is defined by,

$$R(\overline{a}) = \int_0^1 0.5(a_\alpha^L +)d\alpha,$$

Where $(a_\alpha^L + a_\alpha^U)$ is the α -level cut of the fuzzy number $\overline{a}$. In this paper we use this method for ranking the fuzzy numbers. The Robust ranking index R($\overline{a}$) gives the representative value of the fuzzy number $\overline{a}$. It satisfies the linearity and additive property.

4. NUMERICAL EXAMPLE

Consider a M^x/M/1/BD/MV queuing system. The corresponding parameters such as arrival rate, service rate, vacation rate, set up rate, breakdown rate, repairing rate are fuzzy numbers.Let $\overline{\lambda}$ =[.1,.2,.3,.4], $\overline{\mu}$ =[.4,.5,.6,.7], $\overline{\eta}$ =[0.03,0.04,0.05,0.06], $\overline{\gamma}$ =[0.5,0.6,0.7,0.8], $\overline{\alpha}$=[0.2,0.3,0.4,0.5], $\overline{\beta}$=[0.6,0.7,0.8,0.9] whose intervals of confidence are[.1+α,.4-α], [.4+α,.7-α], [0.03+α,0.06-α], [0.5+α,0.8-α], [0.2+α,0.8-α], [0.6+α,0.9-α].

The corresponding cost parameters such as the start up cost, build up cost, holding cost, set up cost, dormant cost, cost for busy, vacation period, breakdown cost, are fuzzy numbers.

Let $\overline{C_y}$= [1000, 1100, 1200, 1300], $\overline{C_{build}}$= [12,14,16,18],$\overline{C_h}$= [0.5,1,1.5,2], $\overline{C_{set}}$ = [400, 450, 500, 550],$\overline{C_{dor}}$=[16,17,18,19],$\overline{C_{busy}}$=[1200,1300,1400,1500],$\overline{C_v}$=[6,7,8,9], $\overline{C_{br}}$ =[20,30,40,50] whose intervals of confidence are[1000+α 1300-α], [12+α 18-α], [0.5+α 2-α], [400+α 550-α], [16+α 19-α], [1200+α 1500-α], [6+α 9-α] respectively. Now we evaluate R(0.1,0.2,0.3,0.4) by applying Robust Ranking Method. The membership function of the Trapezoidal Fuzzy Number (0.1,0.2,0.3,0.4) is

$$\theta(p)=\begin{cases}\frac{p-0.1}{0.1}, 0.1 \le p \le 0.2\\ 0.1\ , 0.2 \le p \le 0.3\\ \frac{0.4-p}{0.1}, 0.3 \le p \le 0.4\\ 0\ \ \ , \text{Otherwise}\end{cases}$$

The α cut of the fuzzy number (0.1,0.2,0.3,0.4) is(p_α^L, p_α^U) =(0.1+0.1α 0.4-0.1α) for which

$R(\bar{\lambda}) = R(.1,.2,.3,.4) = \int_0^1 0.5(.1+.4)d\alpha = \int_0^1 0.5(.5)d\alpha$ =0.25.

By proceeding similarly, the Robust Ranking Indices for the fuzzy numbers $\overline{\mu}$, $\overline{\eta}$, $\overline{\gamma}$, $\overline{\alpha}$, $\overline{\beta}$, $\overline{C_y}$, $\overline{C_{build}}$, $\overline{C_h}$, $\overline{C_{dor}}$, $\overline{C_{busy}}$, $\overline{C_v}$, $\overline{C_{br}}$ are calculated as $R(\overline{\mu})$=0.55, $R(\overline{\eta})$=0.045, $R(\overline{\gamma})$=0.65, $R(\overline{\alpha})$=0.35, $R(\overline{\beta})$=0.75, $R(\overline{C_y})$=1150, $R(\overline{C_{build}})$=15, $R(\overline{C_h})$=1.25, $R(\overline{C_{set}})$=475, $R(\overline{C_{dor}})$=17.5, $R(\overline{C_{busy}})$=1350, $R(\overline{C_v})$=7.5, $R(\overline{C_{br}})$=35.

According to the above condition the average total cost is evaluated and tabulated as

m/n	1	2	3	4	5	6	7	8	9	10	11…	15
1	778.92	778.75	778.43	778.01	777.56	777.12	776.73	776.41	776.17	776.01	775.93	776.31
2		777.84	777.71	777.47	776.16	776.83	776.53	776.25	776.05	775.91	775.84	776.22
3			777.06	776.97	776.79	776.56	776.32	776.11	775.94	775.82	775.76	776.14
4				776.52	776.45	776.31	776.15	775.98	775.85	775.75	775.70	776.07
5					776.15	776.10	776.00	775.88	775.78	775.70	775.66	776.00
6						775.92	775.88	775.81	775.73	775.67	**775.64**	775.96
7							775.79	775.76	775.72	775.68	775.66	775.94
8								775.76	775.74	775.71	775.69	775.94
9									775.79	775.77	775.76	775.97
10										775.88	775.87	776.03
11											776.01	776.13
12												776.26
13												776.43
14												776.65
15												776.90

The above table gives the average or representative value of total expected cost T_c^R(m,N) for different values of m=1,2,3…..15 and N=1,2,3…..15, which is a crisp value. For any system we always look into minimize the cost. In this table we observe that when (m,N) increases and the total average cost decreases we further obtain the least minimum cost at(6,11) which corresponds to 775.64. According to the specified cost parameter we obtain the optimal cost at (6,11). Thus we give input as fuzzy values and get output as crisp values.

5. CONCLUSION

In this paper fuzzy set theory has been applied to bulk arrival queuing system with bi level threshold policy, breakdown and multiple vacation. The arrival rate, service rate, vacation rate, setup rate, repair rate, breakdown rate and cost functions are fuzzy number which are more realistic and general in nature. Moreover the fuzzy problem has been converted into crisp problem by using Robust ranking technique, (i.e) if we give fuzzy inputs we can get crisp outputs. Since the system interest such as the average total cost is crisp value, the management can take the best and optimum decisions. We find that the solution of fuzzy problem can be obtained by applying Robust ranking method efficiently.

6. REFERENCES

[1] Chen.S.P(2005) "Parametric nonlinear programming approach to fuzzy queues with bulk service",European Journal of Operational Research,Vol.163, pp.434-444.

[2] Choobinesh.F and Li.H(1993), " An index for ordering fuzzy numbers",Fuzzy numbers and System,Vol 54,pp.143-161.

[3] Choudhury.G and Paul.M(2006), "A batch arrival queue with a second optional service channel under N-policy", Stochastic Analysis and Application,Vol 24,pp.1-12.

[4] Julia Rose Mary and Shanmugapriya(2014), "Optional operating policy of $FM_{(m,N)}^X$/G_{SOS}/1/MV", International journal of computer application,Vol 1,issue4,pp.199-207.

[5] Julia Rose Mary and Angel Jenitta (2014), "Evaluation of total average cost of $M^x_{(m,N)}$/M/1/BD/SV with fuzzy parameter using Robust Ranking Technique", National Annual Research Congress (2014)

[6] Kao C,Li.C and Chen.S(1993) "Parametric programming to the analysis of fuzzy queues",Fuzzy Sets and System,107,pp.93-100.

[7] Ke.J.C,(2004) "Bi-level control for batch arrival queues with an early setup and un-reliable server" Appl.Math.Modeling,Vol 28, pp.469-485.

[8] Lee.H.S and Srinivasan.M.M(1989), "Control policies for the M^X/G/1 queuing system", Manege.Sci,35(6),pp.708-721.

[9] Lee H.W,Lee S.S and Chae K.C(1994), "Operating Characteristics of $M^X/G/1$ queue with N-policy", Queuing system 15,pp.387-399.

[10] Lee H.W,Lee S.S,Yoon S.H and Chae K.C(1995) "Batch arrival queue with N-policy and single vacations" Coput.Oper.Res,22(2),pp.173-189.

[11] Lee H.W and Park J.O(1997) " Optimal strategy in N-policy Production system with early setup", J.Oper.Res.Soc,48,pp.306-313.

[12] Lee H.W, Park J.O, Kim B.K,Yoon S.H,Ahn B.Y and Park N.I(2004), "Queue length and waiting time analysis of batch arrival queue with bi level control", Appl.Math.Modeling,28,pp.469-485

[13] Li RJ and Lee ES(1989), Analysis of fuzzy queues, Computers and Mathematics with applications;vol 17(7),pp.1143-1147

[14] Nagi D.S and Lee E.S(1992), Analysis and simulation of fuzzy queues, Fuzzy sets and systems,46,pp.321-330

[15] A.Nagoor Gani and V.Ashok Kumar(2009), "A Bulk arrival queuing model with fuzzy parameters and fuzzy varying batch size", vol.2,no.3

[16] Nagarjan & Solairaju(2010), "Computing Improved fuzzy optimal Hungarian assignment problems with fuzzy costs under robust ranking techniques", vol.6,no.13,pp.6-13.

[17] Neuts M.F(1984), "The M/G/1 queue with limited number of admission or a limited admission period during each service time", Technical Report No.978,University of Delawara.

[18] Palapandi.B, Geethamani.G(, "Evaluation of performance of bulk arrival queue with fuzzy parameters using robust ranking technique", International journal of computing engineering research, vol 03,issue 10,pp.53-57.

[19] Rue R.c and Roshan Shine(1981), " Some properties of optimal control for entries to an M/M/1 queue", Naval Research logistic quarterly,28,pp.520-532.

[20] Stidham.S(1985), "Optimal control of admission to a queuing system",IEEE Trans,Automat controal,30,pp.705-713.

[21] Yadin.M and Naor.P(1963), "Queuing system with a removable service station" Oper.Res.Q.,14,pp.393-405.

[22] Yager R.R and Chen S.P(1981) "A procedure for ordering fuzzy subsets of the unit interval,Information Science,vol.24,pp.143-161.

[23] Zimmermann HF(2001).Fuzzy set theory and its application, Fourth edition K luwernijhoff, Boston.