Output Feedback Stabilization of a Class of Non-affine Nonlinear Systems in Discrete Time

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ABSTRACT

In this paper, output feedback control is investigated for a general class of uncertain non-affine nonlinear systems in discrete time. Control system design employs feedback linearization, coupled with a novel filter which is built to estimate the feedback linearization error. Output feedback control is then developed to stabilize the systems by utilizing the estimation. In the control design, implicit function theorem and the mean value theorem are exploited to handle the difficulty of non-affine appearance of the control input. The proposed control is of great significance in engineering practice due to its linear control architecture, high dynamic performance, clear physical meanings and robustness to the modeling errors.

General Terms:

Algorithms, Nonlinear Control Theory

Keywords:

Nonaffine nonlinear systems, discrete-time systems, output feedback control, uncertainty

1. INTRODUCTION

Recently, the control problem of non-affine nonlinear systems has attracted increasing research interests within the control systems community. Many elegant control schemes for continuous-time systems in this area have been obtained, including adaptive neural network (NN) control [1]-[4], adaptive fuzzy control [5], and backstepping control by incorporating the adaptive NN control method [6]. However, due to the the fact that the linearity property of the derivative of a Lyapunov function in continuous-time is not presented in the difference of the Lyapunov function in discrete time [7] and the lack of applicability of Lyapunov techniques [8], many elegant control methods in continuous-time domain may be not suitable for discrete-time systems. Therefore, it is challenging and very important to develop control scheme for discrete-time nonlinear systems.

In [9], the authors considered the control problem of a class of nonlinear discrete-time systems with general relative degree and proposed a stable NN controller through backpropagation such that the closed loop achieved the desired control performance. For strict-feedback nonlinear dynamical systems in discrete time, adaptive NN control schemes were presented by state feedback in [10] and output feedback in [11]. And for pure-feedback nonlinear systems in the discrete-time form adaptive neural output feedback tracking controllers were further investigated in [12, 13]. The common features of these adaptive control approaches are employed online NN to compensate for the unknown continuous functions, of which the tremendous advantage is that the unknown functions can be approximated to an arbitrarily accuracy. A limitation lies that reconstruction errors must occur if the structure of the approximators (i.e., the number of hidden layers and neurons of NN) is not sufficiently rich, and these reconstruction errors are introduced into the closed-loop and deteriorate the performance of the system. In general, the richer the structure is the more favorable performance the system exhibits. However, the richer structure may lead to the complexity of the controller and cause the heavy computational burden.

In practical applications, it is expected that control strategy can reduce the effects of uncertainties to a given accuracy, and has a simple control architecture, high dynamic performance and clear physical meanings. In this paper, output feedback control is investigated for a general class of highly uncertain discrete-time non-affine nonlinear systems. Control system design employs feedback linearization, coupled with a novel filter which is built to estimate the feedback linearization error. Output feedback control is then developed to stabilize the systems by utilizing the estimation. To handle the difficulty of non-affine appearance of the control input, implicit function theorem and the mean value theorem are exploited in the control design. The main contributions of the paper are as follows:

—Novel filter is constructed for a class of non-affine nonlinear systems, by which the feedback linearization error can be estimated without using NN;
—Based on the novel filter, robust output feedback control is developed to stabilize the discrete-time non-affine nonlinear systems;
—The proposed control is of great significance in engineering practice due to its linear control architecture, high dynamic performance, clear physical meanings and robustness to the modeling errors.

Throughout this paper, $\mathbb{Z}_0^+$ stands for nonnegative integers, $|\cdot|$ denotes the absolute value of a scalar, $\|\cdot\|$ denotes the norm of a matrix, $I_{n\times n}$ denotes the $n \times n$ dimension identity matrix,

$a := b$ means that b is defined as a, $[\ ]^T$ denotes the transpose of a vector/matrix, $\sigma(M)$ denotes the spectral radius of a square matrix M, i.e., $\sigma(M) := \max_{1\le i\le n}\{|\lambda_i|\}$, where $\lambda_1, \cdots, \lambda_n$ are the eigenvalues of matrix M, $A := \begin{bmatrix} 0 & & \\ \vdots & I_{(n-1)\times(n-1)} & \\ 0 & \cdots & 0 \end{bmatrix}_{n\times n}$, $B := [0, 0, \cdots, 0, 1]_n^T, C := [1, 0, \cdots, 0, 0]_n^T$. The following lemma will be used for control design and stability analysis in Section 3.

LEMMA 1 **IMPLICIT FUNCTION THEOREM**[14]. *Assume that function $f(x, u) : \mathbb{R}^n \times \mathbb{R} \to \mathbb{R}$ is continuously differentiable $\forall(x, u) \in \mathbb{R}^n \times \mathbb{R}$. If there exists a constant $c > 0$ such that $\partial f/\partial u \ge c > 0, \forall(x, u) \in \mathbb{R}^n \times \mathbb{R}$, then, there exists a continuous function $u(x) : \mathbb{R}^n \to \mathbb{R}$ such that $f(x, u(x)) = 0$.*

2. PROBLEM FORMULATION

Consider the non-affine nonlinear systems in the following form

$$\begin{cases} y_{k+n} &= f(x_k, u_k), k \in \mathbb{Z}_0^+ \\ x_k &= [y_k, y_{k+1}, \cdots, y_{k+n-1}]^T \end{cases} \tag{1}$$

where
$u_k \in \mathbb{R}$ is the control input;
$y_k \in \mathbb{R}$ is the measured output;
$x_k \in \mathbb{R}^n$ is the state vector;
$f : \mathbb{R}^{n+1} \to \mathbb{R}$ is an unknown implicit function;
$\psi : u_k \to f$ is a bijection for every fixed (x_k, u_k);
$n \ge 1$ is system order.

Assume that system function f is continuously differentiable with respect to all the arguments and only the output $y_i, 0 \le i \le k$ are available for feedback at each step k. And without lose of generality, assume that the origin $x = 0$ is an equilibrium point of system (1), i.e., $f(0, 0) = 0$. In order to design a control input u which stabilizes the system, the following assumption is made for the function f.

ASSUMPTION 1. *The sign of $\partial f/\partial u_k$ is known, and $\partial f/\partial u_k \ne 0, \forall(x, u_k) \in \Omega_x \times \mathbb{R}$, where $\Omega_x \subset \mathbb{R}^n$ is a certain controllability region containing the origin.*

The control objective is to design a robust control for the system such that the output is stabilized at the origin, and meanwhile all the signals in the closed-loop system remain semi-globally uniformly ultimately bounded (SGUUB).

REMARK 1. *Assumption 1 implies that $\partial f/\partial u_k$ is strictly either positive or negative. Without lose of generality, assume that $\partial f/\partial u_k > 0, \forall(x, u_k) \in \Omega_x \times \mathbb{R}$. Assumption 1 is reasonable because $\partial f/\partial u_k$ being away from zero is controllable condition of system (1).*

REMARK 2. *In practice, many engineering plants including active magnetic bearing systems [3], vibrating systems [15],continuous stirred-tank reactor systems [16], etc, are reasonably described in the non-affine forms (1). Because, for these plants, they are difficult to be exactly described in affine forms even though the modeling errors are neglected.*

REMARK 3. *Under Assumption 1, the non-affine nonlinear system described by (1) includes a large class of nonlinear systems. It is should be noted that many elegant results in this area have been obtained using the approximators (e.g., NN) to estimate the uncertainties. Next section in this paper will be presented a novel dynamic linear filter which also can be used to estimate the uncertainties. Thus, compared with NN control, the proposed control architecture is quite simple.*

3. ROBUST OUTPUT FEEDBACK CONTROL DESIGN

By adding and subtracting gu in the right-hand side of system (1), feedback linearization is performed by

$$y_{k+n} = gu_k + \Delta_k, \tag{2}$$

where $g > 0$ is a design constant, and

$$\Delta_k = f(x_k, u_k) - gu_k, \tag{3}$$

which is the feedback linearization error. Let the control input be determined as

$$u_k = \frac{1}{g}(u_{d,k} + u_{\Delta,k}), \tag{4}$$

where $u_{d,k}$ is a dynamic feedback controller designed to stabilize linearized dynamics in (2) by assuming $\Delta_k = 0$; $u_{\Delta,k}$ is a filter-based compensator designed to handle the effect of Δ_k, i.e., given a small parameter $\varepsilon > 0$, find a $u_{\Delta,k}$ such that

$$|u_{\Delta,k} + \Delta_k| \le \varepsilon, \forall k \ge k_0, \tag{5}$$

where $k_0 \in \mathbb{Z}_0^+$.

Due to the bijection $\psi : u_k \to f(\cdot, u_k)$, it is noted that the compensator $u_{\Delta,k}$ is designed to deal with Δ_k, whereas Δ_k is a function of $u_{\Delta,k}$ through $f(\cdot, u_k)$ and u_k. Like the result in the continuous time [5, 17], the following lemma is introduced to guarantee the existence and uniqueness of a solution of

$$h(x_k, u_{d,k}, u_{\Delta,k}) = 0 \tag{6}$$

where $h(x_k, u_{d,k}, u_{\Delta,k}) = u_{\Delta,k} + \Delta_k$.

LEMMA 2. *[5, 17] Let constant $g > 0$ satisfies the following inequalities*

$$0 < \frac{1}{2}\left(\frac{\partial f}{\partial u_k}\right) < g, \forall(x, u_k) \tag{7}$$

then (6) has a unique solution over the entire input domain of interest.

3.1 Dynamic feedback controller design

Under the assumption that $\Delta_k = 0$, then we can let $u_{\Delta,k} = 0$ and design $u_{d,k}$. System (2) reduce as

$$y_{k+n} = u_{d,k} \tag{8}$$

which can be stabilized using classical linear control design, such as the dynamic feedback control

$$\xi_{k+1} = A_d\xi_k + B_d y_k, \tag{9a}$$

$$u_{d,k} = C_d^T\xi_k + d_d y_k, \tag{9b}$$

where $A_d \in \mathbb{R}^{(n-1)\times(n-1)}, B_d, C_d \in \mathbb{R}^{n-1}$ and $d_d \in \mathbb{R}$ are appropriately chosen parameters such that matrix $A_s = \begin{bmatrix} A + d_dBC^T & BC_d^T \\ B_dC^T & A_d \end{bmatrix}$ is Schur, i.e., the spectral radius $\sigma(A_s) < 1$.

It should be noticed that it is required dim $\xi_k \geq n-1$ due to the dynamics in (8) having n poles at the origin [18]. Without lose of generality, the minimum dimension is chosen in the sequel. And it is the same reason for considering the dimension of a filter given in the next subsection.

3.2 Compensator design

Substituting (4) and (9b) into (2), we have

$$y_{k+n} = u_{d,k} + u_{\Delta,k} + \Delta_k, \tag{10}$$

To compensate the effect of Δ_k, we construct a filter by using the available output

$$\hat{y}_{k+n} = -\epsilon d_c(y_k - \hat{y}_k) - C_c^T \eta_k + u_{d,k} + u_{\Delta,k}, \tag{11a}$$

$$\epsilon^{-1}\eta_{k+1} = A_c \eta_k + \epsilon B_c(y_k - \hat{y}_k), \tag{11b}$$

where $\eta_k \in \mathbb{R}^{n-1}$, $u_{\Delta,k}$ will be designed in (14) and ϵ is a small parameter to be specified later. By introducing the variable

$$\tilde{y}_k = \epsilon(y_k - \hat{y}_k), \tag{12}$$

and from (10) and (11), we have

$$\epsilon^{-1}\tilde{y}_{k+n} = d_c\tilde{y}_k + C_c^T\eta_k + \Delta_k, \tag{13a}$$

$$\epsilon^{-1}\eta_{k+1} = A_c\eta_k + B_c\tilde{y}_k, \tag{13b}$$

where $A_c \in \mathbb{R}^{(n-1)\times(n-1)}, B_c, C_c \in \mathbb{R}^{n-1}$ and $d_c, \epsilon \in \mathbb{R}$ are appropriately chosen parameters such that not only matrix $A_{ss} = \begin{bmatrix} A + d_c BC^T & BC_c^T \\ B_c C^T & A_c \end{bmatrix}$ is Schur, but also $\tilde{y}_k$ converges in a faster time scale than y_k.

Observing (13a), the compensator is designed as

$$u_{\Delta,k} = d_c\tilde{y}_k + C_c^T\eta_k. \tag{14}$$

from which and combining (13a) the left-hand side of (5) becomes

$$\begin{aligned} &|u_{\Delta,k} + \Delta_k| \\ &= \left|d_c\tilde{y}_k + C_c^T\eta_k + \Delta_k\right| = \epsilon^{-1}\left|\tilde{y}_{k+n}\right|. \end{aligned} \tag{15}$$

Theorem analysis in the next subsection will be shown that, under the proposed controller (4), (9) and (14) and some conditions, the output y_k and $\epsilon^{-1}\tilde{y}_{k+n}$ converge to the origin and all the signals in the closed-loop system remain bounded.

REMARK 4. *In the literatures [4, 18], observers were constructed to estimate the unmeasurable states, then the NNs were employed to approximate the uncertainties. The novel filter proposed in this paper is built based on the available output information and can be directly estimated the uncertainties without using the NNs.*

3.3 Stability analysis

For clarity, let

$$\tilde{x}_k = [\tilde{y}_k, \tilde{y}_{k+1}, \cdots, \tilde{y}_{k+n}]^T,$$

$$X_k = \left[x_k^T, \xi_k^T\right]^T, \tilde{X}_k = \left[\tilde{x}_k^T, \eta_k^T\right]^T,$$

$$\bar{B} = \begin{bmatrix} B \\ 0 \end{bmatrix}, I_\epsilon = \begin{bmatrix} I_{(n-1)\times(n-1)} & 0 \\ 0 & \epsilon I_{n\times n} \end{bmatrix},$$

$$\sigma_{As} = \sigma(A_s), \sigma_{Ass} = \sigma(A_{ss}), \sigma_\epsilon = \sigma(I_\epsilon)$$

$$F_{x,k}(x_k, u_k) = \frac{\partial f(x_k, u_k)}{\partial x_k}, f_{u,k}(x_k, u_k) = \frac{\partial f(x_k, u_k)}{\partial u_k}$$

The compact form of system (13) can be written by

$$I_\epsilon^{-1}\tilde{X}_{k+1} = A_{ss}\tilde{X}_k + \bar{B}\Delta_k \tag{16}$$

Substituting (4), (9) and (14) into (2), the closed-loop takes the compact form of

$$X_{k+1} = A_s X_k + \bar{B}\bar{B}^T\Xi_k, \tag{17}$$

where $\Xi_k = A_{ss}\tilde{X}_k + \bar{B}\Delta_k$.
From the mean value theorem, there exists a $\theta \in (0, 1)$ such that

$$f(x_{k+1}, u_{k+1}) - f(0, 0) = F_{x,k+1}^\theta x_{k+1} + f_{u,k+1}^\theta u_{k+1} \tag{18}$$

where $F_{x,k+1}^\theta = F_{x,k+1}(\theta x_{k+1}, \theta u_{k+1})$, $f_{u,k+1}^\theta = f_{u,k+1}(\theta x_{k+1}, \theta u_{k+1})$.
Combining (3), (16) and (18), we have

$$\begin{aligned} \Xi_{k+1} &= A_{ss}\tilde{X}_{k+1} + \bar{B}\Delta_{k+1} \\ &= A_{ss}I_\epsilon\Xi_k + \bar{B}[f(x_{k+1}, u_{k+1}) - gu_{k+1}] \\ &= A_{ss}I_\epsilon\Xi_k + \bar{B}[F_{x,k+1}^\theta x_{k+1} + (f_{u,k+1}^\theta - g)u_{k+1}] \\ &= A_{ss}I_\epsilon\Xi_k + \alpha X_{k+1} + \beta\tilde{X}_{k+1} \\ &= \bar{A}_{ss}I_\epsilon\Xi_k + \alpha[A_s X_k + \bar{B}\bar{B}^T\Xi_k] \end{aligned} \tag{19}$$

where
$\alpha = \bar{B}\left[F_{x,k+1}^\theta + \frac{f_{u,k+1}^\theta - g}{g} d_d C^T, \frac{f_{u,k+1}^\theta - g}{g} C_d^T\right]$
$\beta = \frac{f_{u,k+1}^\theta - g}{g}\bar{B}\left[d_c C^T, C_c^T\right]$
$\bar{A}_{ss} = A_{ss} + \beta$.

The following theorem shows that the proposed controller stabilizes systems (1) under the assumption that the feedback linearization error $\Delta_k \equiv 0$.

THEOREM 3. *Under the assumption that $\Delta_k \equiv 0$, consider the closed-loop consisting of system (1) satisfying Assumption 1, the filter (11), and the controller (4), (9) and (14). Then, there exists an $\epsilon^* > 0$ such that for all $\epsilon \in (0, \epsilon^*)$, the closed-loop system is globally exponential asymptotic stable.*

PROOF. Combining (17) and (19), we have

$$\begin{bmatrix} X_{k+1} \\ \Xi_{k+1} \end{bmatrix} = \Phi_1 \begin{bmatrix} X_k \\ \Xi_k \end{bmatrix} \tag{20}$$

where $\Phi_1 = \begin{bmatrix} A_s & \bar{B}\bar{B}^T \\ 0 & A_{ss}I_\epsilon \end{bmatrix}$. From linear system theory, we know that the dynamics (20) is stable if and only if $\sigma(\Phi_1) < 1$, i.e., $\sigma_{As}\sigma_{Ass}\sigma_\epsilon < 1$. It is easy to obtain that the system is globally exponential asymptotic stable for all $\epsilon \in (0, \epsilon_1^*)$, where $\epsilon_1^* = 1/(\sigma_{As}\sigma_{Ass})$. □

Based on Assumption 1, the following theorem shows the proposed controller stabilizes systems (1) despite the presence of Δ_k.

THEOREM 4. *Consider the closed-loop consisting of system (1) satisfying Assumption 1, the filter (11), and the controller (4), (9) and (14) and let $\epsilon^* = 1/(\sigma_{As}\sigma_{Ass})$. For all $x(0) \in \Omega_x$ and all $\epsilon \in (0, \epsilon^*)$ which satisfies*

$$\sigma_{As} + \sigma_{Ass}\epsilon + c_1 c_2 < \sqrt{2} \tag{21}$$

then the system output y_k converges to the origin and all signals in the closed-loop system remain SGUUB, where c_1 is an upper boundedness of α, i.e., $\|\alpha\| \leq c_1$ and $c_2 = \|\bar{B}\bar{B}^T\|$.

PROOF. Observing the matrix $\bar{A}_{ss}$ defined in (19). Then there exists a dialog matrix

$$M = \begin{bmatrix} I_{(n-1)\times(n-1)} & 0 & 0 \\ 0 & \frac{f^{\theta}_{u,k+1}}{g} & 0 \\ 0 & 0 & I_{(n-1)\times(n-1)} \end{bmatrix}$$

such that

$$\bar{A}_{ss} = A_{ss}M. \tag{22}$$

From Assumption 1, we have

$$0 < \frac{f^{\theta}_{u,k+1}}{g} < \frac{1}{2}.$$

Thus, $\sigma(M) = 1$. Moreover, since f is a continuously differetiable with respect to all the arguments and Ω_x is a compact set and Assumption 1, then $\partial F/\partial x_k$ and $\partial f/\partial u_k$ are bounded. i.e., there exist a constant $c_1 > 0$ such that $\|\alpha\| \leq c_1$.

Combining (17), (19) and (22), we have

$$\|X_{k+1}\| \leq \sigma_{As}\|X_k\| + c_2\|\Xi_k\| \tag{23}$$

$$\|\Xi_{k+1}\| \leq (\sigma_{Ass}\sigma_{\epsilon} + c_1c_2)\|\Xi_k\| + c_1\sigma_{As}\|X_k\| \tag{24}$$

The compact form of (23) and (23) can be written by

$$\begin{bmatrix} \|X_{k+1}\| \\ \|\Xi_{k+1}\| \end{bmatrix} \leq \Phi_2 \begin{bmatrix} \|X_k\| \\ \|\Xi_k\| \end{bmatrix} \tag{25}$$

where $\Phi_2 = \begin{bmatrix} \sigma_{As} & c_2 \\ c_1\sigma_{As} & \sigma_{Ass}\sigma_{\epsilon} + c_1c_2 \end{bmatrix}$. If $\sigma(\Phi_2) < 1$, then the dynamics (25) is stable. In what follows, we make analysis of the matrix Φ_2.

(i)If the design parameter $1 < \epsilon < 1/(\sigma_{As}\sigma_{Ass})$, then $\sigma_{\epsilon} = \epsilon$ and

$$\Phi_2 = \begin{bmatrix} \sigma_{As} & c_2 \\ c_1\sigma_{As} & \sigma_{Ass}\epsilon + c_1c_2 \end{bmatrix}.$$

Furthermore

$$\sigma(\Phi_2) = \frac{(\sigma_{As} + \sigma_{Ass}\epsilon + c_1c_2)^2}{2} - \sigma_{As}\sigma_{Ass}\epsilon \tag{26}$$

From (26), when ϵ satisfies (21) then $\sigma(\Phi_2) < 1$.

(ii)If $\epsilon \leq 1$, then $\sigma_{\epsilon} = 1$. Clearly, when

$$\sigma_{As} + \sigma_{Ass} + c_1c_2 < \sqrt{2}$$

we remain have $\sigma(\Phi_2) < 1$. □

REMARK 5. *Unlike the previous control approaches using NN and/or fuzzy systems [4, 5, 17], the robust output tracking controller proposed here is easy to implement due to its linear control architecture. The parameters in the proposed control have clear physical meanings and the desired dynamic performance can be achieved by tuning the design parameters.*

4. CONCLUSION

The main contribution of this paper has presented robust design for a general class of nonlinear systems non-affine in control by output feedback. A novel high-gain filter has been constructed to generate a fast time-scale signal to estimate the modeling errors without using NNs and/or fuzzy systems. Compared with the previous adaptive controllers, the proposed controller is of great significance in engineering practice due to its linear control architecture, high dynamic performance and clear physical meanings. In the future, investigation on a general class of nonaffine nonlinear systems will be interesting research topics in this field.

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