Optimizing the Satellite Control Gains with Nonlinear Motion Equations using SQP Method

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ABSTRACT
Two functions of control system are attitude maneuver and attitude stability. The capability to attitude-maneuver a satellite is based on using control torques. In this paper, the torques generated by Euler angle errors and quaternion error vector for small attitude commands are compared and then the same is done for large attitude commands. It's founded that Euler angle errors characterized by a fixed control do not produce the desirable response. The SQP optimization method also used to optimize calculated control gains. This SQP optimization method with the following provisions such as maximum settling time, maximum rise time, maximum overshoot and maximum steady state error causes that the Euler angles error method provides an appropriate responses.

Keywords
Optimizing control gains, SQP, Nonlinear Equations of Motion, Euler Angles Error, Quaternion Error Vector

1. INTRODUCTION
Rigid-body attitude control has studied problems in case of nonlinear control which is addressed in various resources, this is because of complications in its applications such as high performance aircraft, spacecraft, underwater vehicles, and robotics applications. For instance, a rapid performance of large angle maneuvers in satellites is required in order to perform rapid for various missions and it can be shown in multi-target acquisition or in pointing and tracking capabilities. Because of the cross coupling of angular velocity terms these rapid maneuvers are calculated by nonlinear rigid body dynamics. [1]

The attitude of the satellite against external torque disturbances was to stabilize by the control system. Aerodynamic drag effects, solar radiation and solar wind torques, parasitic torques created by the propulsion thrusters produce external torque disturbance. There are similar features for attitude controls that are based on spin: keeping one axis of the satellite inertial stabilized in space is considered as a spin principle. [2]

In this paper, the two basic control rules, i.e. Euler angle errors and quaternion error vector have been used.

Nonlinear equations of motion are considered as follows:

$$\begin{aligned} M_x &= I_x\dot{\omega}_x + \omega_y\omega_z(I_z - I_y),\\ M_y &= I_y\dot{\omega}_y + \omega_x\omega_z(I_x - I_y),\\ M_z &= I_z\dot{\omega}_z + \omega_y\omega_y(I_y - I_x).\end{aligned} \quad (1)$$

where ω_x, ω_y and ω_z are the angular velocity components, I_x, I_y and I_z are the principal moments of inertia, and M_x, M_y and M_z are the total attitude control torques and disturbance torques. [2] [3]

$$\begin{bmatrix}\dot{\phi}\\ \dot{\theta}\\ \dot{\psi}\end{bmatrix} = \frac{1}{c\theta}\begin{bmatrix} c\theta & s\phi s\theta & c\phi s\theta\\ 0 & c\phi c\theta & -s\phi c\theta\\ 0 & s\phi & c\phi\end{bmatrix}\begin{bmatrix}\omega_1\\ \omega_2\\ \omega_3\end{bmatrix} + \frac{n}{c\theta}\begin{bmatrix}s\psi\\ c\theta c\psi\\ s\theta s\psi\end{bmatrix} \quad (2)$$

$$\begin{bmatrix}\dot{q_1}\\ \dot{q_2}\\ \dot{q_3}\\ \dot{q_4}\end{bmatrix} = \frac{1}{2}\begin{bmatrix} 0 & \omega_3 & -\omega_2 + n & \omega_1\\ -\omega_3 & 0 & \omega_1 & \omega_2 + n\\ \omega_2 - n & -\omega_1 & 0 & \omega_3\\ -\omega_1 & -\omega_2 - n & -\omega_3 & 0\end{bmatrix}\begin{bmatrix}q_1\\ q_2\\ q_3\\ q_4\end{bmatrix} \quad (3)$$

For quaternion error vector, the quaternion feedback is used as follows:

$$q_S^{-1}q_T = q_E = \begin{bmatrix} q_{T4} & q_{T3} & -q_{T2} & q_{T1}\\ -q_{T3} & q_{T4} & q_{T1} & q_{T2}\\ q_{T2} & -q_{T1} & q_{T4} & q_{T3}\\ -q_{T1} & -q_{T2} & -q_{T3} & q_{T4}\end{bmatrix}\begin{bmatrix}-q_{S1}\\ -q_{S2}\\ -q_{S3}\\ q_{S4}\end{bmatrix} \quad (4)$$

Where q_E, q_T and q_S are (respectively) the error, target, and spacecraft quaternions.

The simplest control law for stabilizing and attitude-maneuvering such a system may be stated as follows:

$$\begin{aligned} T_{cx} &= 2K_x q_{1E} q_{4E} + K_{xd} p,\\ T_{cy} &= 2K_y q_{2E} q_{4E} + K_{yd} q,\\ T_{cz} &= 2K_z q_{3E} q_{4E} + K_{zd} r.\end{aligned} \quad (5)$$

[2],[4]

In a case at reference (1), the moments of inertia of the satellite are given as I_X=1,000, I_Y = 500, I_Z = 700 kg-m^2 and assuming the closed-loop natural frequencies of ω_n = 1 rad/sec and closed-loop damping factors of ζ = 1, the control gains are obtained as follow:

Table 1. The control gains with ω_n = 1 and ζ = 1

K_X	K_Y	K_Z	K_{dx}	K_{dy}	K_{dz}
1000	500	700	2000	1000	1400

The first for the three small attitude commands ψ_{com} = -6°, θ_{com}= -4° and ϕ_{com} = 4° are calculated the Euler angles, with the assuming step angular commands, starting at t_o = 1 sec and zero Euler angles at initial time. The results of the simulation with the above conditions are shown in figures 1 and 2.

It can be seen that for these comparatively small attitude commands, the time responses for the two control laws are almost identical. These results are for small attitude changes. The direction cosine attitude errors approach the Euler angle errors.

The same simulations were repeated for the much larger attitude commands ψ_{com} = -60°, θ_{com}= -40° and ϕ_{com} = 40°. Comparison between the time responses for the Euler angles control law (Figure 3) and the quaternion control law (Figure 4), that these simulations are obtained by given control gains from reference (1), shows the clear superiority of the latter. [2]

2. OPTIMIZATION

Sequential Quadratic Programming (SQP) is one of the most successful methods for the numerical solution of constrained nonlinear optimization problems. It depend on intense theoretical foundation and provides powerful algorithmic tools for the solution of large-scale technologically appropriate problems. [4]

SQP methods solve a sequence of optimization sub-problems, each of which optimizes a quadratic model of the objective subject to a linearization of the constraints. Solving a set of nonlinear equations by Newton's method and derive the non-linear equations simultaneously using.

Kuhn-Tucker conditions to lagrangian constrain optimization problems, if the problem is unconstrained, then the method reduces to Newton's method for finding a point where the gradient of the objective vanishes. SQP methods have been implemented in many packages, including NPSOL, OPSYC, SNOPT, NLPQL, MATLAB, OPTIMA, GNU Octave and SQP. [5] [6]

In this paper, using SQP optimization method, we calculated the optimal control gains with constraints of settling time less than 15 seconds, rise time of 8 seconds, overshoot less than 1.5% and steady state error of zeros.

The calculated values are shown in table 2:

Table 2. The Optimal Control Gains

K_X	K_Y	K_Z	K_{dX}	K_{dY}	K_{dZ}
139.0598	500	329.6802	999.9351	494.8144	1000

3. SIMULATION

As already noted in reference (1), according to Figure 3, it was concluded that the Euler angles method for large attitude commands have inappropriate response.
But, in this paper, simulated with the large attitude commands ψ_{com} = -60°, θ_{com}= -40° and ϕ_{com} = 40°, and by optimal control gains and the other conditions like reference 1 (Figures 4, 5).

As can be seen in Figure 5, Euler angles variations satisfies the above constraints and also are given very better response than simulation in reference (1) (Figure 3).

The results show that for large Euler angles, the quaternion error vector method necessarily have not given a more appropriate response than the Euler angle error method.

4. CONCLUSION

As shown in the figures 1 and 2, by a fixed control gains in small angles command, the Euler angles error method In comparison with quaternion error vector is not much different; however, when command angles were large, the Euler angles method responses were more inappropriate than quaternion error vector in figures 3 & 4. By using the optimal control gains as shown in the figures 4 and 5, the Euler angles error method have more appropriate response than quaternion error vector. As shown in figures 3 and 5, it is clear that the optimal control gains that optimized by SQP method, the answer is not only better than the assumed control gains by the closed-loop natural frequencies of ω_n = 1 rad/sec and closed-loop damping factors of ζ = 1, even the optimal control gains values are less than the assumed control gains, which indicates that even by the actuator weaker, less weight and energy saving, it obtained a better response.

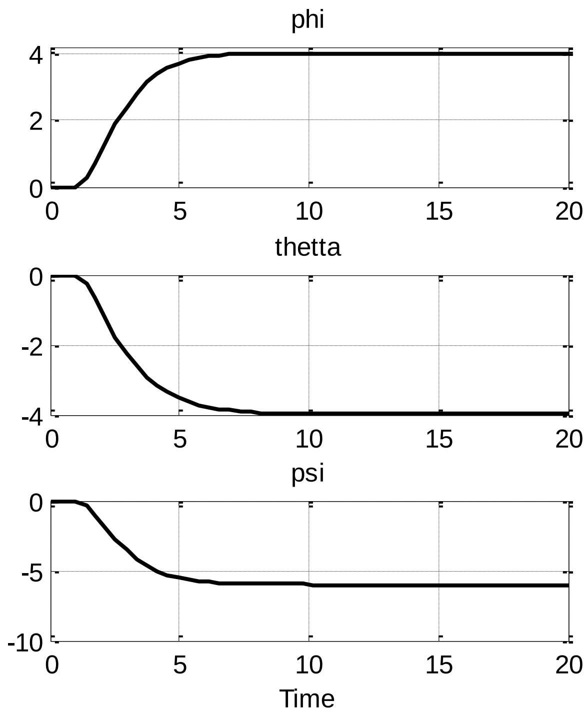

Fig 1: Euler angle step responses using the Euler angle error control law with ω_n = 1, ζ = 1: ψ_{com}= -6°, θ_{com}= -4° and ϕ_{com}= 4°.

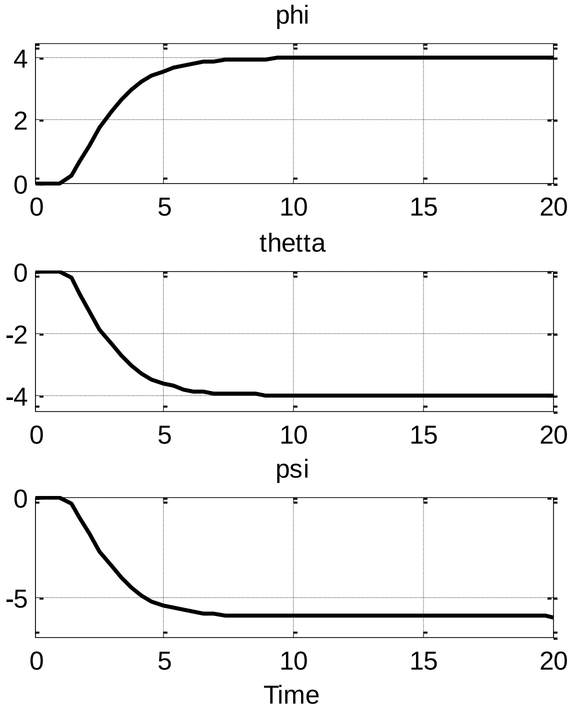

Fig 2: Euler angle step responses using the quaternion error control law with $\omega_n = 1, \zeta = 1$: $\psi_{com} = -6°$, $\theta_{com} = -4°$ and $\phi_{com} = 4°$.

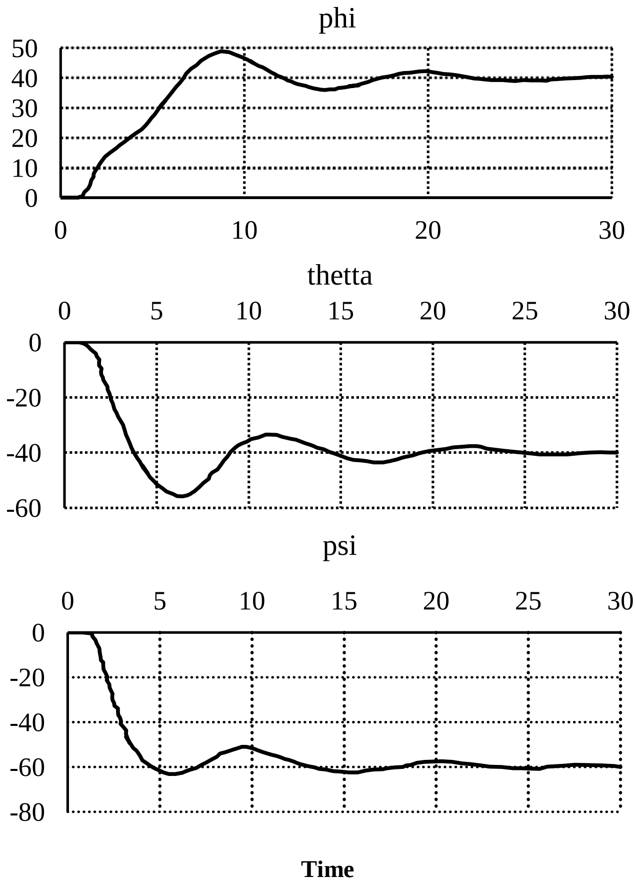

Fig 3: Euler angle step responses using the Euler angle error control law with $\omega_n = 1, \zeta = 1$: $\psi_{com} = -60°$, $\theta_{com} = -40°$ and $\phi_{com} = 40°$.

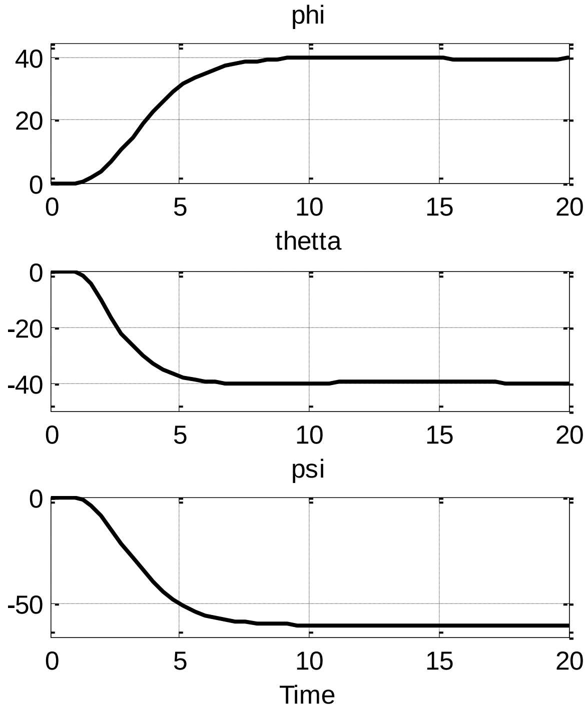

Fig 4: Euler angle step responses using the quaternion error control law with optimal control gains: $\psi_{com} = -60°$, $\theta_{com} = -40°$ and $\phi_{com} = 40°$.

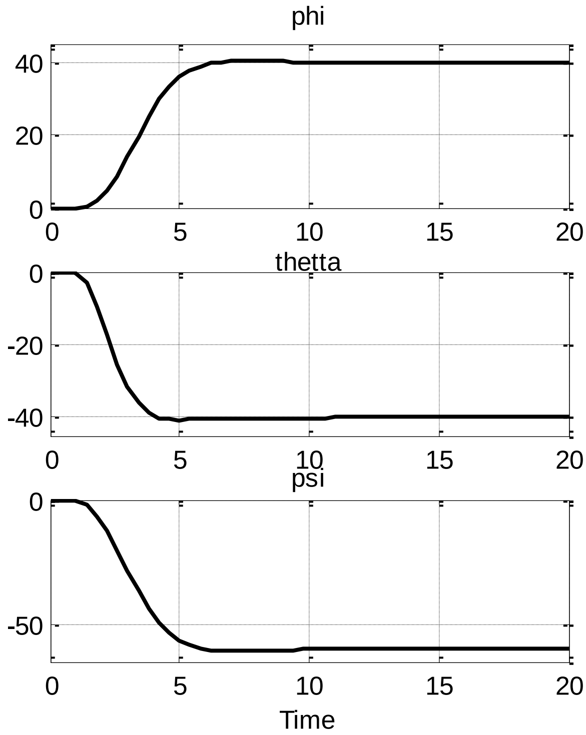

Fig 5: Euler angle step responses using the Euler angle error control law with optimal control gains: $\psi_{com} = -60°$, $\theta_{com} = -40°$ and $\phi_{com} = 40°$.

5. REFERENCES

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