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Numerical Experimentation to Detect the Signal in MIMO

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ABSTRACT
Multiple input and multiple output (MIMO) is a key technology for third and fourth generation wireless communication system. Unique aspect of MIMO is spatial multiplexing which actually increases the data rate by transmitting several information stream in parallel without increasing the transmit power. However, spatial demultiplexing or signal detection at the receiver side is a challenging task for Spatially Multiplex MIMO(SM MIMO) systems. This paper presents numerical experimentation to detect the signal in MIMO using MatLab platform.

General Terms
Wireless communication, MIMO

Keywords
Detection, spatial multiplexing, wireless communication, ZF, V-Blast, MMSE

1. INTRODUCTION
In a MIMO system both transmit and receive antenna combine to give a large diversity order. In which spatial diversity gain can be obtained when multiple antenna are present at either the transmit or the receive side [1]. In a spatial multiplexing system, the data stream to be multiplexed into M_T lower rate stream which are then simultaneously sent from the M_T transmit antennas after coding and modulation and all the transmitted streams occupy the same frequency band(i.e. they are co-channel signals). At the receiver side, each receive antenna observes a superposition of the transmitted signals. The receiver then separates them into constituent data streams and demultiplexes them to recover the original data stream. And this separation step at the receiver needs to determine the performance and computational complexity of the receiver. Algorithms to separate the parallel data streams corresponding to the M_T transmit antenna are of two categories i.e. linear and nonlinear Next sections discusses mathematical representation of the MIMO system and numerical simulations for detection of the signal in MIMO using these algorithms [2].

1.1 Narrowband MIMO System Model
Figure 1 shows N_R x N_T MIMO system Model. In this paper the system model considered has 4 transmit and 4 receive antennas shown in figure 1. The transmitted symbols are taken independently from a Quadrature Amplitude Modulation (QAM) constellation.

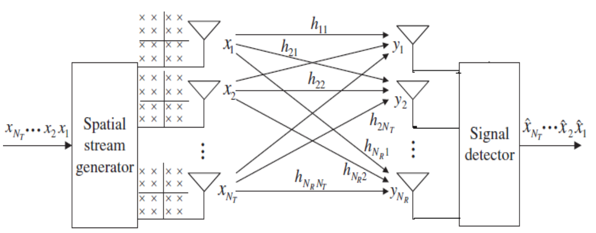

Fig 1:Model of MIMO system

Since each receive antenna is able to receive signals from all the transmit antenna the received signals of all the four antennas are given by

$$\begin{aligned} y_1 &= h_{11}x_1 + h_{12}x_2 + h_{13}x_3 + h_{14}x_4 + n_1 \\ y_2 &= h_{21}x_1 + h_{22}x_2 + h_{23}x_3 + h_{24}x_4 + n_2 \\ y_3 &= h_{31}x_1 + h_{32}x_2 + h_{33}x_3 + h_{34}x_4 + n_3 \\ y_4 &= h_{41}x_1 + h_{42}x_2 + h_{43}x_3 + h_{44}x_4 + n_4 \end{aligned} \quad (1)$$

where h_{ij} ,x_j and n_i represents channel gain from the j^{th} transmit antenna to i^{th} receive antenna, respectively. Let $y = [y_1, y_2, y_3, y_4]^T$ where T is the transpose operation and using the principle of matrix multiplication the received symbol vector is given by

$$y = Hx + n \quad (2)$$

where H is the 4x4 MIMO channel matrix. The channel matrix H is deterministic and assumed to be constant at all times and also known to both the transmitter and the receiver.i.e.the system under consideration is time invariant.

$$H = \begin{bmatrix} h_{11} & h_{12} & h_{13} & h_{14} \\ h_{21} & h_{22} & h_{23} & h_{24} \\ h_{31} & h_{32} & h_{33} & h_{34} \\ h_{41} & h_{42} & h_{43} & h_{44} \end{bmatrix} \quad (3)$$

x is a transmit symbol vector

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \quad (4)$$

and n is noise symbol vector

$$n = \begin{bmatrix} n_1 \\ n_2 \\ n_3 \\ n_4 \end{bmatrix} \quad (5)$$

where the transmit symbol vector and noise vector are denoted $x = [x_1, x_2, x_3, x_4]^T$ and $n = [n_1, n_2, n_3, n_4]^T$, respectively [3]. The capacity of MIMO channel in the above equation can be linearly increase with the minimum of the numbers of transmit and receive antennas under certain conditions where the channel gains , h_{ij} ,are independent zero mean CGSG random variable [4].With this consideration the higher transmission rates can be achieved without increasing the transmission power significantly for a given bandwidth.

In order to detect the transmit symbol vector ,the received symbol vector in equation (2) can be analyze with the numerical simulation in which we consider a MIMO system with 4 transmit and 4 receive antennas without taking into account the symbol energy.

To achieve this we consider a transmit symbol vector and noise symbol vector respectively as

$$x = \begin{bmatrix} -1+i \\ 1-i \\ -1+i \\ 1-i \end{bmatrix} \quad , \quad n = \begin{bmatrix} 0.07+0.20i \\ -0.18-0.27i \\ 0.07+0.20i \\ -1.00+1.55i \end{bmatrix}$$

And the channel matrix H is considered as

$$H = \begin{bmatrix} 0.54+0.32\text{i} & 3.03-0.21\text{i} & -2.26-0.43\text{i} & 2.77-0.06\text{i} \\ 1.83-1.31\text{i} & -1.35+0.71\text{i} & 0.86+0.34\text{i} & 3.58+0.73\text{i} \\ 3.58+0.73\text{i} & 3.58+0.73\text{i} & -1.35+0.71\text{i} & 1.83-1.31\text{i} \\ 2.77-0.06\text{i} & 2.77-0.06\text{i} & 3.03-0.21\text{i} & 0.54+0.32\text{i} \end{bmatrix} \tag{6}$$

The received signal vector in equation (2) becomes

$$y = Hx + n$$

$$y = \begin{bmatrix} 7.43-7.48\text{i} \\ 1.77+2.60\text{i} \\ 1.23-5.00\text{i} \\ -2.96+4.57\text{i} \end{bmatrix} \tag{7}$$

To detect x from y, equation (2) is rearranged as

$$H^{-1}y = H^{-1}Hx + H^{-1}n$$

$$= x + H^{-1}n$$

$$= \begin{bmatrix} -0.9+1i \\ 0.9-0.9i \\ -1+1.i \\ 0.9-1i \end{bmatrix} \tag{8}$$

From equation (8) it can be view that the most likely signal vector for

$$\begin{bmatrix} -0.9+1i \\ 0.9-0.9i \\ -1+1.i \\ 0.9-1i \end{bmatrix}$$

is considered to be

$$\begin{bmatrix} -1+i \\ 1-i \\ -1+i \\ 1-i \end{bmatrix}$$

which is almost same as that of transmitted symbol vector considered. Accordingly the signal detection in MIMO has been successfully carried out. It is also observed that we can have $H^{-1}y \approx x$ if $H^{-1}n$ becomes negligible in terms of the detection performance.

2. LINEAR DETECTION

In linear receivers, the received signal treats all the transmitted signals as interference except for the desired stream from the target transmit antenna and the signal is filtered by linear filter to detect each symbol separately .The matrix equalizer is then computed by using following techniques

2.1 ZF Detection

As given in [5,6] a simple linear receiver is the zero-forcing (ZF) receiver which basically inverts the channel transfer matrix, i.e., assuming that H is invertible an estimate of the M_T x1 transmitted data symbol vector s is obtained as

$$\hat{x} = H^{-1}y \tag{9}$$

It nullifies the Interference by following weight matrix

$$W_{ZF} = (H^H H)^{-1} y \tag{10}$$

where $(.)^H$ denotes the Hermitian transpose operation.

In other words, it inverts the effect of channel as

$$\begin{aligned} \hat{x}_{ZF} &= W_{ZF}\, y \\ &= x + (H^H H)^{-1} H^H n \\ &= x + \bar{n}_{ZF} \end{aligned} \tag{11}$$

where $\bar{n}_{ZF} = W_{ZFn} = (H^H H)^{-1} H^H n$

The ZF technique nullifies the interference by the weight matrix

$$W_{ZF} = (H^H H)^{-1} H \tag{12}$$

The ZF receiver hence perfectly separates the co-channel signals si (i= 0, 1,…, MT -1).For ill conditioned H, the ZF receiver performs well in high SNR regime, whereas in the low SNR regime there will be significant noise enhancement. The zero forcing criteria used in the receiver have the disadvantage that the inverse filter may excessively amplify noise at frequencies where folded channel spectrum has high attenuation. The ZF equalizer thus suffers from noise enhancement since it focuses on canceling the effects of the channel response at the expense of enhancing the noise, and is not often used for wireless link, and it also has poor bit error rate performance.

2.2 MMSE Detection

MMSE (Minimum mean squared error) detection minimizes the overall error due to noise and mutual interference .The weight matrix for MMSE is given as

$$W_{MMSE} = (H^H H + \sigma_n^2 I)^{-1} H^H \qquad (13)$$

where σ_n^2 is the statistical information of noise and I mutual information between the transmitter and receiver. The MMSE receiver is less sensitive to noise at the cost of reduced signal separation quality.

2.3 Performance Analysis and Associated Complexity of ZF and MMSE Algorithms

In [7]-[9], the asymptotic performance of linear receiver in MIMO fading channels had been discuss by considering two cases. First one is for fixed no. of antennas, the limit of error probability in the high–signal to noise ratio (SNR regime) in terms of the diversity–multiplexing tradeoff (DMT), second is the error probability for fixed SNR in the regime of large (but finite) number of antennas.

In comparison with the above two cases, as per as DMT is concerned, the ZF and MMSE receiver achieve the same (DMT), which is largely suboptimal even in the case where outer coding and decoding is performed across the antennas whereas behavior of the ZF and MMSE receivers at finite rate and non asymptotic SNR, is different. The ZF receiver achieves poor diversity at any finite rate, the MMSE receiver error curve slope flattens out progressively, as the coding rate increases. If the second case is considered i.e. when SNR is fixed and the numbers of antennas become large, the mutual information at the output of ZF and MMSE linear receiver has fluctuations that converge in distribution to a Gaussian random variable whose mean and variance can be characterized in closed form.

2.4 OSIC Signal Detection

Ordered successive Interference cancellation (OSIC) is a bank of linear detection techniques which is used to detect one of the parallel data stream ,with the detected signal components successively canceled from the received signal at each stage.

Figure 2 illustrates the OSIC signal detection for four spatial

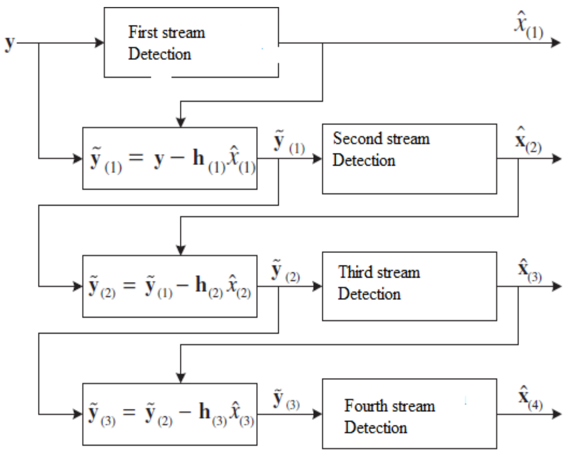

Fig 2: Concept of OSIC signal detection

streams where $\hat{x}_{(1)}, \hat{x}_{(2)}, \hat{x}_{(3)}$ denotes the symbol to be detected which may be different from the transmit vector .The OSIC signal detection can be performed by using either ZF technique or MMSE technique in which QR factorization of channel matrix H is performed[10].In this paper we are using ZF SIC Method to detect the signal for given channel matrix.

2.4.1 ZF SIC

Based on the QR factorization of the channel matrix H the SIC method is introduced in [11] based on this the channel matrix H given in equation (3) is QR factorized as

$$[Q, R] = qr(H) \qquad (14)$$

returns two matrices

$$H = QR$$

$$= Q\begin{bmatrix} r_{11} & r_{12} & r_{13} & r_{14} \\ 0 & r_{22} & r_{23} & r_{24} \\ 0 & 0 & r_{33} & r_{34} \\ 0 & 0 & 0 & r_{44} \end{bmatrix} \qquad (15)$$

where Q is a orthogonal matrix where the inner product of any two columns of Q is zero and the inner product of a column with itself is one and R is an upper triangular matrix in which any element under the main diagonal is zero. By multiplying Q^H equation(3) of received symbol vector can written as

$$s = Q^H y \qquad (16)$$

$$s = Rx + Q^H n \qquad (17)$$

Since $Q^H n$ and n have same statistical properties, $Q^H n$ can is denoted by n and equation (15) can be written as

$$s = Rx + n \qquad (18)$$

which estimated the transmitted symbol vector as

$$s_1 = r_{11}x_1 + r_{12}x_2 + r_{13}x_3 + r_{14}x_4 + n_1$$
$$s_2 = r_{21}x_2 + r_{23}x_3 + r_{24}x_4 + n_2$$
$$s_3 = r_{33}x_3 + r_{34}x_4 + n_3$$
$$s_4 = r_{44}x_4 + n_4 \qquad (19)$$

The values of s1 to s4 can be calculated as follows by considering the same value of H in equation (6).After performing QR decomposition on H we get Q & R as follows

$$Q = \begin{bmatrix} -0.10-0.06i & 0.60-0.09i & -0.36-0.04i & 0.56+0.38i \\ -0.35+0.25i & -0.60+0.40i & -0.24+0.05i & 0.44+0.11i \\ -0.69-0.14i & 0.21+0.09i & -0.41+0.05i & 0.41-0.30i \\ -0.53+0.01i & 0.17+0.03i & 0.77-0.15i & 0.22-0.02i \end{bmatrix}$$

and

$$R=\begin{bmatrix} -5.14+0.00i & -3.72+0.30i & -0.75-1.04i & -2.74+0.00i \\ 0 & 4.31+0.00i & -1.40-0.89i & 0.21-2.10i \\ 0 & 0 & 3.64+0.00i & -2.29+0.55i \\ 0 & 0 & 0 & 2.97+0.00i \end{bmatrix}$$

and symbol vector s is estimated as

$$s=\begin{bmatrix} 1.22+1.09i \\ 4.64-6.43i \\ -6.43+7.46i \\ 2.64-2.73i \end{bmatrix}$$

From this vector s we can observe that the most likely signal vector for

$$x=\begin{bmatrix} -1.41+0.71i \\ 0.90-0.33i \\ -1.23+0.90i \\ 1.22-0.44i \end{bmatrix}$$

is considered to be

$$\begin{bmatrix} -1+i \\ 1-i \\ -1+i \\ 1-i \end{bmatrix}$$

which is same as that of transmitted one. Thus, the detection has performed successfully.

2.4.2 MMSE-SIC

In MMSE-SIC to improve the performance the background noise can be taken into account for linear filtering and two schemes are used to perform the detection. One is by extended channel matrix and another is by adopting the MMSE estimator.

2.4.3 Ordering

In the SIC detection, the ordering of symbol detection plays a key role in mitigating the error propagation, where the overall performance would be decided by the magnitude of the diagonal terms in **R**. The following four techniques are reported in the literature for ordering of symbol detection[12].

1. SINR - Based Ordering
2. SNR - Based Ordering
3. Column - Norm Based Ordering
4. Received Signal Based Ordering

3. NONLINEAR DETECTION

In contrast to linear data detection, where all layers are detected jointly, a tree search approach is used in the non linear receivers. Following are the different algorithms which are use for Non linear data detection.

1. VBLAST (Vertical Blast laboratories layered space-time)
2. Maximum Like hood detection (ML Detection)

3.1 VBLAST (Vertical Blast laboratories layered space-time)

An attractive alternative to ZF and MMSE receivers which in general yields improvement performance at the cost of increased computational complexity is called V-BLAST algorithm[13,14] In V-BLAST rather than jointly decoding all the transmit signals, first decode the strongest signal ,then subtract this strongest signal from the received signal, proceed to decode the strongest signal of the remaining transmit signals, and so on The optimum detection order in such a nulling and cancellation strategy is from the strongest to the weakest signal.

The V-BLAST algorithm show slightly better performance, but suffers from error propagation and is still suboptimal.

3.2 ML Detection

Maximum Likelihood detection yields the best performance in terms of error rate. However, this detection algorithm also has the highest computational complexity which moreover exhibits exponential growth in the number of transmit antennas[15].Maximum likelihood (ML) detection calculates the Euclidean distance between the received signal vector and the product of all possible transmitted signal vectors with the given channel H and finds the one with the minimum distance. This is carried out by exhaustively searching for all the candidate vector and selecting the maximum likely one with the smallest error probability.

$$\hat{s}_{ML}=\arg_s \min\left\| y-Hx \right\|^2 \qquad (20)$$

As we are using 4 transmit and receive antenna and 16 QAM for bit signaling, the possible candidate vectors are 4^{16}=65536. Among this we are using sixteen combinations to perform the signal detection using ML .These sixteen combinations for transmit symbol vector are represented in Table 1. By using the values of H and y in equation (6) and (7)respectively and substituting the candidate vectors from Table 1 in equation(20).

$$d_{ml}=\left\| y-Hx \right\|^2 \qquad (21)$$

Table 1.List of sixteen possible combinations

SN	**x (transmit symbol vector combinations)**	**dml Corresponding to x**
1	-1-i,-1-i,-1-i,-1-i	559.37
2	-1-i,-1-i,-1-i,-1+i	518.84
3	-1-i,-1-i,-1-i,1-i	349.17
4	-1-i,-1-i,-1-i,1+i	308.64
5	-1+i,1-i,-1+i,1-i	3.61
6	-1+i,-1-i,-1-i,-1+i	490.54
7	-1+i,-1-i,-1-i,1-i	207.74
8	-1+i,-1-i,-1-i,1+i	280.35

9	1-i,-1-i,-1-i,-1-i	450.97
10	1-i,-1-i,-1-i,-1+i	410.44
11	1-i,-1-i,-1-i,1-i	353.92
12	1-i,-1-i,-1-i,1+i	313.39
13	1+i,-1-i,-1-i,-1-i	309.53
14	1+i,-1-i,-1-i,-1+i	382.15
15	1+i,-1-i,-1-i,1-i	212.48
16	1+i,-1-i,-1-i,1+i	285.10

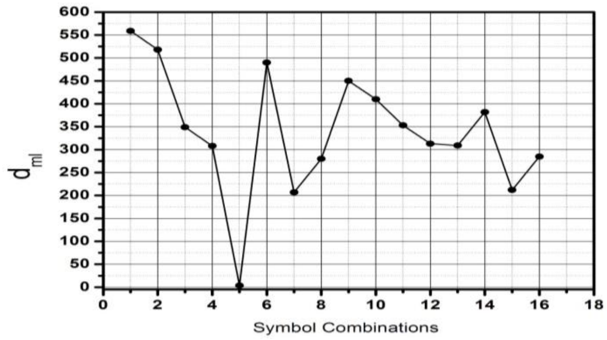

Fig 3: Symbol Detection using ML

In table 1 and corresponding figure 3, since the fifth d_{ml} = 3.61 becomes the smallest one the corresponding S_{ml} is the transmitted symbol vector

$x = \begin{bmatrix} -1+i \\ 1-i \\ -1+i \\ 1-i \end{bmatrix}$ is chosen as the ML solution. Therefore the symbol vector has been detected correctly with the ML detection.

4. CONCLUSION

MIMO detection is to estimate the unknown transmitted signal vector, x for given received signal vector, y, and the channel gain H.To realize this we exploit several existing approaches reported in the literature. In this paper, we presented numerical experimentation for successful detection of 4 x 4 MIMO system using Linear and Nonlinear techniques. During numerical analysis it has been found that at the receiver, detection is performed that computes and delivers the estimates of the transmitted data bits. The detector operates on the the received signal in each separate transmission symbol interval and produces a number or a set of numbers that represent an estimate of a transmitted Quadrature Amplitude Modulation. It is also observed that in linear receivers, the received signal vector is linearly transformed by a matrix equalizer and all the layers are detected jointly whereas in the non linear receiver different tree search approaches are used. Among the two linear algorithms i.e .ZF and MMSE, it has been found that MMSE is having better performance than the ZF detection. In the non linear receiver algorithms the V-BLAST gives slightly better performance than the linear MMSE algorithm, but suffers from error propagation. The Maximum likelihood detection gives best performance in terms of error rate, but the computational complexity of this approach is usually too high for complex constellation and for the large no of antennas, making it impossible to implement for large array sizes and high order digital modulation schemes.

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